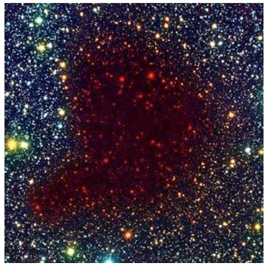
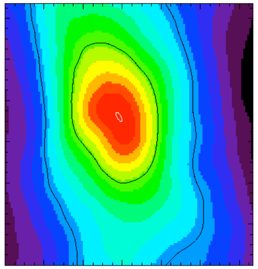
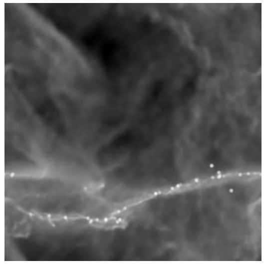
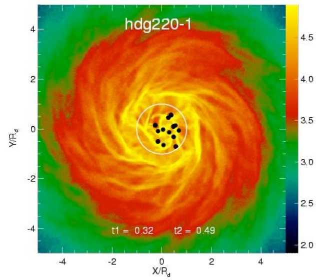
ISM dynamics: theoretical considerations









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Institut für Theoretische Astrophysik

Zentrum für Astronomie Heidelberg
ARI+ITA+LSW

ISM dynamics:
theoretical considerations

- phenomenology
- derivation of the hydrodynamic equations
- virial theorem
- Jeans criterion: critical mass for gravitational collapse
- Bonnor-Ebert spheres: pressure-bounded gas sphere in hydrostatic equilibrium

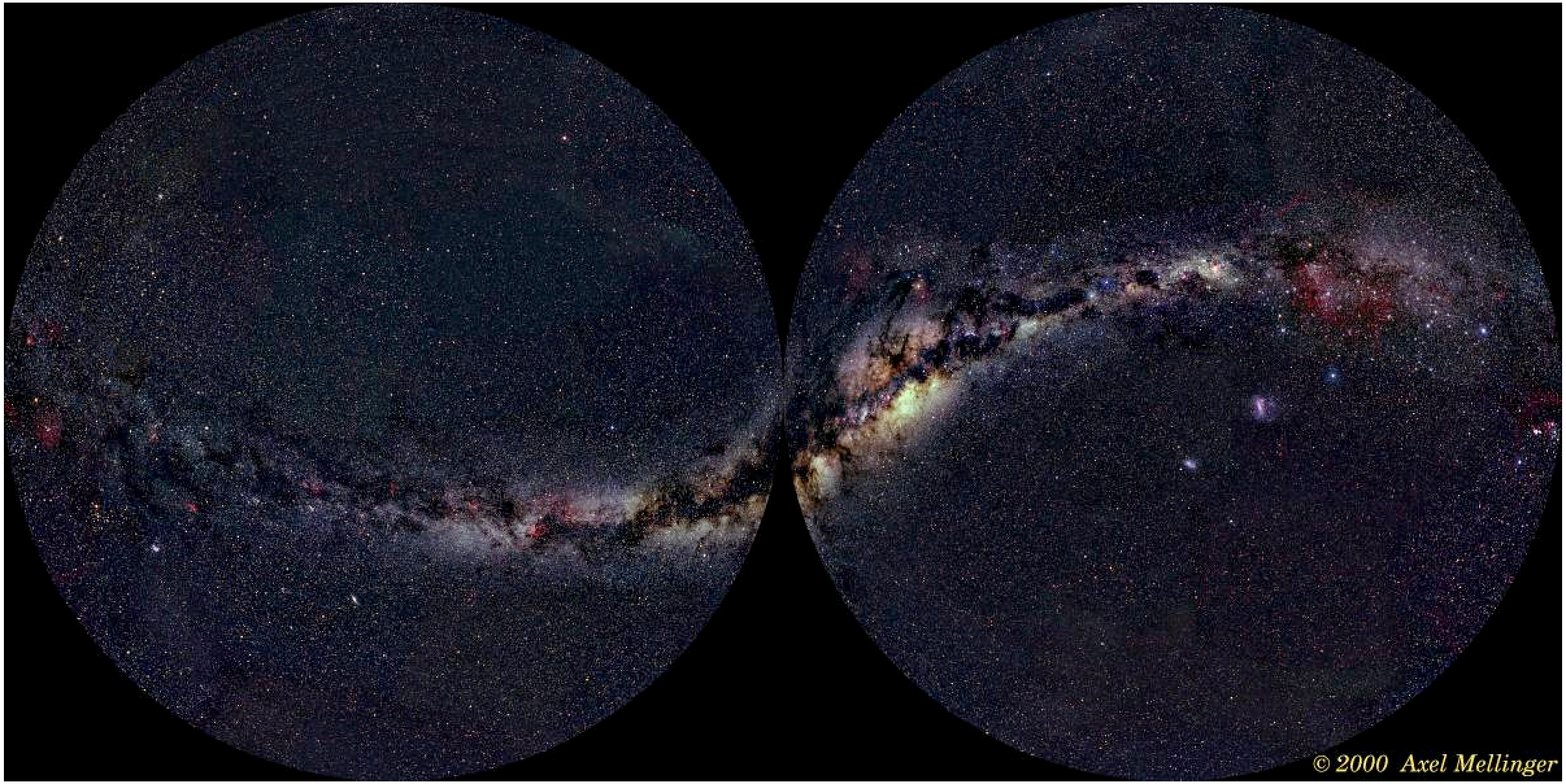
Star formation in "typical" spiral:

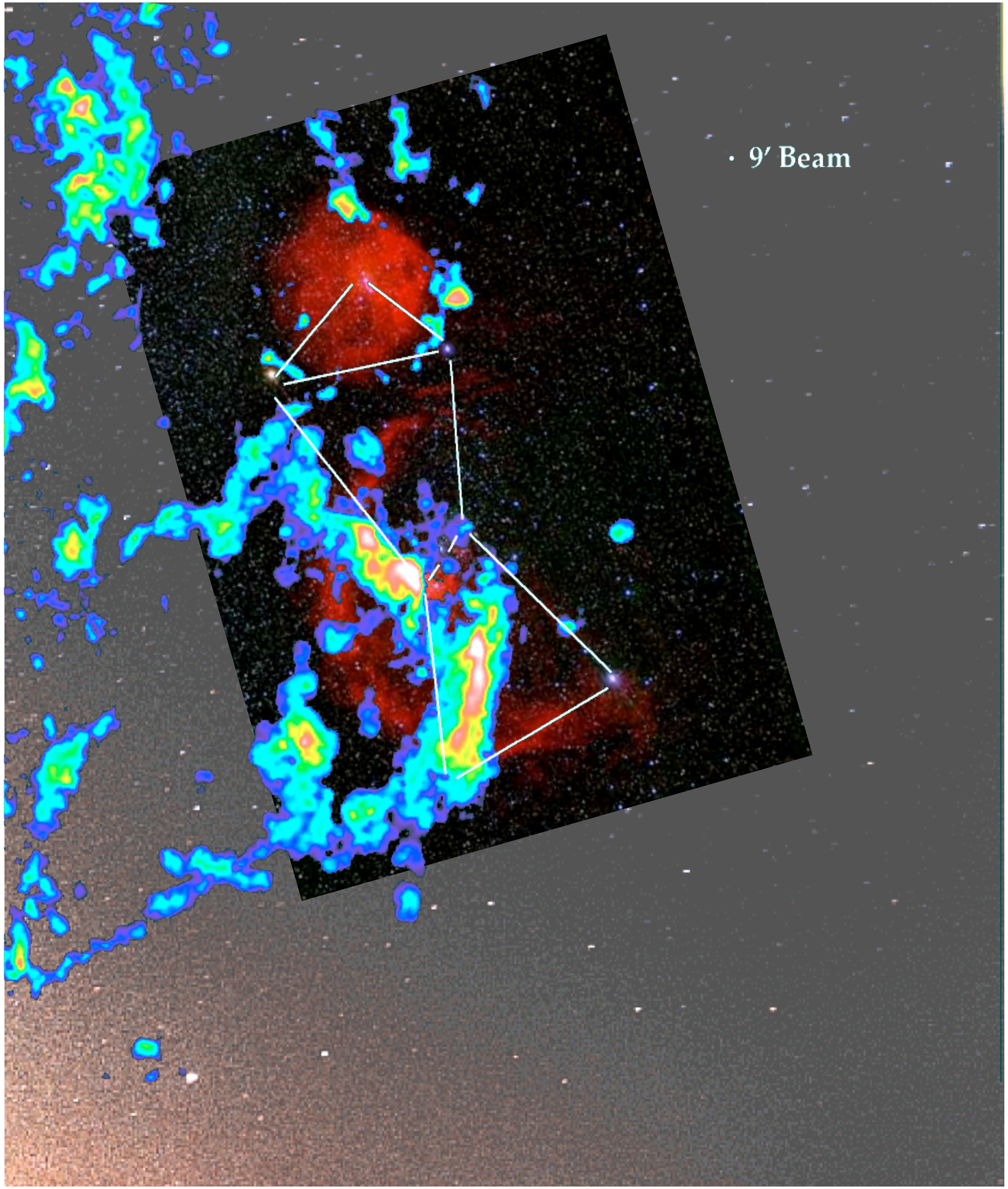

(from the Hubble Heritage Team)

NGC4622

- Star formation *always* is associated with *clouds of gas and dust*.
- Star formation is essentially a ***local phenomenon*** (on ~pc scale)
- HOW is star formation is *influenced* by ***global*** properties of the galaxy?

Star forming clouds in the Milky Way



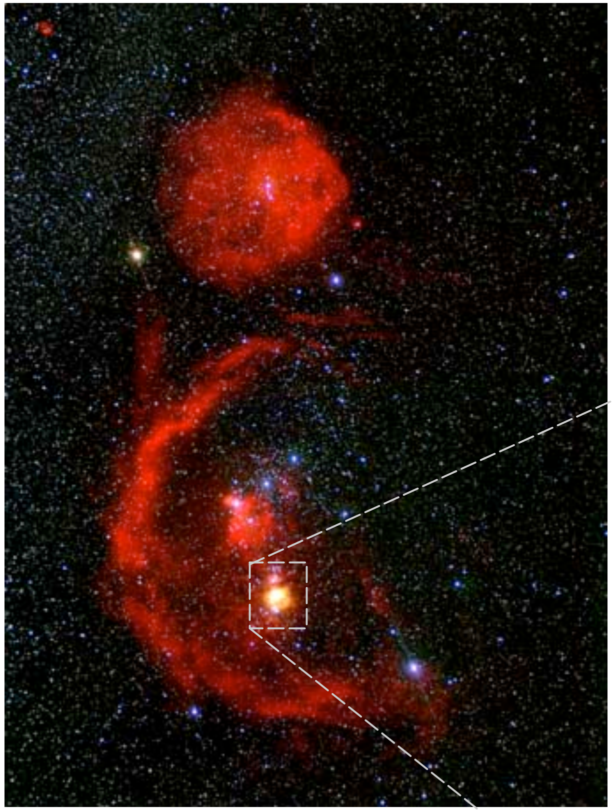


Star formation in Orion

We see

- *stars* (in optical light)
- *atomic hydrogen* (in Hα -- red)
- *molecular hydrogen H_2* (radio -- color coded)

