Assignment #12: due Tuesday, January 26, 2010

Theoretical Astrophysics

Winter 2009/2010

Ralf Klessen & Ingo Berentzen, ZAH/ITA, Albert-Ueberle-Str. 2, 69120 Heidelberg

---

Note, this is voluntary homework. You are not required to hand it in.
All points on this sheet are bonus points.

---

1. **Virial Theorem in Stellar Dynamics and Negative Heat Capacity**
25 bonus points

Consider a system of N particles in which the potential between particles i and j, with $i, j \in N$, has the form

$$\Phi(\vec{r}_i, \vec{r}_j) = \frac{C}{|\vec{r}_i - \vec{r}_j|^\alpha} \,, \tag{1}$$

with constant C and exponent α.

(a) Show that the scalar virial theorem has the form

$$2K + \alpha W = 0 \,, \tag{2}$$

where K is the kinetic energy and W is the potential energy.

(b) Which values of α lead to a negative heat capacity?

(c) Speculate about the long-term evolution of systems with negative heat capacity. What is the difference between stars and stellar clusters? What is the role of close binary systems in stellar clusters?

2. **Stability of Circular Orbits** 15 bonus points

Show that circular orbits in a given potential are unstable if the angular momentum per unit mass on a circular orbit decreases outward.

Hint: Use cylindrical coordinates and look for stable trajectories with L_z = constant first.