Assignment #1: due Thursday, Oct. 21, 2010

Theoretical Astrophysics

Winter 2010/2011
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1. **Number of stars in the Milky Way** 5 pt

   Does the Milky Way Galaxy contain more stars than there are grains of sand in the beach volleyball court at the Neckarwiese? Please justify your answer using simple order-of-magnitude estimates.

2. **Timescale estimates for the Sun** 20 pt

   (a) *Dynamical timescale:* The collapse timescale for a self-gravitating object is given by $t_{\rm dyn} \approx 1/\sqrt{G\,\rho}$. Calculate it for the Sun assuming a mean density of $\rho = 1.4\,{\rm g\,cm^{-3}}$

   (b) *Sound crossing time:* The sound speed in the solar core is roughly $350\,{\rm km\,sec^{-1}}$. How long does a sound wave need to cross the Sun assuming a constant sound speed throughout the Sun (the solar radius is $6.96 \times 10^5\,{\rm km}$).

   (c) *Nuclear timescale:* The Sun's energy is produced by the process of fusion of hydrogen into helium. If 10% of the solar mass is consumed in this process during the Sun's lifetime, how long does the Sun's energy production persist if the Sun's energy loss (i.e. luminosity: $L_\odot = 3.846 \times 10^{33}\,{\rm erg\,s^{-1}}$) is constant during that time. Use the formula $\tau = E/\dot{E}$ to estimate the nuclear time scale. Note that 0.7% of the hydrogen rest mass is turned into energy in the fusion process.

   (d) *Kelvin-Helmholtz timescale:* The Kelvin-Helmholtz timescale is the ratio of the gravitational energy of an object to its luminosity. Calculate the Kelvin-Helmholtz timescale for the Sun. Assume that the Sun has a constant density.

3. **Molecular excitation** 15 pt

   Giant molecular clouds (GMCs) are composed almost entirely of molecular hydrogen (H_2), but also contain small quantities of tracer molecules. The most important of these tracers is carbon monoxide (CO). Assume that the molecular hydrogen in a GMC has a Maxwell-Boltzmann velocity distribution. Compute the temperature at which an H_2 molecule with a kinetic energy equal to the mean kinetic energy of the distribution can excite a CO molecule from its ground state to the:

(a) $J = 1$ excited rotational level ($\Delta E = 4.76 \times 10^{-4}$ eV)

(b) $v = 1$ excited vibrational level ($\Delta E = 0.266$ eV)

(c) B $^1\Sigma^+$ excited electronic level ($\Delta E = 10.5$ eV)

[Note: 1 eV $\simeq 1.6 \times 10^{-12}$ erg]. Ignore the effects of any internal excitation of the H_2 molecules, and the contribution of the CO to the mean molecular weight of the gas.

In a typical GMC, the temperature of the gas is in the range 10–20 K. Which of these levels will be excited?

4. Relaxation to equilibrium (bonus points 20 pt)

Consider an ideal gas with a distribution function $f = f_0 + g$, where f_0 is the Maxwell distribution function and g is a small perturbation.

(a) Give an expression for the collision term $\dot{f}_c$ in the Boltzmann equation in terms of f_0 and g. [Hint: Use the kinetic theory of elastic encounters and consider only terms up to first order in the perturbation term.].

(b) A coarse approximation for $\dot{f}_c$ can be obtained by using the collision approximation *(Stoßzahlansatz)* as discussed in the lecture and by looking at the functional form of the individual terms in $\dot{f}_c$. Take one as being representative and show that

$$\dot{f}_c \sim -g n \sigma_{\text{tot}} \bar{u}_{\text{rel}}, \tag{1}$$

where n is the number density of particles, σ_{tot} is the total collision cross-section and $\bar{u}_{\text{rel}}$ is the mean relative velocity between the particles.

(c) Using Eq. 1, show that the Boltzmann equation can be written (approximately) as:

$$\frac{\partial f}{\partial t} + \vec{w} \cdot \vec{\nabla}_x f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_w f = -\frac{f - f_0}{\tau}. \tag{2}$$

where m is the particle mass and $\tau = 1/(n \sigma_{\text{tot}} \bar{u}_{\text{rel}})$. Discuss the physical interpretation of τ.