Assignment #9: due Tuesday, Dec. 22

Theoretical Astrophysics

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1. **Thermal radiation** 30 pt

   A spherical gas cloud of radius R and temperature T emits thermal radiation at a rate $P(\nu)$ (power per unit volume and frequency range). Its distance from the Earth is d ($d \gg R$).

   (a) First assume the cloud is optically *thin*. What is the brightness of the cloud measured on Earth? Assume the cloud is viewed along a parallel ray which has a distance b from the cloud center.

   (b) What is the effective temperature of the cloud?

   (c) What is the flux F_ν measured at the Earth coming from the entire cloud?

   (d) How does the measured brightness temperature compare with the cloud's temperature? The brightness temperature T_b is defined by the equation

   $$I_\nu = B_\nu(T_b) \tag{1}$$

   where B_ν is the black body spectrum.

   (e) What are the above answers for an optically *thick* cloud?

2. **Eddington limit** 20 pt

   (a) Derive the conditions under which a star with luminosity L_* can disperse its surrounding, optically thin gas with total mass M. The result is $M/L < \kappa/(4\pi\, G\, c)$, where κ is the frequency independent mass absorption coefficient.

   (b) Calculate the terminal velocity of the gas in this case. Assume the gas is accelerated away from the center in the gravitational potential of the star.

   (c) Calculate the Eddington luminosity of the star, i.e. the critical luminosity at which a central source starts to disperse its environment. Use the minimum value of κ which you can estimated from Thomson scattering off free electrons in a fully ionized hydrogen plasma. Express your result as a function of the stellar mass in units of $M_\odot$.

3. **Strömgren radius** 10 pt

The Strömgren sphere is defined as the sphere of fully ionized gas around a (massive) star.

(a) Derive the expression for the radius R_S of the Strömgren sphere in hydrogen gas, given that the number of recombinations per unit volume per second can be written as $\alpha\, n_e\, n_p$, where $\alpha = 3.1 \times 10^{-13}\,\mathrm{cm}^3\ \mathrm{sec}^{-1}$ and n_e and n_p are the electron and proton number density, respectively. Inside R_S the number of recombinations is equal to the number of ionising photons from the star.

(b) Calculate the Strömgren radius for an O5 star ($T_\mathrm{eff} = 54.000\,K$, $L = 2 \times 10^5\,L_\odot$) embedded in a homogeneous cloud of atomic hydrogen with number density $n_\mathrm{H} = 10^4\,\mathrm{cm}^{-3}$. To calculate the number of ionising photons from the star use Wien's law and assume for simplicity that all photons are emitted at the peak frequency of the spectrum.