Assignment #4: due Thursday, Nov. 11, 2010

Theoretical Astrophysics

Winter 2010/2011

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1. **Scalar Virial Theorem** 15 pt

   (a) Globular clusters are gravitationally bound, dense stellar systems with half-light radii of typically $r_{1/2} \sim 5\,\mathrm{pc}$ and (one-dimensional) velocity dispersions $\sigma_{1/2}$ of roughly $6\,\mathrm{km\,s^{-1}}$. The total bolometric luminosity is $\sim 2.4 \times 10^{38}\,\mathrm{erg\,s^{-1}}$. Using the scalar virial theorem calculate the mass-to-light ratio Υ for a globular cluster in units of the solar mass-to-light ratio $\Upsilon_\odot = M_\odot/L_\odot$.

   (b) The dwarf-spheroidal galaxy Draco - a satellite galaxy of our Milky Way - has a distance of about $d \approx 72\,\mathrm{kpc}$ and an estimated half-mass radius $r_{1/2}$ of about $5.7'$ on the sky. Draco has a luminosity of $L = 2.6 \times 10^5\,L_\odot$ and a (one-dimensional) dispersion velocity of $\sigma \approx 13.2\,\mathrm{km\,s^{-1}}$. Calculate the mass-to-light ratio Υ of the Draco galaxy.

   (c) Compare the result to the one found by Irwin & Hatzidimitriou (1995, Monthly Notices of The Royal Astronomical Society, 277, 1354). Speculate about the difference? The high mass-to-light ratio suggests that Draco is dark matter dominated. Can you think of an alternative explanation for the high value of Υ derived above?

2. **Parker wind solution** 40 pt

   Consider a steady, radial flow of an ideal gas in the gravitational field of a star. Assume a polytropic equation of state,

   $$P = K\,\rho^\gamma\,, \tag{1}$$

   where K is constant along the streamlines and $\gamma < 5/3$.

   (a) Show that the continuity equation can be written as

   $$4\pi\,r^2\,\rho\,v = \dot{M}\,, \tag{2}$$

   where v is the radial velocity and $\dot{M}$ is the constant rate of change of mass. Derive the relevant Euler equation for this spherically symmetric system.

   (b) Show that a smooth solution containing both sub- and supersonic regions of the flow exists only if

   $$v^2 = c_s^2 \quad \text{at} \quad r = \frac{G\,M}{2c_s^2}\,, \tag{3}$$

   where $c_s = \sqrt{\gamma P/\rho}$ is the speed of sound (which depends on r!).

(c) Imposing this condition and the boundary conditions

$$\rho = \rho_* \quad \text{and} \quad c_s = c_* \tag{4}$$

at the surface $r = r_*$ of the star, find the mass loss rate in the wind in the limit of low surface velocity $v_* \ll c_*$, given that the surface temperature is large compared to the "virial" temperature, i.e. $c_*^2 \gg G\,M/r_*$.

(d) Find the location of the sonic point and discuss the behavior of the solutions as $\gamma \to 5/3$.