Assignment #11: due Thursday, Jan. 8

Theoretical Astrophysics

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1. **H I 21cm line emission** 30 pt

   The ground state of atomic hydrogen is split into two hyperfine levels, 0 and 1, with statistical weights $g_0 = 1$ and $g_1 = 3$. Radiative transitions from upper level 1 to lower level 0 produce emission at a frequency $\nu_{10} = 1420.40575$ MHz – the famous 21 cm hydrogen line. The spontaneous transition probability for this line is $A_{10} = 2.9 \times 10^{-15}$ s^{-1}.

   If we can ignore the effects of indirect radiative pumping, then the number densities of atoms in levels 0 and 1, n_0 and n_1 are related by

   $$(C_{01} n_{\rm H} + B_{01} I_{10}) n_0 = (C_{10} n_{\rm H} + B_{10} I_{10} + A_{10}) n_1, \qquad (1)$$

   where I_{10} is the specific intensity at ν_{10} and C_{01} and C_{10} are the rate coefficients for the collisional excitation and de-excitation of level 1, which are given approximately by

   $$C_{10} = 2.7 \times 10^{-13} T^{1.4} \qquad (2)$$
   $$C_{01} = 3\, C_{10} \exp\left(\frac{-E_{10}}{kT}\right), \qquad (3)$$

   for kinetic temperatures in the range $20 < T < 60\, K$, where $E_{10} = h\nu_{10}$.

   (a) An interstellar cloud of cold atomic hydrogen with kinetic temperature T and number density $n_{\rm H}$ is illuminated by an external radiation field with brightness temperature $T_{\rm b}$ at frequency ν_{10}. Calculate the excitation temperature $T_{\rm ex}$ of the cloud if

   (i) $C_{10} n_{\rm H} \ll A_{10}$;

   (ii) $C_{10} n_{\rm H} \gg A_{10}$.

   Assume that the opacity τ_{10} of the cloud is negligible.

   (b) Calculate the brightness temperature of the cloud in terms of $T_{\rm b}$ and $T_{\rm ex}$ for the case where the opacity τ_{10} is not negligible.

   (c) Consider a sheet-like cloud of thickness 20 pc, temperature $T = 60$ K and number density $n_{\rm H} = 10\,{\rm cm}^{-3}$. Compute the brightness temperature of this cloud when:

   (i) $T_{\rm b} = 100\,{\rm K}$

   (ii) $T_{\rm b} = 10\,{\rm K}$

2. **Radiation from an orbiting charge** 30 pt

A particle of mass m and charge e moves at constant, non-relativistic speed v in a circle of radius R.

(a) What is the power emitted per unit solid angle in a direction at an angle θ to the axis of the circle?

(b) Suppose that the particle is moving non-relativistically in a constant magnetic field B. Show that the frequency of circular motion is $\omega_B = eB/mc$, and that the total emitted power is

$$P = \frac{2}{3} r_0^2 c \left(\frac{v_\perp}{c} \right)^2 B^2, \tag{4}$$

where $r_0 = e^2/mc^2$ and $v_\perp$ is the velocity perpendicular to the field. Show also that the radiated power is emitted solely at the frequency ω_B.

(c) Find the differential and total cross sections for Thomson scattering of circularly polarized radiation. Use these results to find the cross sections for unpolarized radiation.