Assignment #2: due Tuesday, Nov. 03, 2009

Theoretical Astrophysics

Winter 2009/2010

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1. Equation of hydrostatic balance — 10 pt

Using the equation of hydrostatic balance, obtain a crude estimate of the central temperature of the Sun. Hint: Approximate the differential operator by the finite difference between the solar surface and center.

2. Boltzmann equation with external potential — 30 pt

Consider a gas at constant temperature T in an external gravitational potential $\Phi(\vec{x})$. Assume that the distribution function can be separated in the form $f = g(\vec{x})\, f_0(\vec{w})$, where

$$f_0(\vec{w}) = \left(\frac{m}{2\pi\, k\, T}\right)^{3/2} \exp\left(-\frac{\mathrm{m}\,\vec{w}^2}{2k\,T}\right) \tag{1}$$

is the Maxwell distribution function. Determine $g(\vec{x})$ from the Boltzmann transport equation.

3. Viscosity — 20 pt

(a) Consider a gas flowing with a mean velocity u_x in the x direction. What is its equilibrium velocity distribution? [Note: assume that there are no external forces acting.]

(b) Now suppose that there is a mean velocity gradient in the z direction such that $\partial u_\mathrm{x}/\partial z \neq 0$. Solve for the velocity distribution function f, assuming that this velocity gradient is a small perturbation.

(c) Show that the zx component of the stress tensor, S_{zx}, can be written in this case as

$$S_{zx} = -\eta \frac{\partial u_x}{\partial z}, \tag{2}$$

and give an expression for η.