Assignment #10: due Thursday, Jan. 13, 2011

Theoretical Astrophysics

Winter 2010/2011

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1. **Thermal radiation** 30 pt

   A spherical gas cloud of radius R and temperature T emits thermal radiation at a rate $P(\nu)$ (power per unit volume and frequency range). Its distance from the Earth is d ($d \gg R$).

   (a) First assume the cloud is optically *thin*. What is the brightness of the cloud measured on Earth? Assume the cloud is viewed along a parallel ray which has a distance b from the cloud center.

   (b) What is the effective temperature of the cloud?

   (c) What is the flux F_ν measured at the Earth coming from the entire cloud?

   (d) How does the measured brightness temperature compare with the cloud's temperature? The brightness temperature T_b is defined by the equation

   $$I_\nu = B_\nu(T_b) \tag{1}$$

   where B_ν is the black body spectrum.

   (e) What are the above answers for an optically *thick* cloud?

2. **Eddington limit** 20 pt

   (a) Derive the conditions under which a star with luminosity L_* and total mass M can disperse optically thin gas in its surrounding. The result is $M/L < \kappa/(4\pi\, G\, c)$, where κ is the frequency independent mass absorption coefficient.

   (b) Calculate the terminal velocity of the gas in this case. Assume the gas is accelerated away from the center in the gravitational potential of the star.

   (c) Calculate the Eddington luminosity of the star, i.e. the critical luminosity at which a central source starts to disperse its environment. Use the minimum value of κ which you can estimated from Thomson scattering off free electrons in a fully ionized hydrogen plasma. Express your result as a function of the stellar mass in units of $M_\odot$.

3. **H I 21cm line emission** 30 pt

The ground state of atomic hydrogen is split into two hyperfine levels, 0 and 1, with statistical weights $g_0 = 1$ and $g_1 = 3$. Radiative transitions from upper level 1 to lower level 0 produce emission at a frequency $\nu_{21\text{cm}} = 1420.40575$ MHz – the famous 21 cm hydrogen line. The spontaneous transition probability for this line is $A_{10} = 2.9 \times 10^{-15}\ \text{s}^{-1}$.

If we can ignore the effects of indirect radiative pumping, then the number densities of atoms in levels 0 and 1, n_0 and n_1 are related by

$$(C_{01} n_\text{H} + B_{01} I_{21\text{cm}}) n_0 = (C_{10} n_\text{H} + B_{10} I_{21\text{cm}} + A_{10}) n_1, \tag{2}$$

where $I_{21\text{cm}}$ is the specific intensity at $\nu_{21\text{cm}}$ and C_{01} and C_{10} are the rate coefficients for the collisional excitation and de-excitation of level 1, which are given approximately by

$$C_{10} = 2.7 \times 10^{-13} T^{1.4} \tag{3}$$

$$C_{01} = 3\, C_{10} \exp\left(-\frac{\Delta E}{kT}\right), \tag{4}$$

for kinetic temperatures in the range $20 < T < 60\ K$, where $\Delta E = h\nu_{21\text{cm}}$.

(a) An interstellar cloud of cold atomic hydrogen with kinetic temperature T and number density n_H is illuminated by an external radiation field with brightness temperature T_b at frequency $\nu_{21\text{cm}}$. Calculate the excitation temperature T_ex of the cloud if

(i) $C_{10} n_\text{H} \ll A_{10}$;

(ii) $C_{10} n_\text{H} \gg A_{10}$.

Assume that the opacity $\kappa_{21\text{cm}}$ of the cloud is negligible.

(b) Calculate the brightness temperature of the cloud in terms of T_b and T_ex for the case where the opacity $\kappa_{21\text{cm}}$ is not negligible.

(c) Consider a sheet-like cloud of thickness 20 pc, temperature $T = 60$ K and number density $n_\text{H} = 10\ \text{cm}^{-3}$. Compute the brightness temperature of this cloud when:

(i) $T_\text{b} = 100$ K

(ii) $T_\text{b} = 10$ K

Happy Winter Holidays and All the Best for the New Year 2011 !