Homework Assignment #12 is due Wednesday, January 20, 2016

Theoretical Astrophysics (MKTP2)

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1. Inverse Compton Process *20 points*

   Consider a cloud of non-relativistic electrons with kinetic temperature T. The cloud is permeated by large numbers of photons with energies $h\nu_{\mathrm{i}} \ll kT$ which are continuously replenished. (A possible example are CMB photons.) The cloud is optically think with respect to electron-photon scattering ($\tau_{\mathrm{es}} \gg 1$) and optically thin with respect to the photon absorption ($\tau_{\mathrm{abs}} \ll 1$). As a result of the inverse Compton process the low-energy photons gain energy and leave the cloud through the surface with energies $h\nu_{\mathrm{f}} \gg h\nu_{\mathrm{i}}$. The energy gain depends on the number of inverse Compton scattering events that occur within the cloud, this in turn depends on the optical depth τ_{es}. The energy transfer saturates when a critical value of τ_{sat} is reached.

   (a) Calculate the energy gain, $\nu_{\mathrm{f}}/\nu_{\mathrm{i}}$, as function of the optical depth of electron-photon scattering τ_{es} and the electron temperature T.

   (b) Find an approximate expression for the optical depth τ_{sat} at which this process saturates.

2. Synchrotron Radiation *10 points*

   Ultrarelativistic electrons moving through a magnetized region emit synchrotron radiation. Consider for simplicity that the magnetic field is homogeneous with a value $\vec{B} =$ constant and that the electron is moving in some arbitrary direction with initial velocity $\vec{\beta}_0$ ($|\vec{\beta}_0| \approx 1$).

   (a) Show that the electron loses energy, i.e. that the Lorentz factor $\gamma(t) = [1 - \beta(t)^2]^{-1/2}$ decreases, as

   $$\gamma(t) = \frac{\gamma_0}{1 + A\gamma_0 t} , \quad \text{with} \quad A = \frac{2e^4 B_\perp^2}{3m^3c^5}$$

   and with γ_0 being the initial Lorentz factor.

   (b) Derive and expression for the time $t_{1/2}$ it takes the electron to lose half of its initial energy.

3. Doppler Shift of the Cosmic Microwave Background *10 points*

Earth's motion introduces a noticeable dipole distortion in the observed cosmic microwave background.

(a) Show that an observer moving with velocity v with respect to a blackbody radiation field of temperature T will see blackbody radiation with temperature T' that depends on the angle θ' with respect to the direction of motion according to

$$T' = \frac{(1-\beta^2)^{1/2}}{1+\beta\cos\theta'}\, T\ , \tag{1}$$

with $\beta = v/c$.

(b) The temperature shift in the observed microwave background due to the motion of the Earth is $\Delta T = \pm 3.3\,\mathrm{mK}$. Compared to the average temperature of $T = 2.73\,\mathrm{K}$ this deviation from isotropy is only of order of 10^{-3}. Calculate the corresponding velocity.