

Assignment #5: due Tuesday, Nov. 24

Theoretical Astrophysics

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1. Accretion Disk

total: 60 + 10 pt

Consider a thin (non-selfgravitating), axisymmetric disk in cylindrical coordinates, (R, θ, z) , whose evolution can be described by the continuity equation and the Navier-Stokes equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \vec{g} - \frac{1}{\rho} \nabla P + \vec{\eta} \quad (1)$$

(here given in Cartesian coordinates), where $\vec{\eta}$ is the viscous friction and \vec{g} is the gravitational acceleration due to the central star with mass M . Assume that the velocity does not vary with disk height z and $v_z = 0$.

- (a) Use the surface density $\Sigma = \int \rho dz$ to rewrite the continuity equation in the form

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} (R \Sigma v_R) = 0 \quad (2)$$

and derive the equation which determines the evolution of the angular momentum

$$\frac{\partial}{\partial t} (\Sigma R^2 \Omega) + \frac{1}{R} \frac{\partial}{\partial R} (\Sigma R^3 \Omega v_R) = T \quad (3)$$

where $\Omega = v_\theta/R$ is the angular velocity and the function T involves the viscous terms. (20 + 10 pt)

- (b) Assume that the viscous force per unit area f_{visc} is proportional to the rotational shear $A = R d\Omega/dR$, i.e.

$$f_{\text{visc}} = \nu \rho A \quad (4)$$

where ν is the kinetic viscosity, derive the viscous term as

$$T = \frac{1}{R} \frac{\partial}{\partial R} \left(\nu \Sigma R^3 \frac{d\Omega}{dR} \right). \quad (5)$$

(10 pt)

- (c) Now assume that the disk matter is in Keplerian orbits around the central star, i.e. $\Omega = \sqrt{GM/R^3}$. Show that the disk evolution is then governed by the equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left(R^{1/2} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2}) \right). \quad (6)$$

(10 pt)

- (d) Consider a steady state, thin accretion disk, i.e. $|\nabla P/\rho| \ll GM/r^2$, $\partial \vec{v}/\partial t = 0$. Show that the mass infall rate $\dot{M} = -2\pi R \Sigma v_R$ is given by

$$\dot{M} = 3\pi \nu \Sigma \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right]^{-1} \quad (7)$$

where R_* are the radii with vanishing shear (say the stellar surface or the inner disk radius). (10 pt)

- (e) Given that the energy dissipation per unit area of the disk surface is given by

$$\dot{E} = -\nu \Sigma R^2 \left(\frac{d\Omega}{dR} \right)^2, \quad (8)$$

show that the total luminosity emitted from the accretion disk is then

$$L = \frac{GM\dot{M}}{2R_*}. \quad (9)$$

Compare this to the total potential energy released and discuss possible discrepancies. (10 pt)