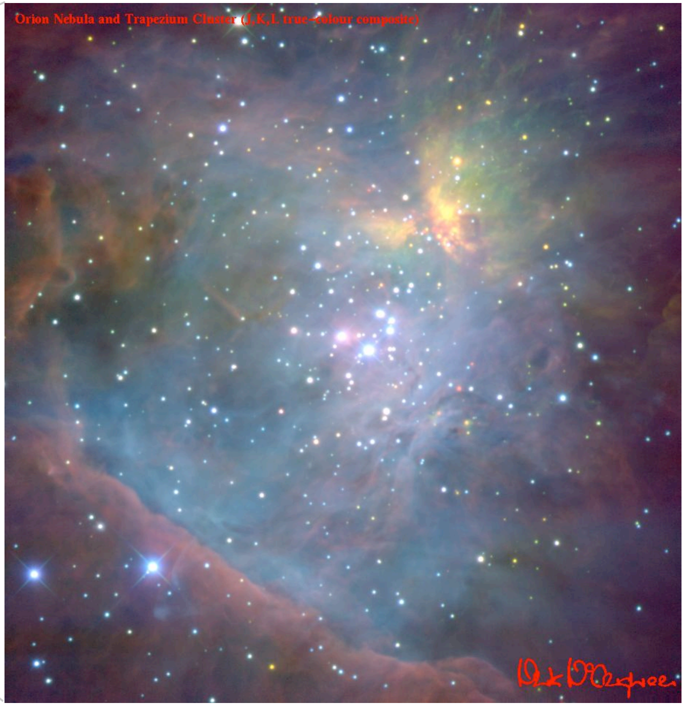
Local star forming region: The Trapezium Cluster in Orion



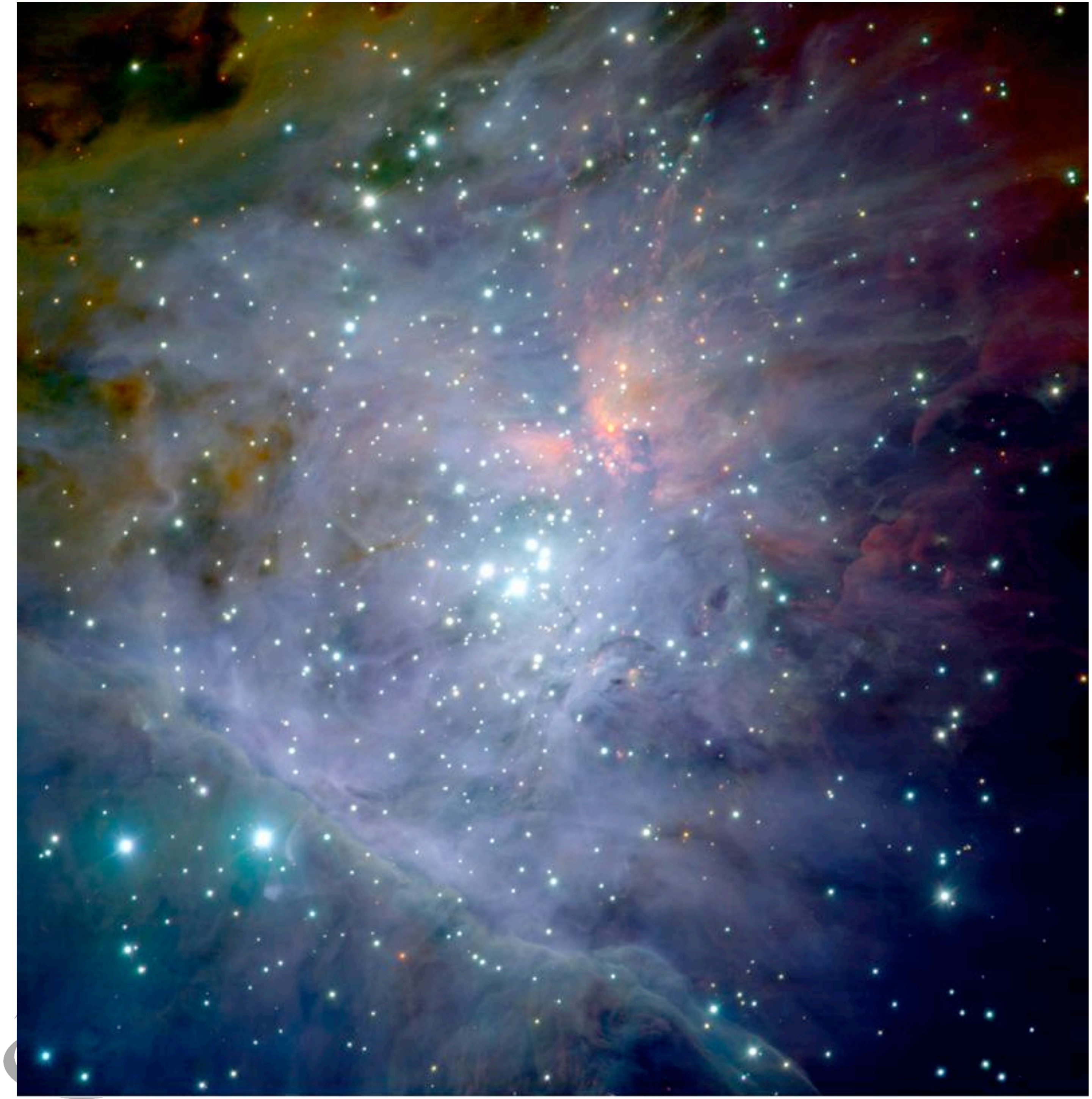
Orion molecular cloud

The Orion molecular cloud is the birth- place of several young embedded star clusters.

The Trapezium cluster is only visible in the IR and contains about 2000 newly born stars.



Trapezium cluster

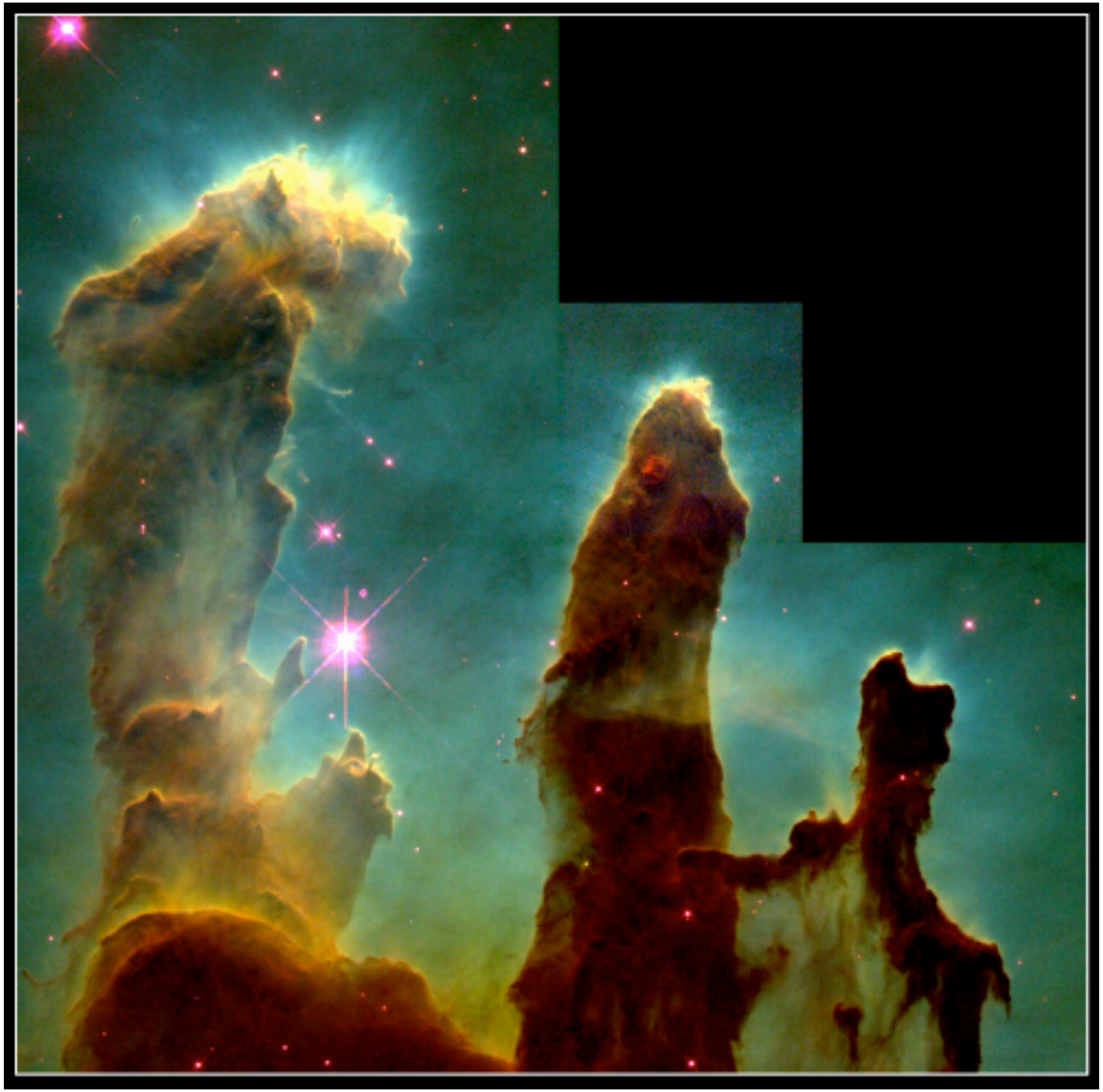


Trapezium Cluster

(detail)

- stars form in clusters
- stars form in molecular clouds
- (proto)stellar feedback is important

(color composite J,H,K by M. McCaughrean, VLT, Paranal, Chile)



HST Aufnahme

Pillars of God (in Eagle Nebula): Formation of small groups of young stars in the tips of the columns of gas and dust ….

Infrared
observation

IR observation with ESO-VLT

Pillars of God (in Eagle Nebula): Formation of small groups of young stars in the tips of the columns of gas and dust ….

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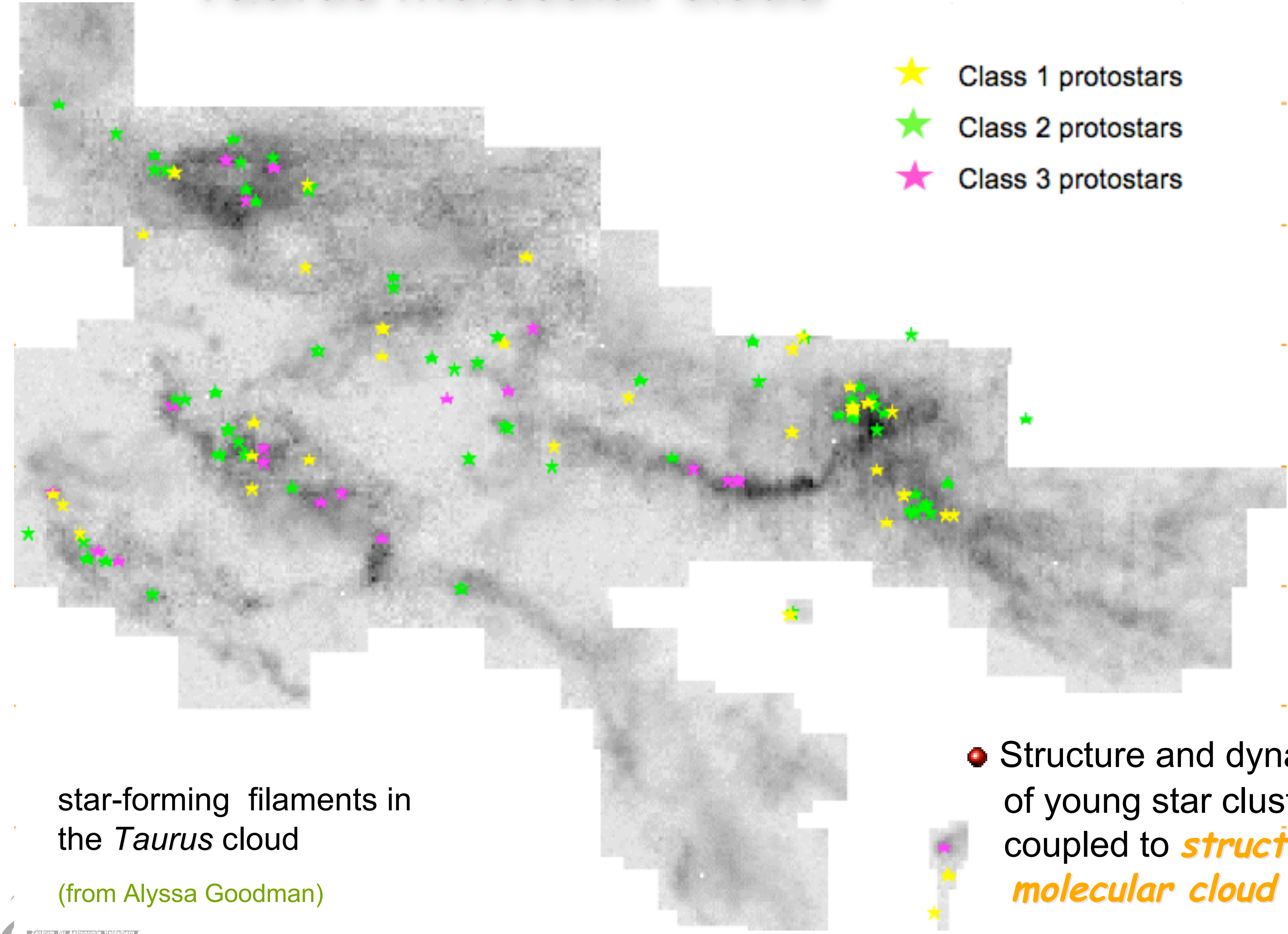
IR observation with ESO-VLT

