Homework Assignment #1 is due Wednesday, Oct. 21, 2015

Theoretical Astrophysics (MKTP2)

Winter Semester 2015/2016

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1. **Number of stars in the Milky Way** *5 pt*

   Does the Milky Way Galaxy contain more stars than there are grains of sand in the beach volleyball court at the Neckarwiese? Please justify your answer using simple order-of-magnitude estimates.

2. **Timescale estimates for the Sun** *20 pt*

   (a) *Dynamical timescale:* The collapse timescale for a self-gravitating object is given by $t_{\rm dyn} \approx (G\rho)^{-1/2}$. Calculate it for the Sun assuming a mean density of $\rho = 1.4\,{\rm g\,cm^{-3}}$

   (b) *Sound crossing time:* The sound speed in the solar core is roughly $350\,{\rm km\,s^{-1}}$. How long does a sound wave need to cross the Sun assuming a constant sound speed throughout the Sun (the solar radius is $6.96 \times 10^5\,{\rm km}$).

   (c) *Nuclear timescale:* The Sun's energy is produced by the process of fusion of hydrogen into helium. If 10% of the solar mass is consumed in this process during the Sun's lifetime, how long does the Sun's energy production persist if the Sun's energy loss (i.e. luminosity: $L_\odot = 3.846 \times 10^{33}\,{\rm erg\,s^{-1}}$) is constant during that time. Use the formula $t_{\rm nuc} \approx E/\dot{E}$ to estimate the nuclear time scale. Note that 0.7% of the hydrogen rest mass is turned into energy in the fusion process.

   (d) *Kelvin-Helmholtz timescale:* The Kelvin-Helmholtz timescale is the ratio of the gravitational energy of an object to its luminosity. Calculate the Kelvin-Helmholtz timescale for the Sun. Assume that the Sun has a constant density.

3. **Angular momentum** *15 pt*

(a) Estimate the total angular momentum of a molecular cloud core with a radius of 0.1 pc (pc $= 3.08 \times 10^{18}$ cm), a mean density of $\rho = 1.67 \times 10^{-20}\,\mathrm{g\,cm^{-3}}$ and a constant angular velocity of $\Omega = 10^{-14}\,\mathrm{rad\,s^{-1}}$.

(b) Assuming the above cloud collapses to a single solar type star, what would be the rotational velocity at its surface (solar mass $M_\odot = 2 \times 10^{33}$ g)?

(c) Could gravity hold this object together? Discuss this result.

(d) Calculate the total angular momentum of the Sun assuming a mean rotational period of 30 days.

4. **Relaxation to equilibrium** *(bonus points 15 pt)*

Consider an ideal gas with a distribution function $f = f_0 + g$, where f_0 is the Maxwell distribution function and g is a small perturbation.

(a) Give an expression for the collision term $\dot{f}_c$ in the Boltzmann equation in terms of f_0 and g. [Hint: Use the kinetic theory of elastic encounters and consider only terms up to first order in the perturbation term.].

(b) A coarse approximation for $\dot{f}_c$ can be obtained by using the collision approximation *(Stoßzahlansatz)* as mentioned in the lecture and by looking at the functional form of the individual terms in $\dot{f}_c$. Take one as being representative and show that

$$\dot{f}_c \sim -gn\sigma_{\mathrm{tot}}\bar{u}_{\mathrm{rel}}, \tag{1}$$

where n is the number density of particles, σ_{tot} is the total collision cross-section and $\bar{u}_{\mathrm{rel}}$ is the mean relative velocity between the particles.

(c) Using Eq. 1, show that the Boltzmann equation can be written (approximately) as:

$$\frac{\partial f}{\partial t} + \vec{w} \cdot \vec{\nabla}_x f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_w f = -\frac{f - f_0}{\tau}. \tag{2}$$

where m is the particle mass and $\tau = 1/(n\sigma_{\mathrm{tot}}\bar{u}_{\mathrm{rel}})$. Discuss the physical interpretation of τ.