Einführung in die Astronomie und Astrophysik 2

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Physical Processes in the ISM — Hand in on May 12, 2011

4.1 Angular momentum

1. Estimate the total angular momentum of a molecular cloud core with a radius of 0.1 pc ($\mathrm{pc} = 3.08 \times 10^{18}\,\mathrm{cm}$), a mean density of $\rho = 1.67 \times 10^{-20}\,\mathrm{g\,cm^{-3}}$ and a constant angular velocity of $\Omega = 10^{-14}\,\mathrm{rad\,sec^{-1}}$.
2. Assuming the above cloud collapses to a single solar type star, what would be the rotational velocity at its surface (solar mass $M_\odot = 2 \times 10^{33}\,\mathrm{g}$)?
3. Could gravity hold this object together? Discuss this result.
4. Calculate the total angular momentum of the Sun assuming a mean rotational period of 30 days. (4 points)

4.2 H I 21cm line emission

The ground state of atomic hydrogen is split into two hyperfine levels, 0 and 1, with statistical weights $g_0 = 1$ and $g_1 = 3$. Radiative transitions from upper level 1 to lower level 0 produce emission at a frequency $\nu_{10} = 1420.40575$ MHz – the famous 21 cm hydrogen line. The spontaneous transition probability for this line is $A_{10} = 2.9 \times 10^{-15}\ \mathrm{s^{-1}}$.

If we can ignore the effects of indirect radiative pumping, then the number densities of atoms in levels 0 and 1, n_0 and n_1 are related by

$$(C_{01} n_\mathrm{H} + B_{01} I_{10}) n_0 = (C_{10} n_\mathrm{H} + B_{10} I_{10} + A_{10}) n_1, \tag{1}$$

where I_{10} is the specific intensity at ν_{10} and C_{01} and C_{10} are the rate coefficients for the collisional excitation and de-excitation of level 1, which are given approximately by

$$C_{10} = 2.7 \times 10^{-13} T^{1.4} \tag{2}$$

$$C_{01} = 3\, C_{10} \exp\left(\frac{-E_{10}}{kT}\right), \tag{3}$$

for kinetic temperatures in the range $20 < T < 60\ K$, where $E_{10} = h\nu_{10}$.

1. An interstellar cloud of cold atomic hydrogen with kinetic temperature T and number density n_H is illuminated by an external radiation field with brightness temperature T_b at frequency ν_{10}. Recall that the brightness temperature is defined as $I_{10} = B_{\nu_{10}}(T_\mathrm{b})$, where B_ν is the Planck function. Calculate the excitation temperature T_ex (i.e. the *effective* temperature which would correspond to a thermal population with T_ex) of the cloud if

   (i) $C_{10} n_\mathrm{H} \ll A_{10}$;

   (ii) $C_{10} n_\mathrm{H} \gg A_{10}$.

   Assume that the opacity τ_{10} of the cloud is negligible.

2. Calculate the brightness temperature of the cloud in terms of T_b and T_ex for the case where the opacity τ_{10} is not negligible. (6 points)