Pillars of God (in Eagle Nebula): Formation of small groups of young stars in the tips of the columns of gas and dust ….

Taurus molecular cloud

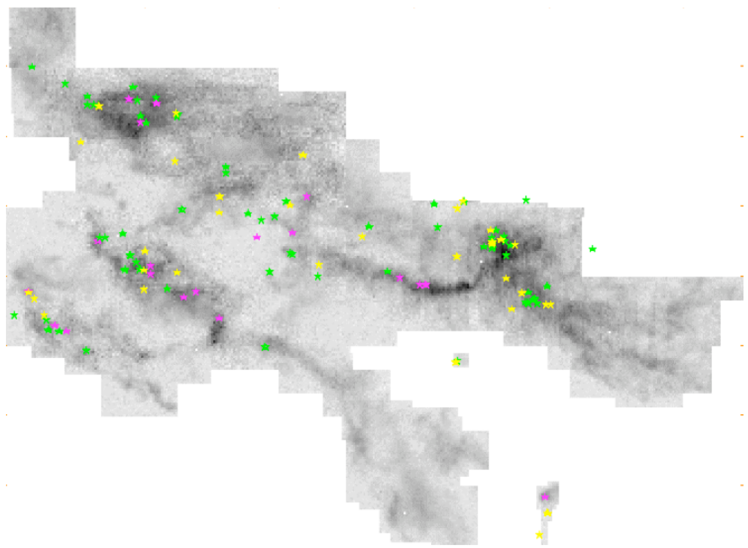


star-forming filaments in the *Taurus* cloud

(from Alyssa Goodman)

- Structure and dynamics of young star clusters is coupled to ***structure of molecular cloud***

Taurus molecular cloud

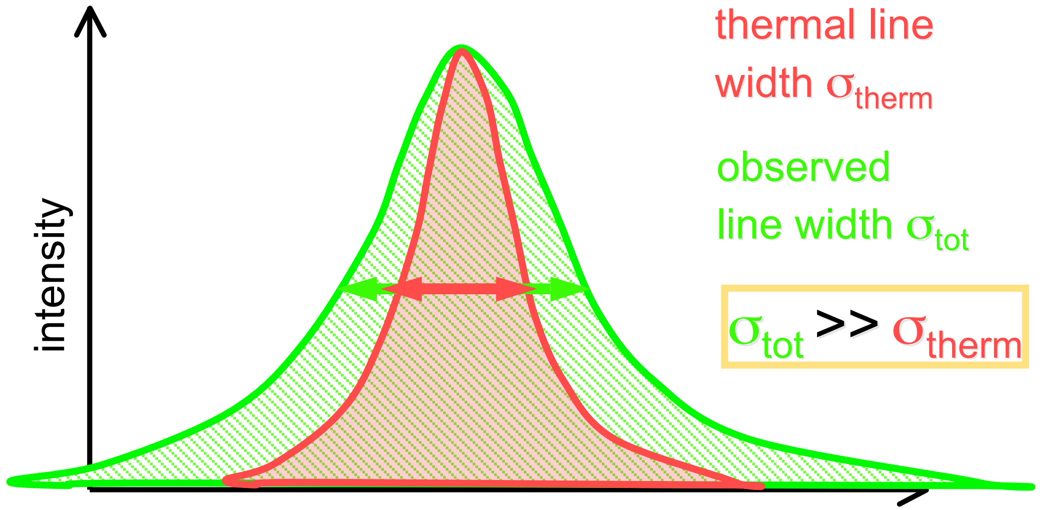


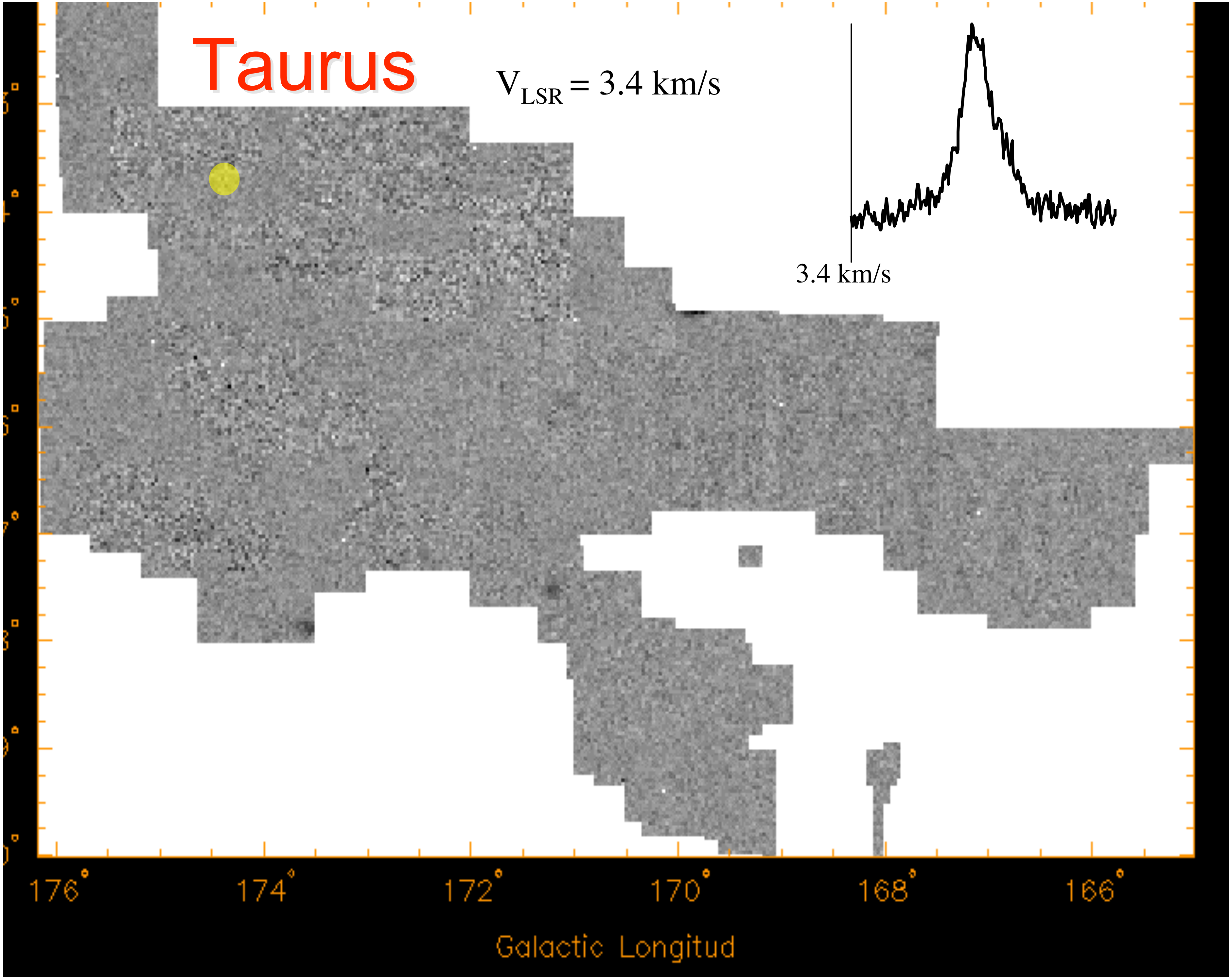
Star-forming filaments in *Taurus* cloud

(from Hartmann 2002)

- Strukture and dynamics of ***molekular cloud*** is determined by ***supersonic turbulence***

- Structure and dynamics of young star clusters is coupled to ***structure of molecular cloud***





