Assignment #2: due Thursday, Oct. 28, 2010

Theoretical Astrophysics

Winter 2010/2011

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1. **Validity of the hydrodynamic approach** 20 pt

   Giant molecular clouds (GMCs) in the Milky Way have densities of $\sim 100\,\mathrm{H_2}$ molecules per cubic centimeter and sizes of several tens of parsec. The typical temperature is 10 K which translates into a sound speed of about $0.2\,\mathrm{km\,s^{-1}}$. Consider a volume element in the 6-dimensional phase space $d^3x d^3w$, where the spatial part d^3x is the size of the Earth and the velocity part $d^3w \sim 10^{-9}\bar{w}$, where $\bar{w}$ is the average velocity of the gas particles.

   (a) What is the approximate number of gas particles in $d^3x d^3w$?

   (b) What is the average distance H_2 particles travel between two collisions?

   (c) In the star formation group at ITA the dynamical evolution of GMCs is usually studied with hydrodynamic simulations? Discuss whether this theoretical approach is justified or not.

   (d) The interstellar medium is threaded by magnetic fields. Do your arguments change if magnetic fields are taken into account.

   Now consider the air you are currently breathing. The density is about $1.2\times10^{-3}\,\mathrm{g\,cm^{-3}}$.

   (e) What is the number of gas particles in a phase space volume element with $d^3x \sim 10^{-6}\,\mathrm{cm^3}$ and $d^3w \sim 10^{-6}\bar{w}$? And how does $(d^3x)^{1/3}$ compare to the typical mean free path of the molecules in air?

   It is sufficient to provide *very rough* order of magnitude estimates. For (c) and (d) consider the fundamental assumptions of hydrodynamics and discuss their validity.

2. **Boltzmann equation with external potential** 30 pt

   Consider a gas at constant temperature T in an external gravitational potential $\Phi(\vec{x})$. Assume that the distribution function can be separated in the form $f = g(\vec{x})\, f_0(\vec{w})$, where

   $$f_0(\vec{w}) = \left(\frac{m}{2\pi\, k\, T}\right)^{3/2} \exp\left(-\frac{\mathrm{m}\,\vec{w}^2}{2k\,T}\right) \tag{1}$$

   is the Maxwell distribution function. Determine $g(\vec{x})$ from the Boltzmann transport equation.