Assignment #11: due Tuesday, Jan. 19

Theoretical Astrophysics

Winter 2009/2010
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1. **Inverse Compton scattering** 10 pt
   Electrons with an energy of 100 GeV are observed at the top of the Earth's atmosphere. Inverse Compton scattering on photons of the cosmic microwave background places a lower limit on the energy loss-rate of these particles. Estimate the maximum time that can have elapsed between their acceleration and their detection. [Hint: Assume that their initial energy was $\gg 100$ GeV.] Approximately how many scattering events occur in this time?

2. **Synchrotron radiation: beaming, retardation** 25 pt
   An electron (charge q and mass m) moving in a uniform magnetic field B with pitch angle α emits synchrotron radiation in a forwardly directed cone of opening angle $\Delta\theta \approx 1/\gamma \ll \alpha$, where γ is the Lorentz factor.

   (a) Find the radius of curvature R of the trajectory, using the definition
   $$\frac{1}{R} = \frac{1}{c\beta}\left|\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\vec{\beta}}{\beta}\right)\right| , \tag{1}$$
   where $c\vec{\beta}$ is the particle velocity.

   (b) Show that the time t_{obs} during which a distant observer is illuminated is at most
   $$t_{\mathrm{obs}} \approx \frac{1}{\omega_{\mathrm{B}}\gamma^3 \sin\alpha} \tag{2}$$
   (see script, Sec. 9.7.2), where $\omega_{\mathrm{B}} = |q|\,B/\gamma mc$ is the gyro frequency of the electron.

   (c) Find the distance travelled by the electron whilst emitting radiation to the observer.

3. **Synchrotron radiation: maximum frequency** 25 pt
   Assume the Lorentz factor of an electron γ ($\gg 1$) is determined by the equation
   $$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \dot{\gamma}_{\mathrm{acc}} + \dot{\gamma}_{\mathrm{synch}} , \tag{3}$$
   where the first term describes an acceleration process operating on the timescale of a gyro period:
   $$\frac{\dot{\gamma}_{\mathrm{acc}}}{\gamma} = \lambda\frac{eB}{\gamma mc} \tag{4}$$

with λ being a constant, and the second term describes the energy lost to synchrotron radiation

$$\frac{\dot{\gamma}_{\rm synch}}{\gamma} = -\frac{2\sigma_{\rm T}}{mc}\frac{B^2}{8\pi}\gamma \sin^2\alpha \ . \tag{5}$$

Show that the Lorentz factor of a particle injected at $\gamma = \gamma_0$ tends to a constant value as $t \to \infty$. Assuming $\lambda \approx 1$ and $\sin\alpha \approx 1$, find the characteristic energy (in MeV) of a synchrotron photon emitted by the electron at large t.