Taurus
V_LSR = 3.4 km/s
3.4 km/s
176°
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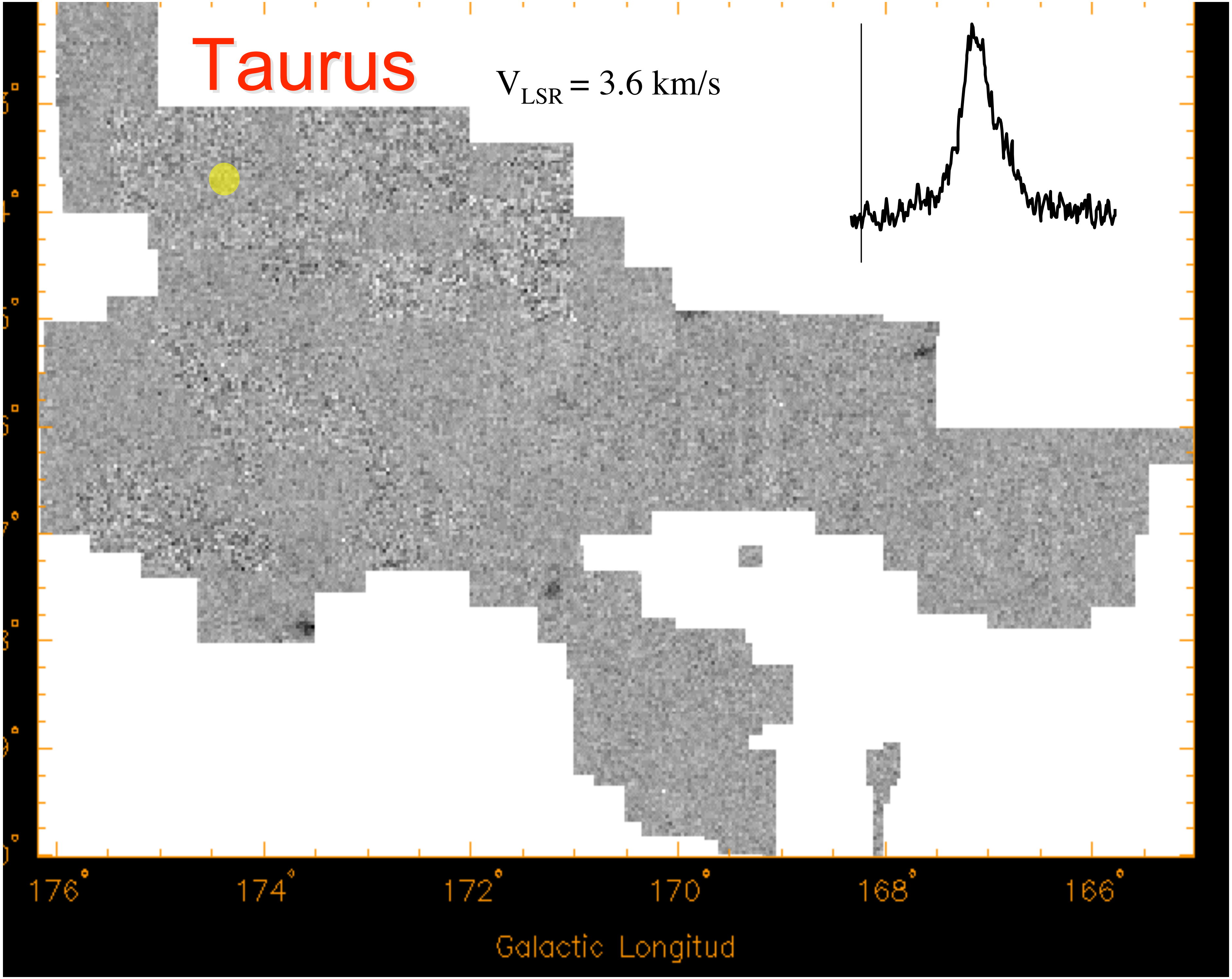
Galactic Longitud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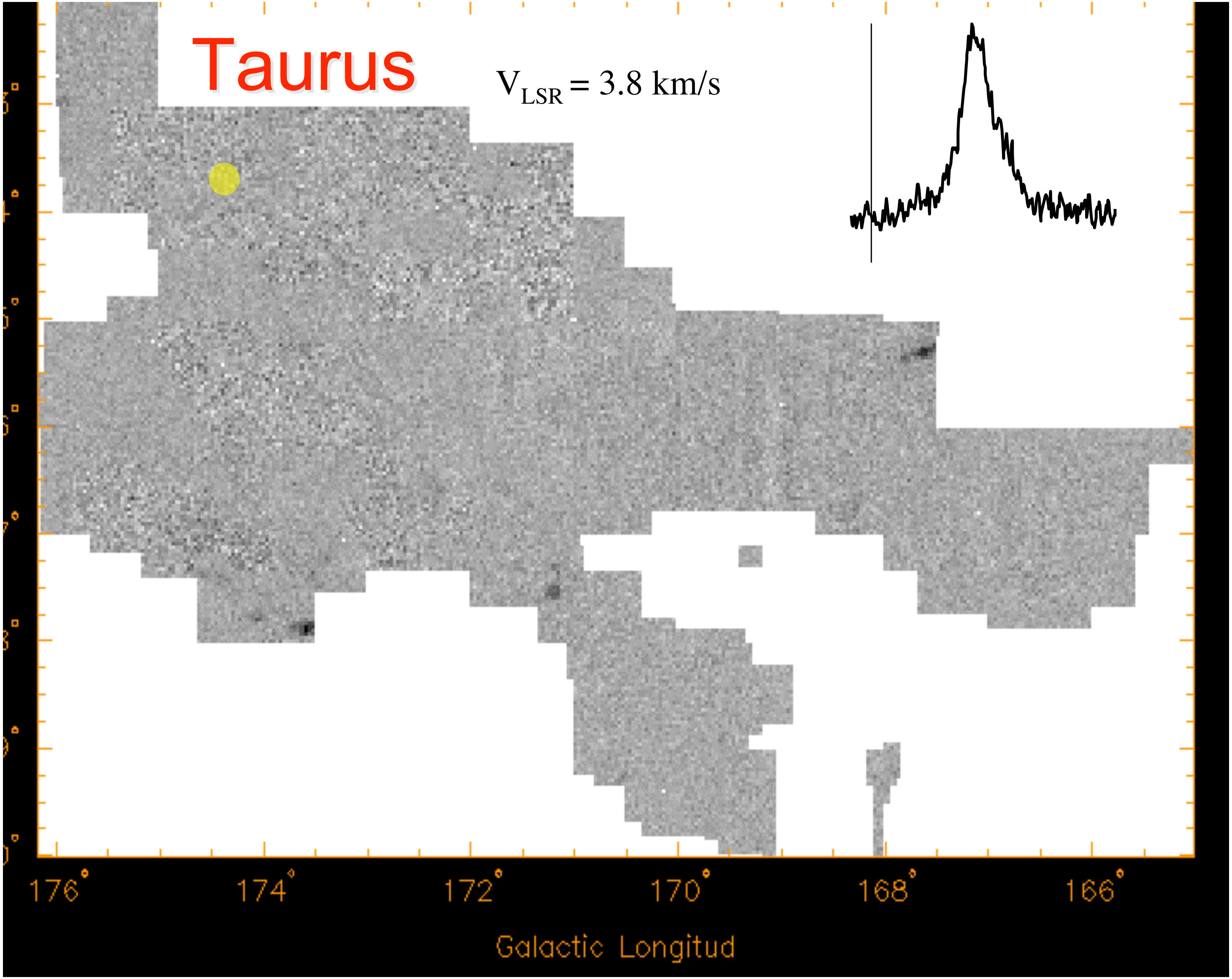
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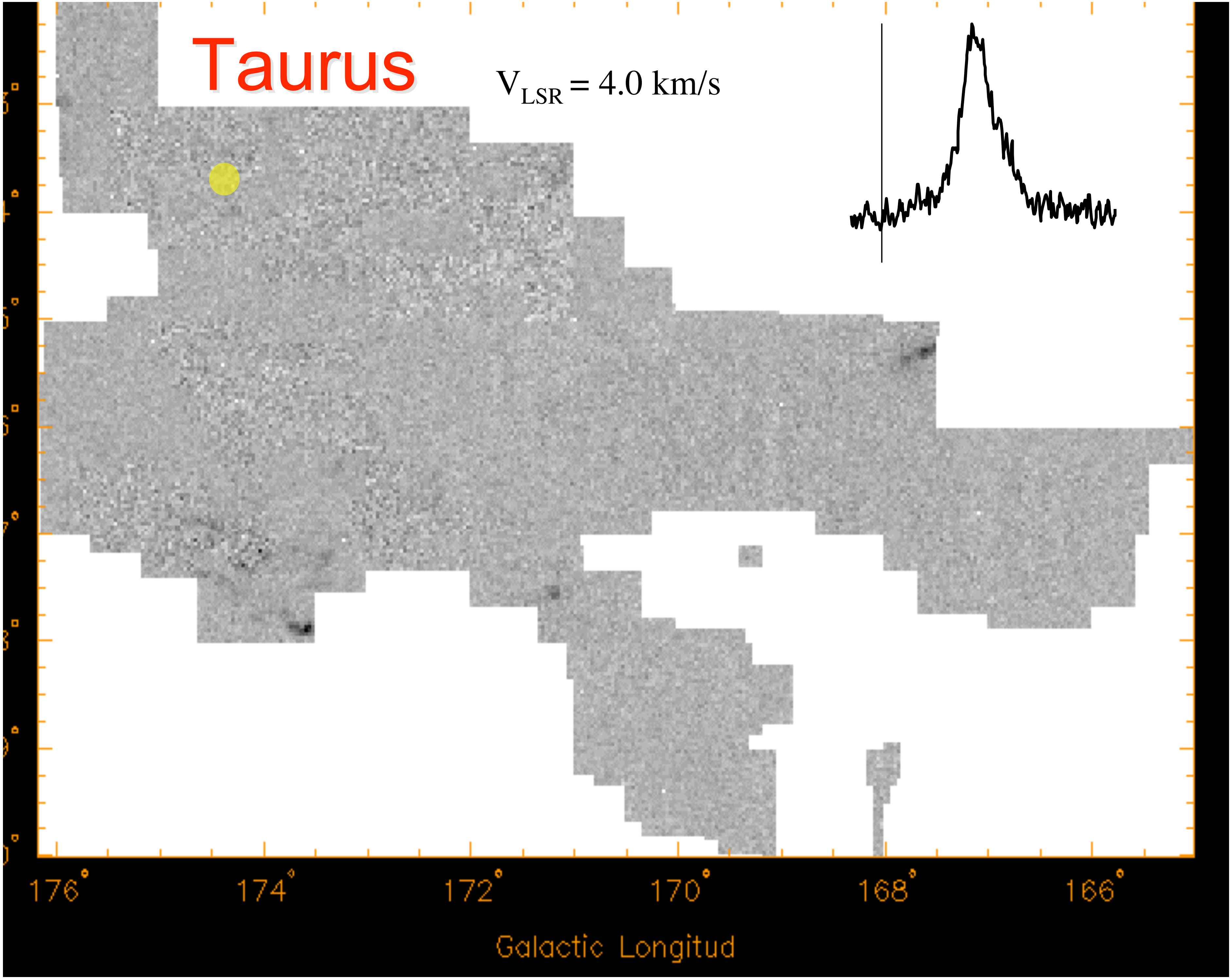
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V_{LSR} = 4.0 km/s
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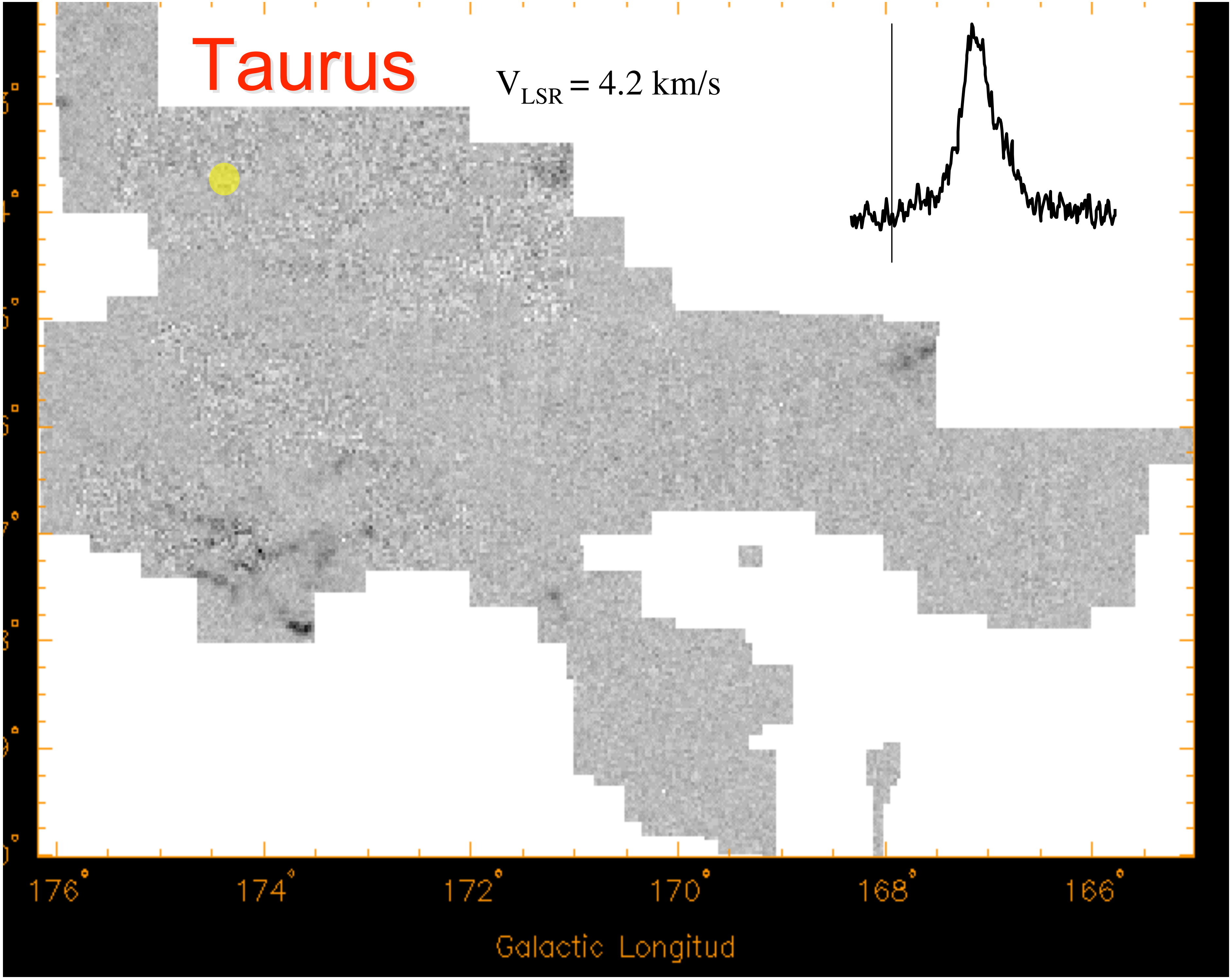

