Assignment #1: due Tuesday, Oct. 27, 2009

Theoretical Astrophysics

Winter 2009/2010

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1. **Number of stars in the Milky Way** 5 pt

   Does the Milky Way Galaxy contain more stars than there are grains of sand in the beach volleyball court at the Neckarwiese? Please justify your answer using simple order-of-magnitude estimates.

2. **Timescale estimates for the Sun** 20 pt

   (a) *Dynamical timescale:* The collapse timescale for a self-gravitating object is given by $t_{\rm dyn} \approx 1/\sqrt{G\,\rho}$. Calculate it for the Sun assuming a mean density of $\rho = 1.4\,{\rm g\,cm^{-3}}$

   (b) *Sound crossing time:* The sound speed in the solar core is roughly $350\,{\rm km\,sec^{-1}}$. How long does a sound wave need to cross the Sun assuming a constant sound speed throughout the Sun (the solar radius is $6.96 \times 10^5\,{\rm km}$).

   (c) *Nuclear timescale:* The Sun's energy is produced by the process of fusion of hydrogen into helium. If 10% of the solar mass is consumed in this process during the Sun's lifetime, how long does the Sun's energy production persist if the Sun's energy loss (i.e. luminosity: $L_\odot = 3.846 \times 10^{33}\,{\rm erg\,s^{-1}}$) is constant during that time. Use the formula $\tau = E/\dot{E}$ to estimate the nuclear time scale. Note that 0.7% of the hydrogen rest mass is turned into energy in the fusion process.

   (d) *Kelvin-Helmholtz timescale:* The Kelvin-Helmholtz timescale is the ratio of the gravitational energy of an object to its luminosity. Calculate the Kelvin-Helmholtz timescale for the Sun. Assume that the Sun has a constant density.

3. **Relaxation to equilibrium** 20 pt

   Consider an ideal gas with a distribution function $f = f_0 + g$, where f_0 is the Maxwell distribution function and g is a small perturbation.

   (a) Give an expression for the collision term $\dot{f}_c$ in terms of f_0 and g. [Hint: use the kinetic theory of elastic encounters].

   (b) Show that $\dot{f}_c$ can be written approximately as:

   $$\dot{f}_c = -gn\sigma_{\rm tot}\bar{u}_{\rm rel}, \tag{1}$$

   where n is the number density of particles, $\sigma_{\rm tot}$ is the total collision cross-section and $\bar{u}_{\rm rel}$ is the mean relative velocity between the particles.

(c) Using Eq. 1, show that the Boltzmann equation can be written (approximately) as:

$$\frac{\partial f}{\partial t} + \vec{w} \cdot \vec{\nabla}_x f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_w f = -\frac{f - f_0}{\tau}. \tag{2}$$

where m is the particle mass and $\tau = 1/(n\sigma_{\rm tot}\bar{u}_{\rm rel})$. Discuss the physical interpretation of τ.

4. **Molecular excitation** 15 pt

Giant molecular clouds (GMCs) are composed almost entirely of molecular hydrogen (H_2), but also contain small quantities of tracer molecules. The most important of these tracers is carbon monoxide (CO). Assume that the molecular hydrogen in a GMC has a Maxwell-Boltzmann velocity distribution. Compute the temperature at which an H_2 molecule with a kinetic energy equal to the mean kinetic energy of the distribution can excite a CO molecule from its ground state to the:

(a) $J = 1$ excited rotational level ($\Delta E = 4.76 \times 10^{-4}$ eV)

(b) $v = 1$ excited vibrational level ($\Delta E = 0.266$ eV)

(c) B $^1\Sigma^+$ excited electronic level ($\Delta E = 10.5$ eV)

[Note: 1 eV $\simeq 1.6 \times 10^{-12}$ erg]. Ignore the effects of any internal excitation of the H_2 molecules, and the contribution of the CO to the mean molecular weight of the gas.

In a typical GMC, the temperature of the gas is in the range 10–20 K. Which of these levels will be excited?