Assignment #1: due Thursday, Oct. 16

Theoretical Astrophysics

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1. **Number of stars in the Milky Way** 5 pt

   Does the Milky Way Galaxy contain more stars than there are grains of sand in the beach volleyball court at the Neckarwiese? Please justify your answer using simple order-of-magnitude estimates.

2. **Timescale estimates for the Sun** 20 pt

   (a) *Dynamical timescale:* The collapse timescale for a self-gravitating object is given by $t_{\rm dyn} \approx 1/\sqrt{G\,\rho}$. Calculate it for the Sun assuming a mean density of $\rho = 1.4\,{\rm g\,cm^{-3}}$

   (b) *Sound crossing time:* The sound speed in the solar core is roughly $350\,{\rm km\,sec^{-1}}$. How long does a sound wave need to cross the Sun assuming a constant sound speed throughout the Sun (the solar radius is $6.96 \times 10^5\,{\rm km}$).

   (c) *Nuclear timescale:* The Sun's energy is produced by the process of fusion of hydrogen into helium. If 10% of the solar mass is consumed in this process during the Sun's lifetime, how long does the Sun's energy production persist if the Sun's energy loss (i.e. luminosity: $L_\odot = 3.846 \times 10^{33}\,{\rm erg\,s^{-1}}$) is constant during that time. Use the formula $\tau = E/\dot{E}$ to estimate the nuclear time scale. Note that 0.7% of the hydrogen rest mass is turned into energy in the fusion process.

   (d) *Kelvin-Helmholtz timescale:* The Kelvin-Helmholtz timescale is the ratio of the gravitational energy of an object to its luminosity. Calculate the Kelvin-Helmholtz timescale for the Sun. Assume that the Sun has a constant density.

3. **Angular momentum** 10 pt

   (a) Estimate the total angular momentum of a 0.1 pc ($1\,{\rm pc} = 3.085 \times 10^{18}\,{\rm cm}$) size molecular cloud core with a mean density of $\rho = 1.67 \times 10^{-20}\,{\rm g\,cm^{-3}}$ and a constant angular velocity of $\Omega = 10^{-14}\,{\rm rad\,s^{-1}}$.

   (b) Assuming the above cloud collapses to a single solar type star, what would be the rotational velocity at its surface (solar mass $M_\odot = 1.989 \times 10^{33}\,{\rm g}$)? Could gravity hold this object together? Discuss this result.

   (c) Calculate the total angular momentum of the Sun assuming a mean rotational period of 30 days.

4. **Molecular excitation** 15 pt

Giant molecular clouds (GMCs) are composed almost entirely of molecular hydrogen (H_2), but also contain small quantities of tracer molecules. The most important of these tracers is carbon monoxide (CO). Assume that the molecular hydrogen in a GMC has a Maxwell-Boltzmann velocity distribution. Compute the temperature at which an H_2 molecule with a kinetic energy equal to the mean kinetic energy of the distribution can excite a CO molecule from its ground state to the:

(a) $J = 1$ excited rotational level ($\Delta E = 4.76 \times 10^{-3}\,\mathrm{eV}$)

(b) $v = 1$ excited vibrational level ($\Delta E = 0.266\,\mathrm{eV}$)

(c) B $^1\Sigma^+$ excited electronic level ($\Delta E = 10.5\,\mathrm{eV}$)

[Note: $1\,\mathrm{eV} \simeq 1.6 \times 10^{-12}\,\mathrm{erg}$]. Ignore the effects of any internal excitation of the H_2 molecules, and the contribution of the CO to the mean molecular weight of the gas.

In a typical GMC, the temperature of the gas is in the range 10–20 K. Which of these levels will be excited?