Assignment #2: due Thursday, Oct. 23

Theoretical Astrophysics

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1. **Boltzmann equation with external potential** 30 pt

Consider a gas at constant temperature T in an external gravitational potential $\Phi(\vec{x})$. Assume that the distribution function can be separated in the form $f = g(\vec{x})\, f_0(\vec{w})$, where

$$f_0(\vec{w}) = \left(\frac{m}{2\pi\, k\, T}\right)^{3/2} \exp\left(-\frac{\mathrm{m}\, \vec{w}^2}{2k\, T}\right) \tag{1}$$

is the Maxwell distribution function. Determine $g(\vec{x})$ from the Boltzmann transport equation.

2. **Relaxation to equilibrium** 20 pt

Consider an ideal gas with a distribution function $f = f_0 + g$, where f_0 is the Maxwell distribution function and g is a small perturbation.

(a) Give an expression for the collision term $\dot{f}_c$ in terms of f_0 and g. [Hint: use the kinetic theory of elastic encounters].

(b) Show that $\dot{f}_c$ can be written approximately as:

$$\dot{f}_c = -g n \sigma_{\mathrm{tot}} \bar{u}_{\mathrm{rel}}, \tag{2}$$

where n is the number density of particles, σ_{tot} is the total collision cross-section and $\bar{u}_{\mathrm{rel}}$ is the mean relative velocity between the particles.

(c) Using Eq. **??**, show that the Boltzmann equation can be written (approximately) as:

$$\frac{\partial f}{\partial t} + \vec{w} \cdot \vec{\nabla}_x f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_w f = -\frac{f - f_0}{\tau}. \tag{3}$$

where m is the particle mass and $\tau = 1/(n \sigma_{\mathrm{tot}} \bar{u}_{\mathrm{rel}})$. Discuss the physical interpretation of τ.

3. **Viscosity** 20 pt

(a) Consider a gas flowing with a mean velocity u_x in the x direction. What is its equilibrium velocity distribution? [Note: assume that there are no external forces acting.]

(b) Now suppose that there is a mean velocity gradient in the z direction such that $\partial u_\mathrm{x}/\partial z \neq 0$. Solve for the velocity distribution function f, assuming that this velocity gradient is a small perturbation.

(c) Show that the zx component of the stress tensor, S_{zx}, can be written in this case as

$$S_{zx} = -\eta \frac{\partial u_x}{\partial z}, \tag{4}$$

and give an expression for η.