Assignment #11: due Thursday, Jan. 20, 2011

Theoretical Astrophysics

Winter 2010/2011
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1. **Mean free path of gas particles and photons** 20 pt

   (a) Calculate the mean free path of the nitrogen molecules you are currently breathing in and out. Take the radius of N_2 to be 1 Å and assume a (room) temperature of 20 °C. What is the average time between collisions?

   (b) Calculate how far you could see if the atmosphere here in Heidelberg had the opacity of the solar photosphere. Use the simple value for electron scattering, $\kappa \approx 0.2\,\mathrm{cm}^2\,\mathrm{g}^{-1}$.

   (c) What is the average mean free path of a photon in the Sun if we assume a mean density of $1.4\,\mathrm{g\,cm}^{-3}$ and again take only electron scattering into account?

   (d) How long does it take a photon released at the center of the Sun to escape through the surface? Assume the photons "diffuse" outwards, so that the succession of collisions (actually, absorption and reemission events) can be described as a random walk.

2. **Strömgren radius** 10 pt

   The Strömgren sphere is the region of fully ionized gas surrounding a massive star.

   (a) Derive the expression for the radius R_S of the Strömgren sphere in hydrogen gas. Assume that the surrounding gas is homogeneous and calculate the number of recombinations per unit volume per second as $\alpha\, n_e\, n_p$, where $\alpha = 3.1 \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{sec}^{-1}$ and n_e and n_p are the electron and proton number density, respectively. Inside R_S the number of recombinations is equal to the number of ionizing photons from the star.

   (b) Calculate the Strömgren radius for an O5 star ($T_\mathrm{eff} = 54.000\,K$, $L = 2 \times 10^5\,L_\odot$) embedded in a cloud of atomic hydrogen with number density $n_\mathrm{H} = 10^4\,\mathrm{cm}^{-3}$. To calculate the number of ionizing photons from the star use Wien's law and assume for simplicity that all photons are emitted at the peak frequency of the spectrum.

3. **Radiation from an orbiting charge** 30 pt

A particle of mass m and charge e moves at constant, non-relativistic speed v in a circle of radius R.

(a) What is the power emitted per unit solid angle in a direction at an angle θ to the axis of the circle?

(b) Suppose that the particle is moving non-relativistically in a constant magnetic field B. Show that the frequency of circular motion is $\omega_B = eB/mc$, and that the total emitted power is

$$P = \frac{2}{3} r_0^2 c \left(\frac{v_\perp}{c} \right)^2 B^2, \tag{1}$$

where $r_0 = e^2/mc^2$ and $v_\perp$ is the velocity perpendicular to the field. Show also that the radiated power is emitted solely at the frequency ω_B.

(c) Find the differential and total cross sections for Thomson scattering of circularly polarized radiation. Use these results to find the cross sections for unpolarized radiation.