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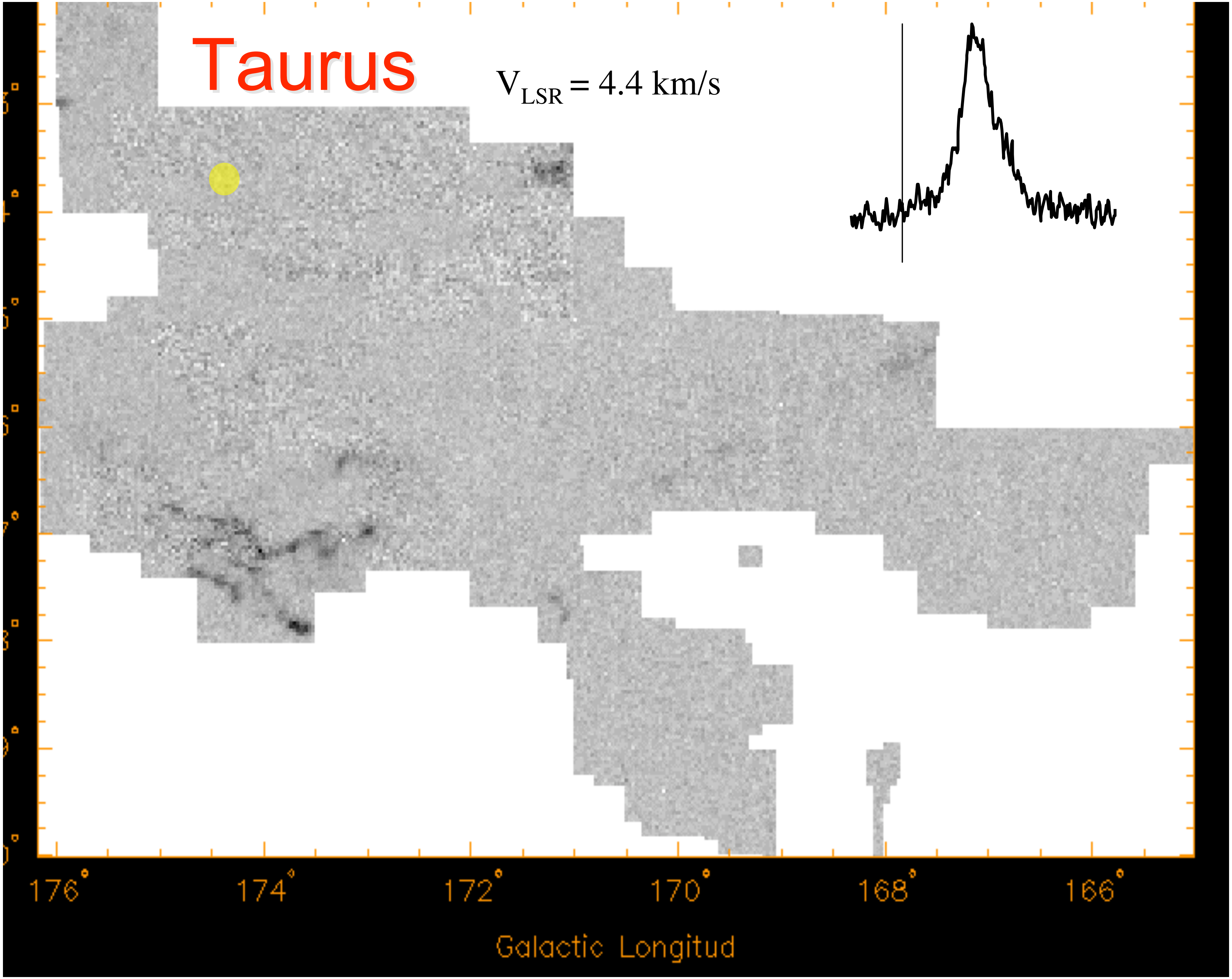
Galactic Longitud



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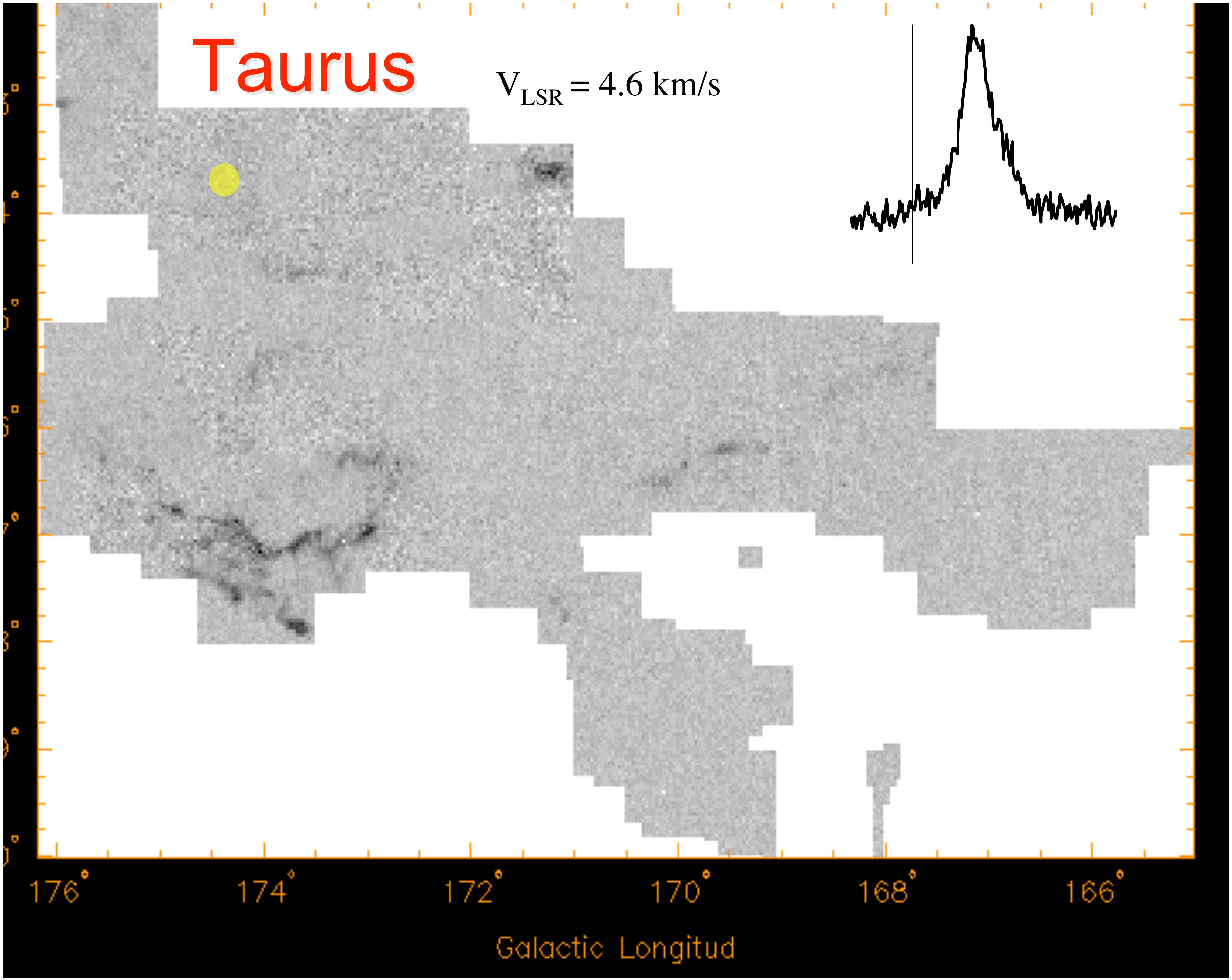

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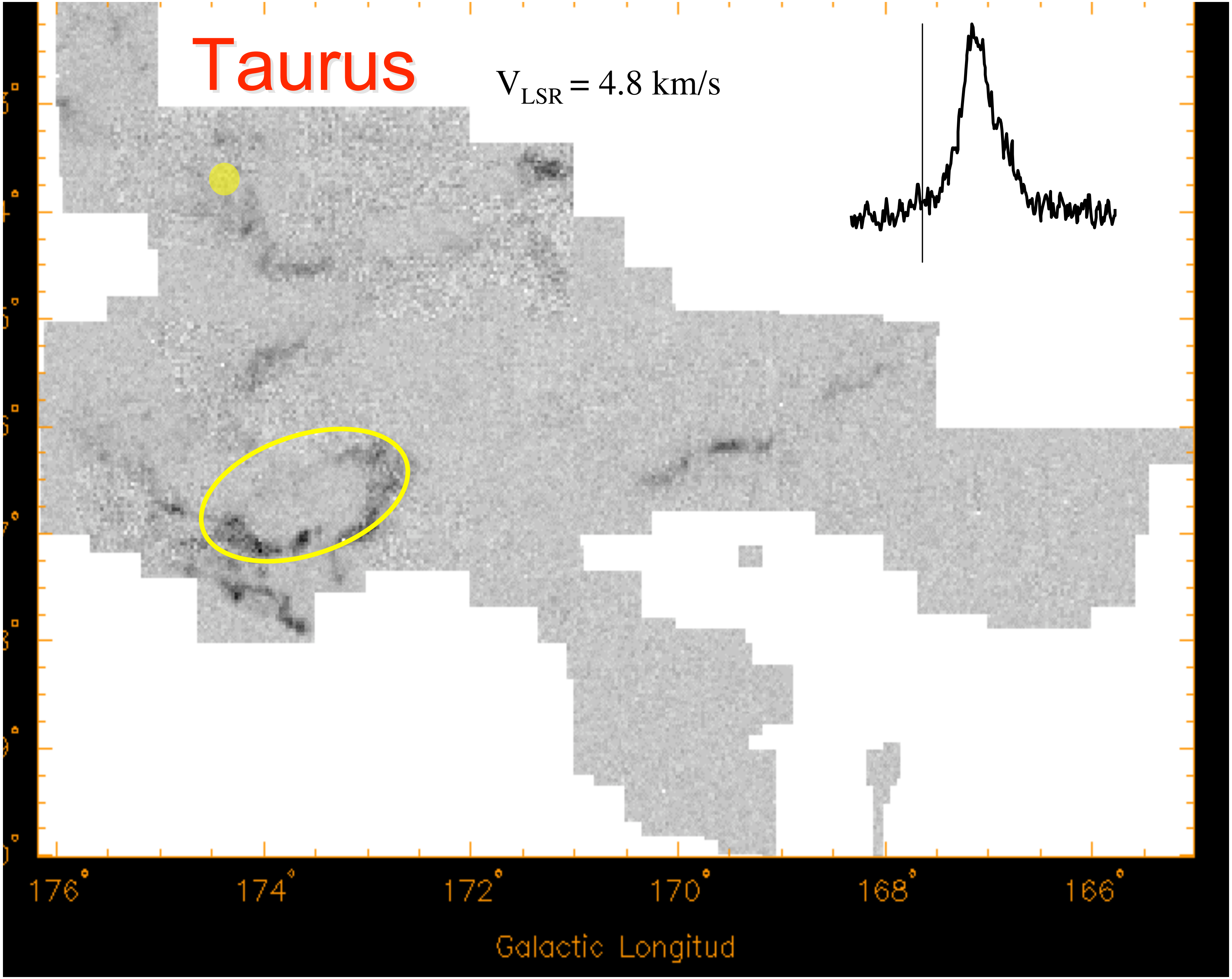

