Homework Assignment #11 is due Wednesday, January 13, 2016

Theoretical Astrophysics (MKTP2)

Winter Semester 2015/2016

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1. **H I 21cm line emission** *30 points*

The ground state of atomic hydrogen is split into two hyperfine levels, 0 and 1, with statistical weights $g_0 = 1$ and $g_1 = 3$. Radiative transitions from upper level 1 to lower level 0 produce emission at a frequency $\nu_{21\mathrm{cm}} = 1420.40575$ MHz – the famous 21 cm hydrogen line. The spontaneous transition probability for this line is $A_{10} = 2.9 \times 10^{-15}\ \mathrm{s}^{-1}$.

If we can ignore the effects of indirect radiative pumping, then the number densities of atoms in levels 0 and 1, n_0 and n_1 are related by

$$(C_{01} n_\mathrm{H} + B_{01} I_{21\mathrm{cm}}) n_0 = (C_{10} n_\mathrm{H} + B_{10} I_{21\mathrm{cm}} + A_{10}) n_1, \tag{1}$$

where $I_{21\mathrm{cm}}$ is the specific intensity at $\nu_{21\mathrm{cm}}$ and C_{01} and C_{10} are the rate coefficients for the collisional excitation and de-excitation of level 1, which are given approximately by

$$C_{10} = 2.7 \times 10^{-13} T^{1.4} \tag{2}$$

$$C_{01} = 3\, C_{10} \exp\left(-\frac{\Delta E}{kT}\right), \tag{3}$$

for kinetic temperatures in the range $20 < T < 60\ K$, where $\Delta E = h\nu_{21\mathrm{cm}}$.

(a) An interstellar cloud of cold atomic hydrogen with kinetic temperature T and number density n_H is illuminated by an external radiation field with brightness temperature T_b at frequency $\nu_{21\mathrm{cm}}$. Calculate the excitation temperature T_ex of the cloud if

(i) $C_{10} n_\mathrm{H} \ll A_{10}$,

(ii) $C_{10} n_\mathrm{H} \gg A_{10}$.

Assume that the opacity $\kappa_{21\mathrm{cm}}$ of the cloud is negligible.

(b) Calculate the brightness temperature of the cloud in terms of T_b and T_ex for the case where the opacity $\kappa_{21\mathrm{cm}}$ is not negligible.

(c) Consider a sheet-like cloud of thickness 20 pc, temperature $T = 60\,\mathrm{K}$ and number density $n_\mathrm{H} = 10\,\mathrm{cm}^{-3}$. Compute the brightness temperature of this cloud for

(i) $T_\mathrm{b} = 100\,\mathrm{K}$,

(ii) $T_\mathrm{b} = 10\,\mathrm{K}$.

2. **Strömgren radius** *8 points*

The Strömgren sphere is the region of fully ionized gas surrounding a massive star.

(a) Derive the expression for the radius R_S of the Strömgren sphere in hydrogen gas. Assume that the surrounding gas is homogeneous and calculate the number of recombinations per unit volume per second as $\alpha\, n_e\, n_p$, where $\alpha = 3.1 \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{sec}^{-1}$ and n_e and n_p are the electron and proton number density, respectively. Inside R_S the number of recombinations is equal to the number of ionizing photons from the star.

(b) Calculate the Strömgren radius for an O5 star ($T_\mathrm{eff} = 54.000\,\mathrm{K}$, $L = 2 \times 10^5\,L_\odot$) embedded in a cloud of atomic hydrogen with number density $n_\mathrm{H} = 10^4\,\mathrm{cm}^{-3}$. To calculate the number of ionizing photons from the star use Wien's law and assume for simplicity that all photons are emitted at the peak frequency of the spectrum.

3. **Mean free path of gas particles and photons** *17 points*

(a) Calculate the mean free path of the nitrogen molecules you are currently breathing in and out. Take the radius of N_2 to be 1 Å and assume a (room) temperature of 20 °C. What is the average time between collisions?

(b) Calculate how far you could see if the atmosphere here in Heidelberg had the opacity of the solar photosphere. Use the simple value for electron scattering, $\kappa \approx 0.2\,\mathrm{cm}^2\,\mathrm{g}^{-1}$.

(c) What is the average mean free path of a photon in the Sun if we assume a mean density of $1.4\,\mathrm{g\,cm}^{-3}$ and again take only electron scattering into account?

(d) How long does it take a photon released at the center of the Sun to escape through the surface? Assume the photons "diffuse" outwards, so that the succession of collisions (actually, absorption and reemission events) can be described as a random walk.

Happy Winter Holidays and All the Best for the New Year 2016 !