

Assignment #7: due Thursday, Nov. 27

Theoretical Astrophysics

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1. Parker instability

30 pt

Consider an isothermal gas in the galactic disk which is threaded with a horizontal magnetic field. Assume a constant gravitational field perpendicular to the disk plane in the z direction, i.e. $\vec{g} = -\hat{z}g$ and a magnetic field parallel to the disk plane x which varies only with z , i.e. $B = \hat{x}B(z)$. For simplicity study the system in two dimensions using cartesian coordinates.

- (a) Assume that the system is in hydrostatic equilibrium with a constant ratio of the magnetic to thermal pressure, i.e.

$$\alpha \equiv \frac{B^2}{8\pi P} = \text{const.} \quad (1)$$

What is the pressure distribution as a function of z ? Use the relation $P = c_s^2 \rho$ where c_s is the constant speed of sound and the scale height $H = (1 + \alpha) c_s^2/g$ to express the result.

Now consider this system slightly perturbed out of its equilibrium. Then, from the linear perturbation analysis one gets the following dispersion relation in the xz -plane,

$$n^4 + c_s^2 \left[(1 + 2\alpha) \left(k^2 + \frac{k_0^2}{4} \right) \right] n^2 + k_x^2 c_s^4 \left[2\alpha k^2 + k_0^2 \left[\left(1 + \frac{3\alpha}{2} \right) - (1 + \alpha)^2 \right] \right] = 0, \quad (2)$$

(you might want to derive this relation in your spare time), where $n = i\omega$, $k_0 = H^{-1}$, and the Fourier modes in the x and z direction for the perturbed quantities are

$$\exp(i\omega t - ik_x x) \quad , \quad \exp(i\omega t - ik_z z) \quad (3)$$

with $k^2 = k_x^2 + k_z^2$.

- (b) Show that in the absence of a magnetic field all roots (in terms of n^2) of this dispersion relation are negative, i.e. $n^2 < 0$. What is the physical implication of this result regarding the instability?

- (c) In the case of a non-vanishing magnetic field derive the instability criterion for the Parker instability (magnetic Rayleigh-Taylor instability)

$$\left(\frac{k}{k_0/2}\right)^2 < 2\alpha + 1. \quad (4)$$

Hint: Use the roots of n^2 to find at least one unstable mode, i.e. $n^2 > 0$.

- (d) Show that the instability criterion is equivalent to

$$\lambda_x > \Lambda_x \equiv 4\pi H \left[\frac{1}{2\alpha + 1} \right]^{1/2} \quad (5)$$

$$\lambda_z > \Lambda_z \equiv \frac{\Lambda_x}{\left(1 - (\Lambda_x/\lambda_x)^2\right)^{1/2}}, \quad (6)$$

with the wavelengths $\lambda_x = 2\pi/k_x$, and $\lambda_z = 2\pi/k_z$.