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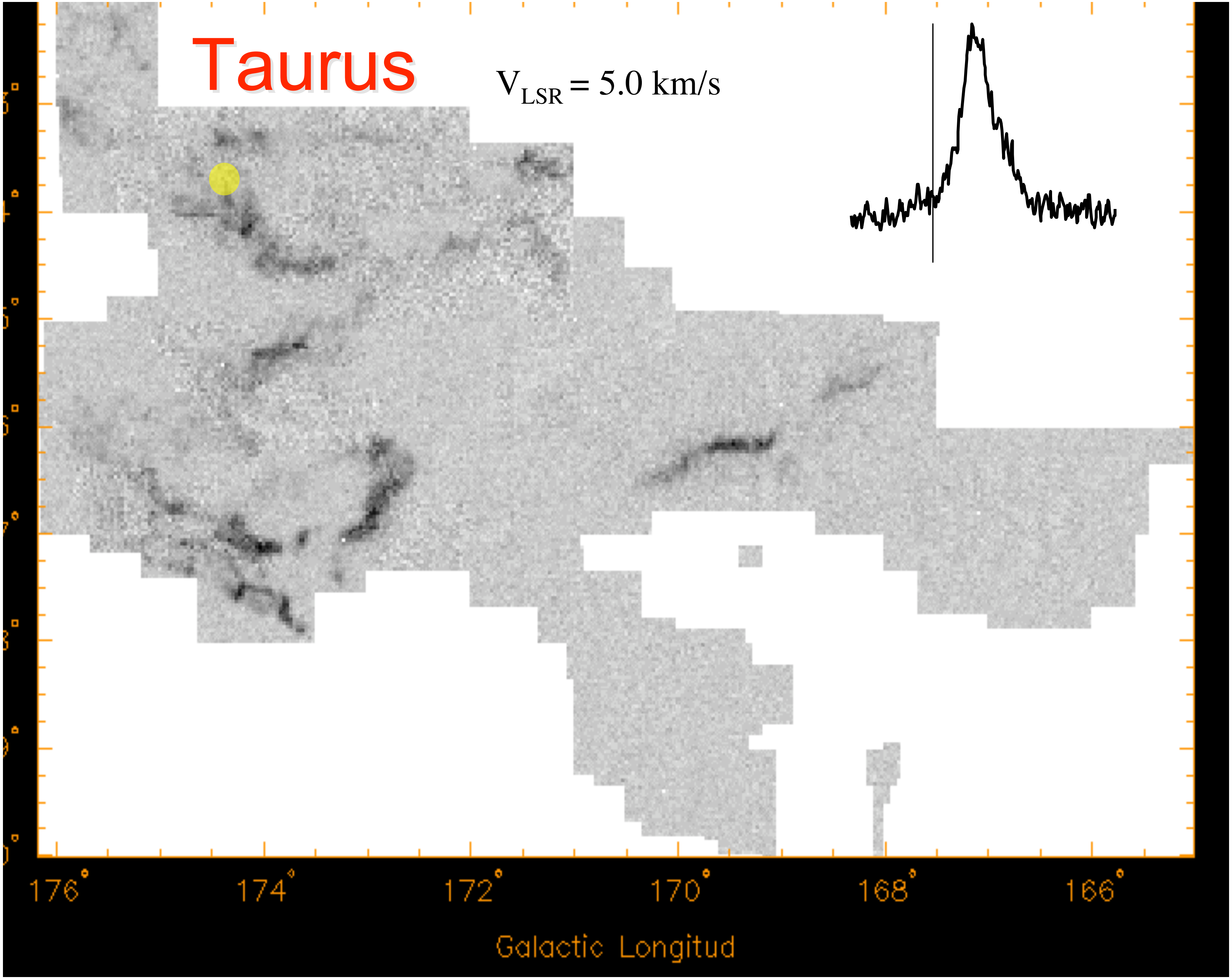

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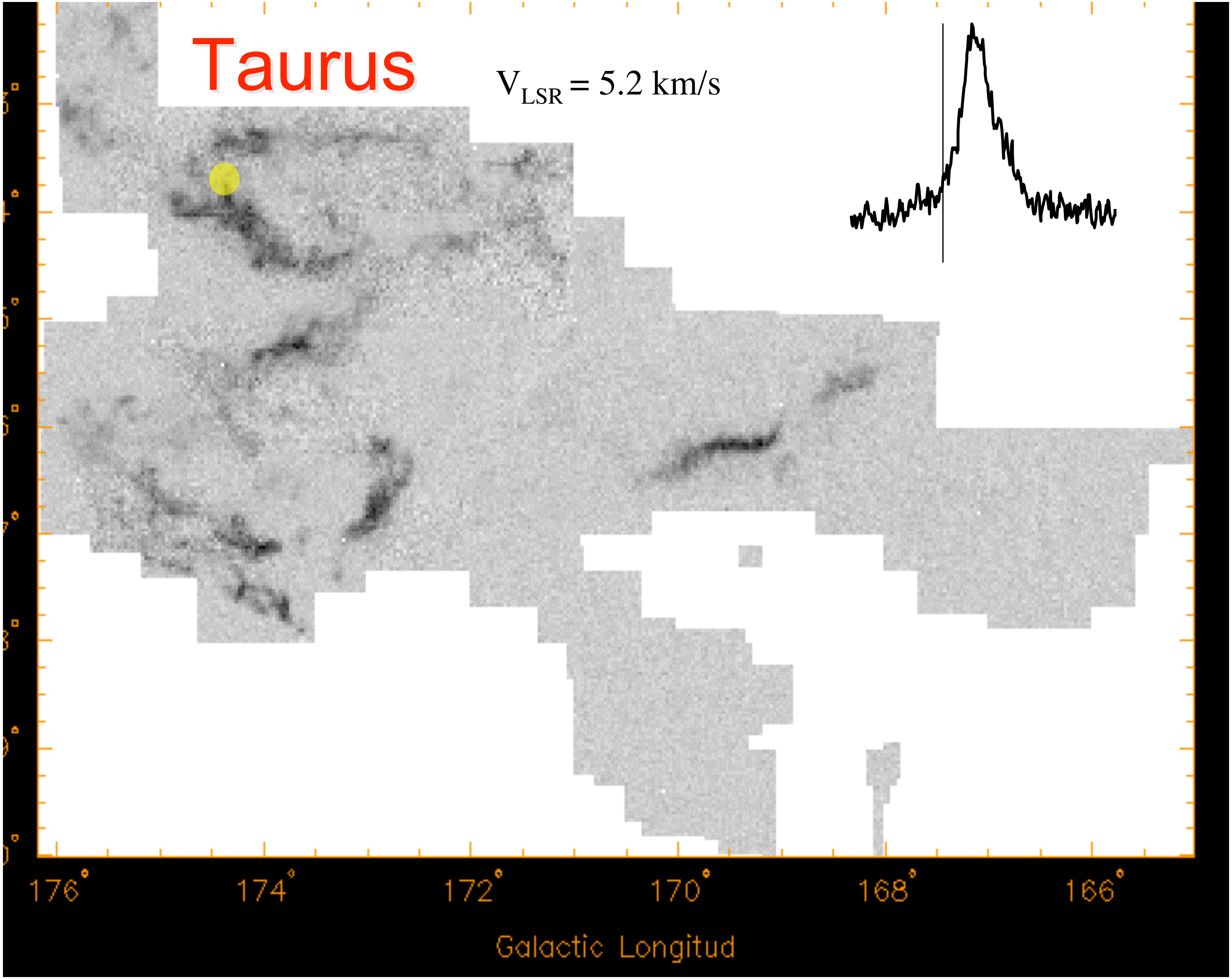
Galactic Longitud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V_LSR = 5.2 km/s
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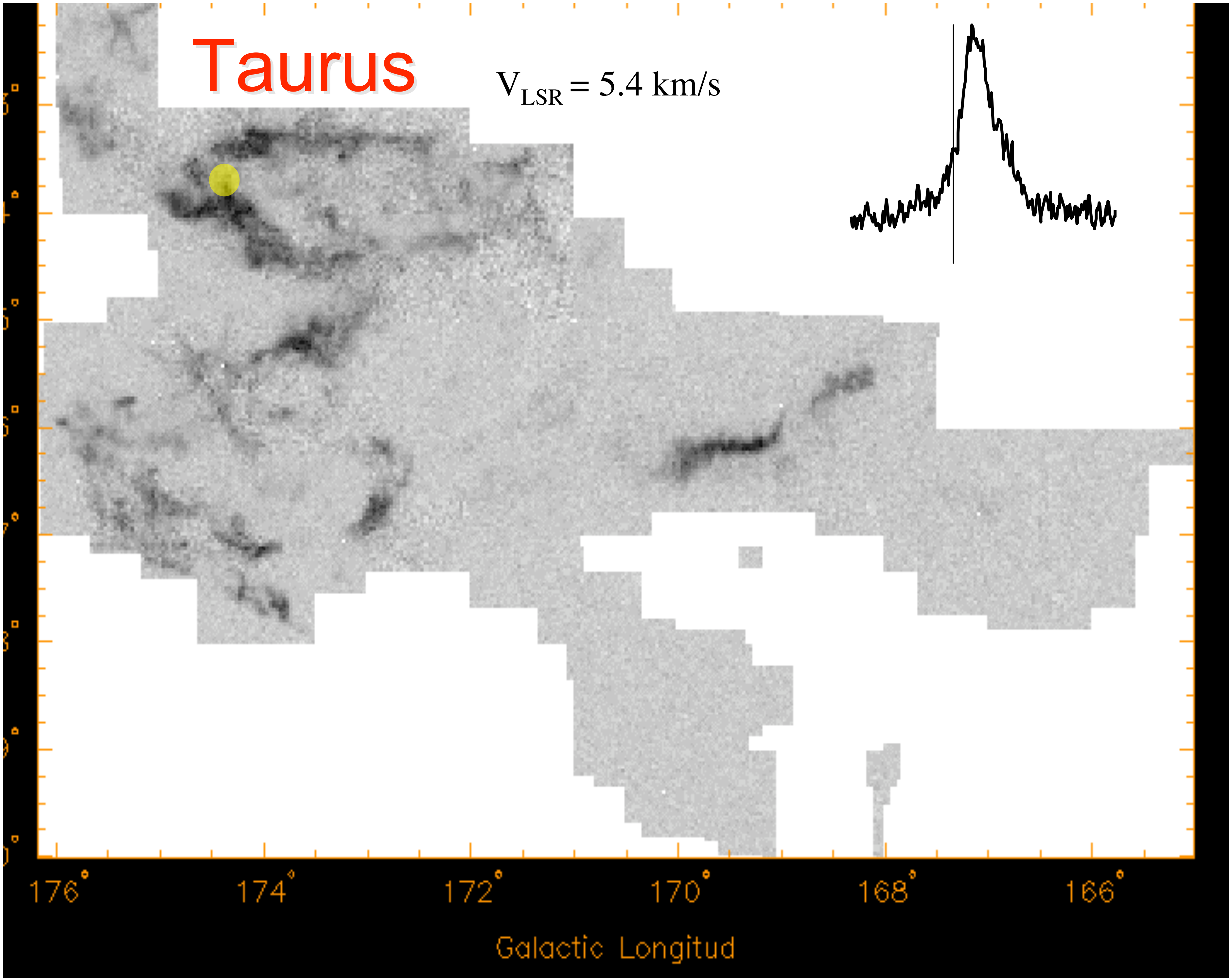

