Assignment #4: due Tuesday, Nov. 17

Theoretical Astrophysics

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1. **Shell model for supernova remnants** 5 pt

   Consider the expansion of a supernova remnant in its "adiabatic" phase. The supernova explosion creates a rapidly expanding hot bubble surrounded by a strong shock that sweeps up material from the ambient medium into a thin shell. Estimate the thickness ΔR of this shell in terms of the radius R of the hot bubble. To do so, use the fact that the Rankine-Hugoniot jump conditions for a strong adiabatic shock in an ideal gas of adiabatic index $\gamma = 5/3$ give a compression ratio of four. Then assume (1) the density in the entire shell is equal to the density immediately after the shock front, (2) all the mass in the shell is swept up ambient material, and (3) the ambient gas is homogeneous with density ρ_1.

2. **Shock of supernova blast wave** 15 pt

   A supernova explosion drives a strong shock that moves with velocity v_1 into the interstellar medium, assumed to be at rest. Assuming further, that the medium behaves as a monatomic ideal gas so that the density jumps by a factor of four across the shock, use the conservation of mass flux ($\rho\vec{v}$) and momentum flux ($P + \rho\vec{v}^2$) across the shock to find the temperature T_2 of the gas immediately behind the shock front. Express T_2 in terms of the shock velocity v_1, the atomic mass m and Boltzmann's constant k. Recall the ideal gas law $P = (\rho/m)kT$.

3. **Parker wind solution** 40 pt

   Consider a steady, radial flow of an ideal gas in the gravitational field of a star. Assume a polytropic equation of state,

   $$P = K\,\rho^{\gamma}\,, \tag{1}$$

   where K is constant along the streamlines and $\gamma < 5/3$.

   (a) Show that the continuity equation can be written as

   $$4\pi\, r^2\,\rho\, v = \dot{M}\,, \tag{2}$$

   where v is the radial velocity and $\dot{M}$ is the constant rate of change of mass. Derive the relevant Euler equation for this spherically symmetric system.

(b) Show that a smooth solution containing both sub- and supersonic regions of the flow exists only if

$$v^2 = c_s^2 \quad \text{at} \quad r = \frac{G\,M}{2c_s^2}\,, \tag{3}$$

where $c_s = \sqrt{\gamma P/\rho}$ is the speed of sound (which depends on r).

(c) Imposing this condition and the boundary conditions

$$\rho = \rho_* \quad \text{and} \quad c_s = c_* \tag{4}$$

at the surface $r = r_*$ of the star, find the mass loss rate in the wind in the limit of low surface velocity $v_* \ll c_*$, given that the surface temperature is large compared to the "virial" temperature, i.e. $c_*^2 \gg G\,M/r_*$.

(d) Find the location of the sonic point and discuss the behavior of the solutions as $\gamma \to 5/3$.