# Assignment #10: due Tuesday, Jan. 12

# Theoretical Astrophysics

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1. **HI 21cm line emission** 30 pt

The ground state of atomic hydrogen is split into two hyperfine levels, 0 and 1, with statistical weights $g_0 = 1$ and $g_1 = 3$. Radiative transitions from upper level 1 to lower level 0 produce emission at a frequency $\nu_{21\mathrm{cm}} = 1420.40575$ MHz – the famous 21 cm hydrogen line. The spontaneous transition probability for this line is $A_{10} = 2.9 \times 10^{-15}$ s$^{-1}$.

If we can ignore the effects of indirect radiative pumping, then the number densities of atoms in levels 0 and 1, $n_0$ and $n_1$ are related by

$$(C_{01} n_\mathrm{H} + B_{01} I_{21\mathrm{cm}}) n_0 = (C_{10} n_\mathrm{H} + B_{10} I_{21\mathrm{cm}} + A_{10}) n_1, \tag{1}$$

where $I_{21\mathrm{cm}}$ is the specific intensity at $\nu_{21\mathrm{cm}}$ and $C_{01}$ and $C_{10}$ are the rate coefficients for the collisional excitation and de-excitation of level 1, which are given approximately by

$$C_{10} = 2.7 \times 10^{-13} T^{1.4} \tag{2}$$

$$C_{01} = 3\, C_{10} \exp\left(-\frac{\Delta E}{kT}\right), \tag{3}$$

for kinetic temperatures in the range $20 < T < 60\ K$, where $\Delta E = h\nu_{21\mathrm{cm}}$.

(a) An interstellar cloud of cold atomic hydrogen with kinetic temperature $T$ and number density $n_\mathrm{H}$ is illuminated by an external radiation field with brightness temperature $T_\mathrm{b}$ at frequency $\nu_{21\mathrm{cm}}$. Calculate the excitation temperature $T_\mathrm{ex}$ of the cloud if

(i) $C_{10} n_\mathrm{H} \ll A_{10}$;

(ii) $C_{10} n_\mathrm{H} \gg A_{10}$.

Assume that the opacity $\kappa_{21\mathrm{cm}}$ of the cloud is negligible.

(b) Calculate the brightness temperature of the cloud in terms of $T_\mathrm{b}$ and $T_\mathrm{ex}$ for the case where the opacity $\kappa_{21\mathrm{cm}}$ is not negligible.

(c) Consider a sheet-like cloud of thickness 20 pc, temperature $T = 60$ K and number density $n_\mathrm{H} = 10\,\mathrm{cm}^{-3}$. Compute the brightness temperature of this cloud when:

(i) $T_\mathrm{b} = 100\,\mathrm{K}$

(ii) $T_\mathrm{b} = 10\,\mathrm{K}$

2. **Radiation from an orbiting charge** 30 pt

A particle of mass $m$ and charge $e$ moves at constant, non-relativistic speed $v$ in a circle of radius $R$.

(a) What is the power emitted per unit solid angle in a direction at an angle $\theta$ to the rotational axis?

(b) Suppose that the particle is moving non-relativistically in a constant magnetic field $B$. Show that the frequency of circular motion is $\omega_B = eB/mc$, and that the total emitted power is

$$P = \frac{2}{3} r_0^2 c \left( \frac{v_\perp}{c} \right)^2 B^2, \tag{4}$$

where $r_0 = e^2/mc^2$ and $v_\perp$ is the velocity perpendicular to the field. Show also that the radiated power is emitted solely at the frequency $\omega_B$.

(c) Find the differential and total cross sections for Thomson scattering of circularly polarized radiation. Use these results to find the cross sections for unpolarized radiation.