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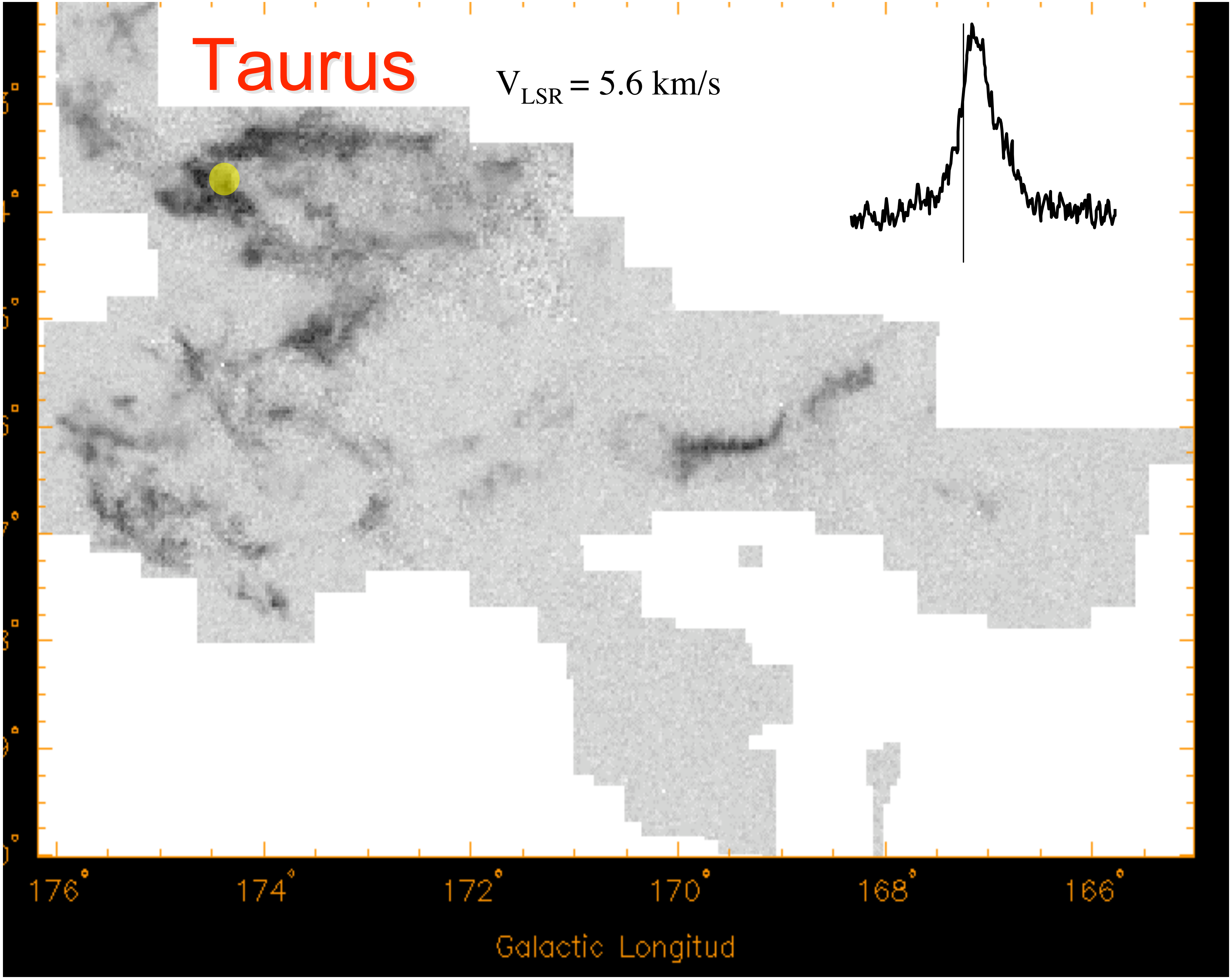

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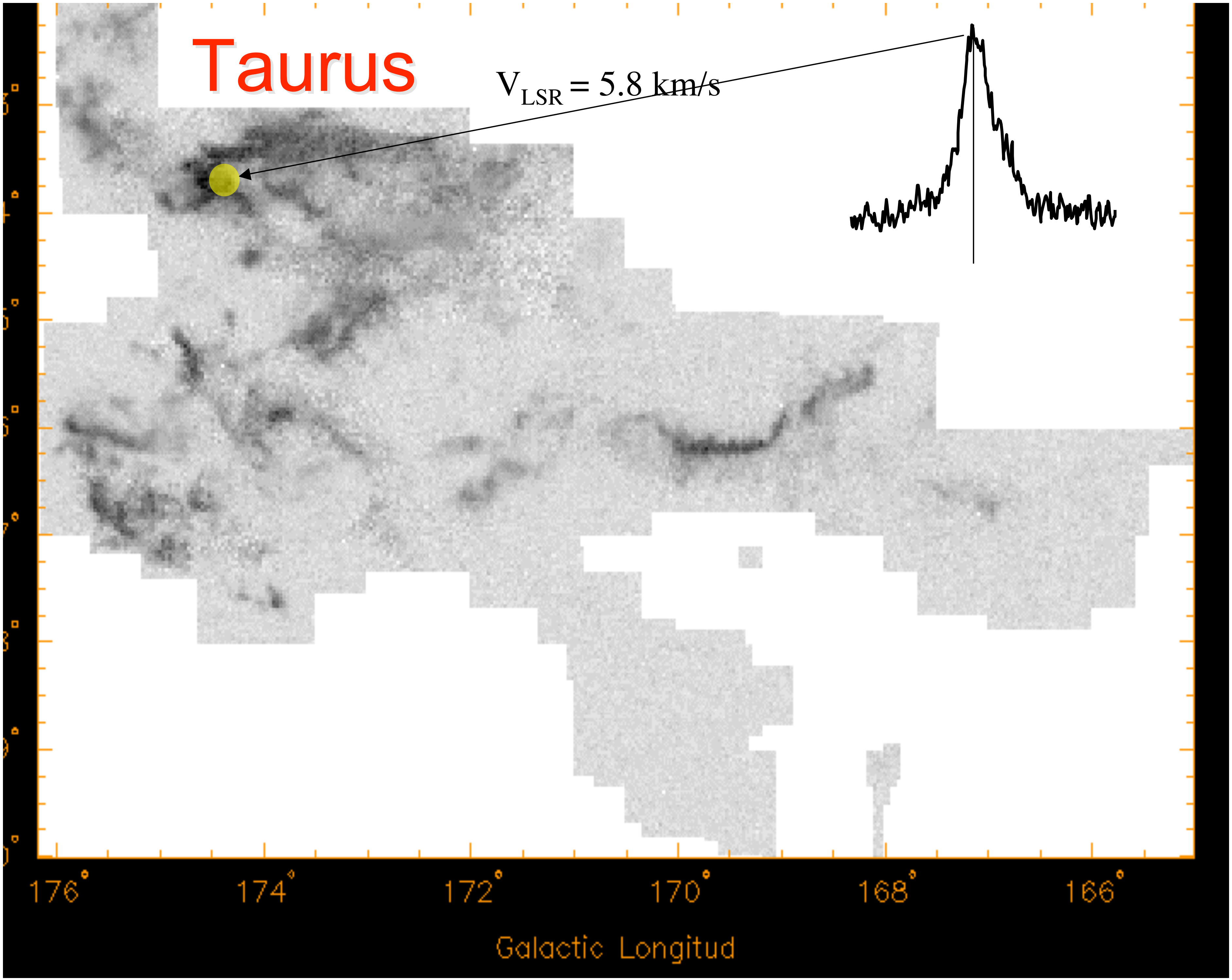
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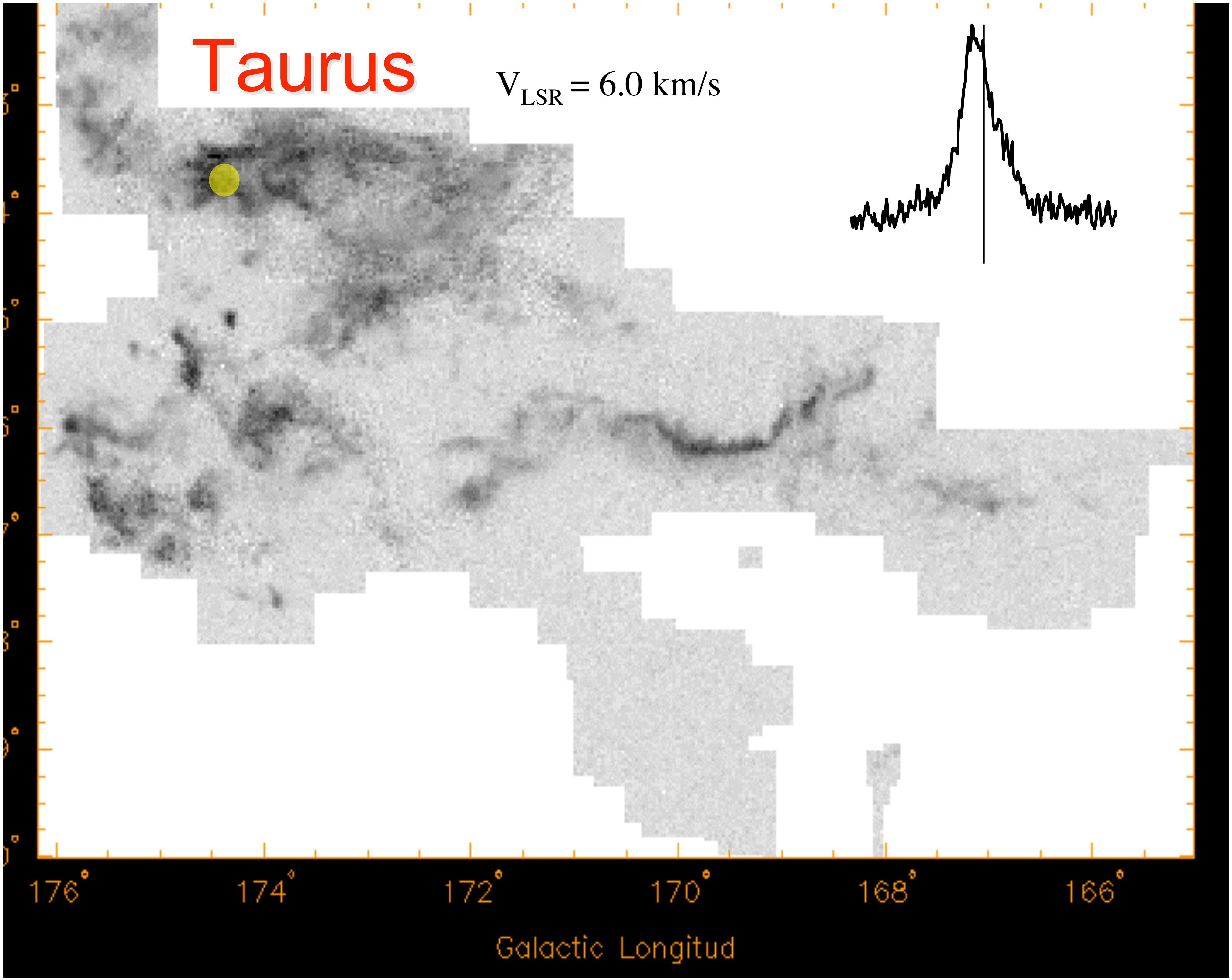
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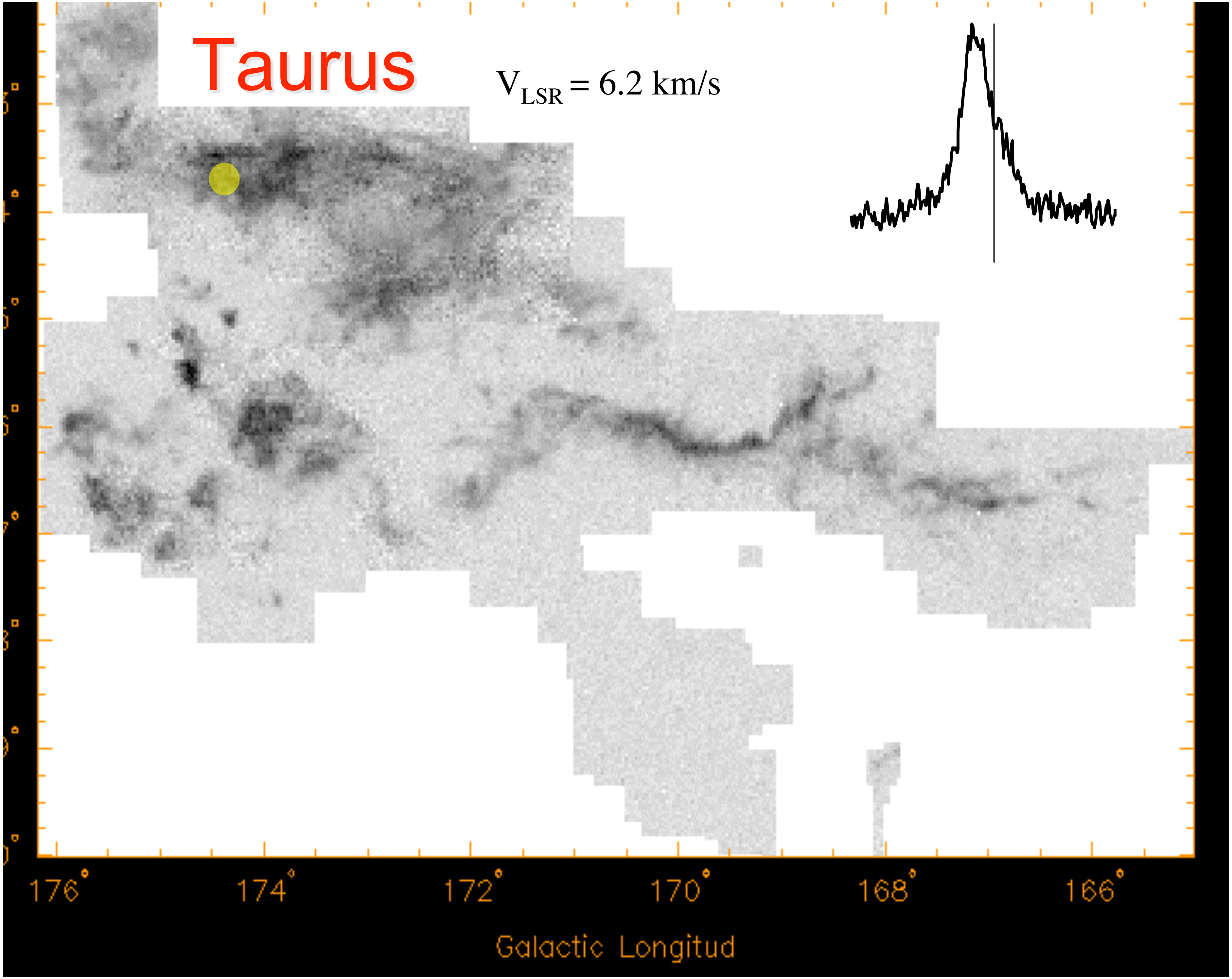
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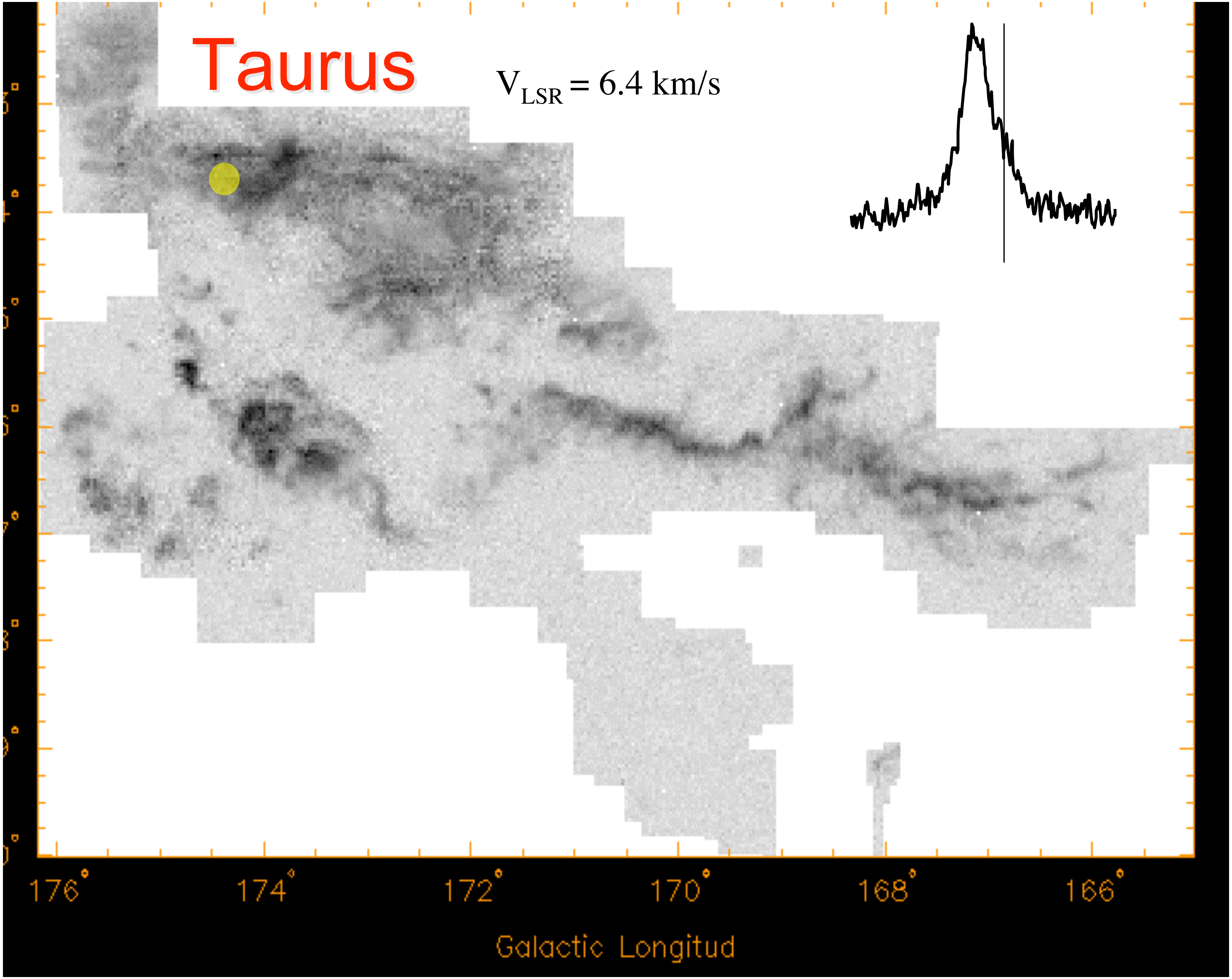
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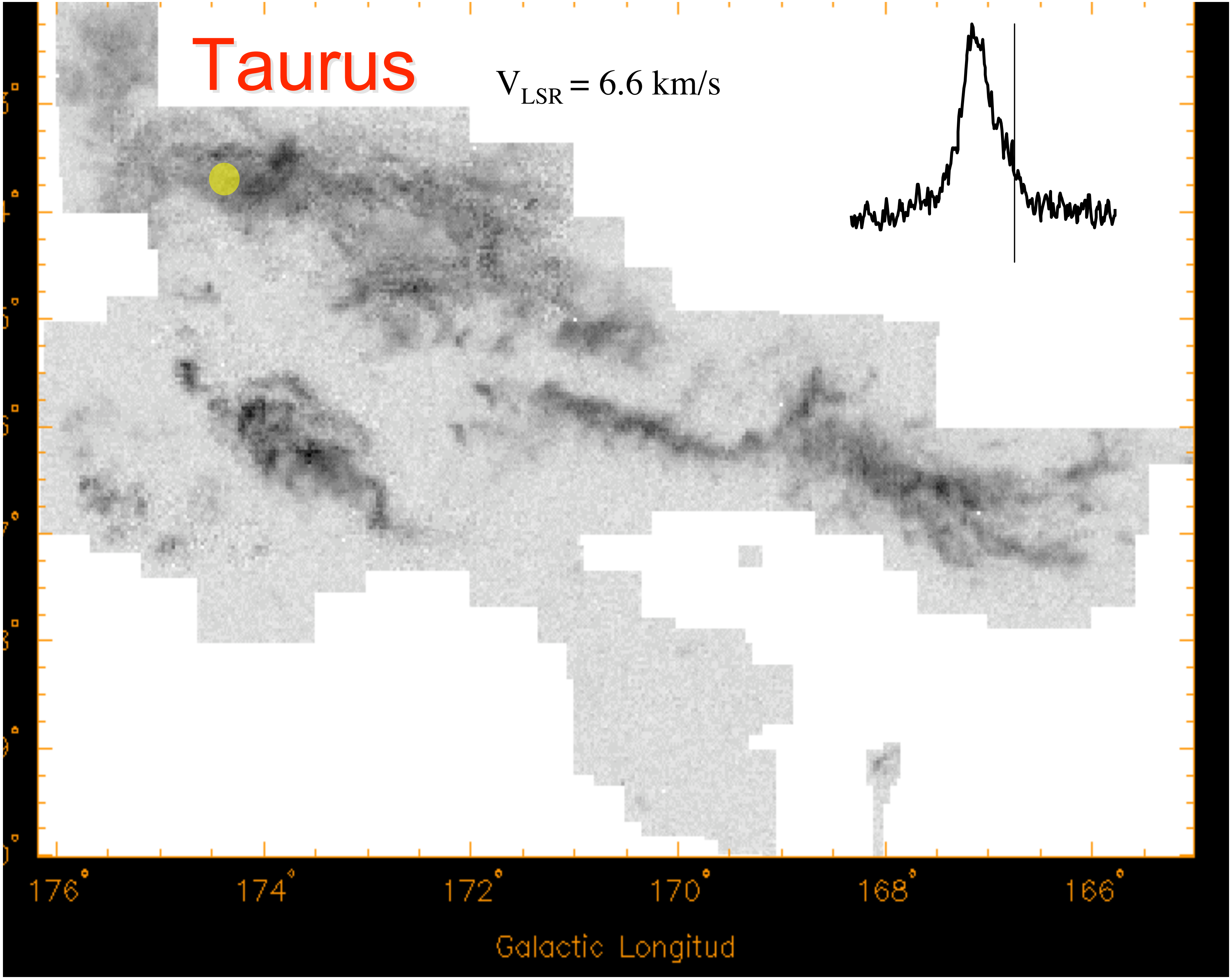

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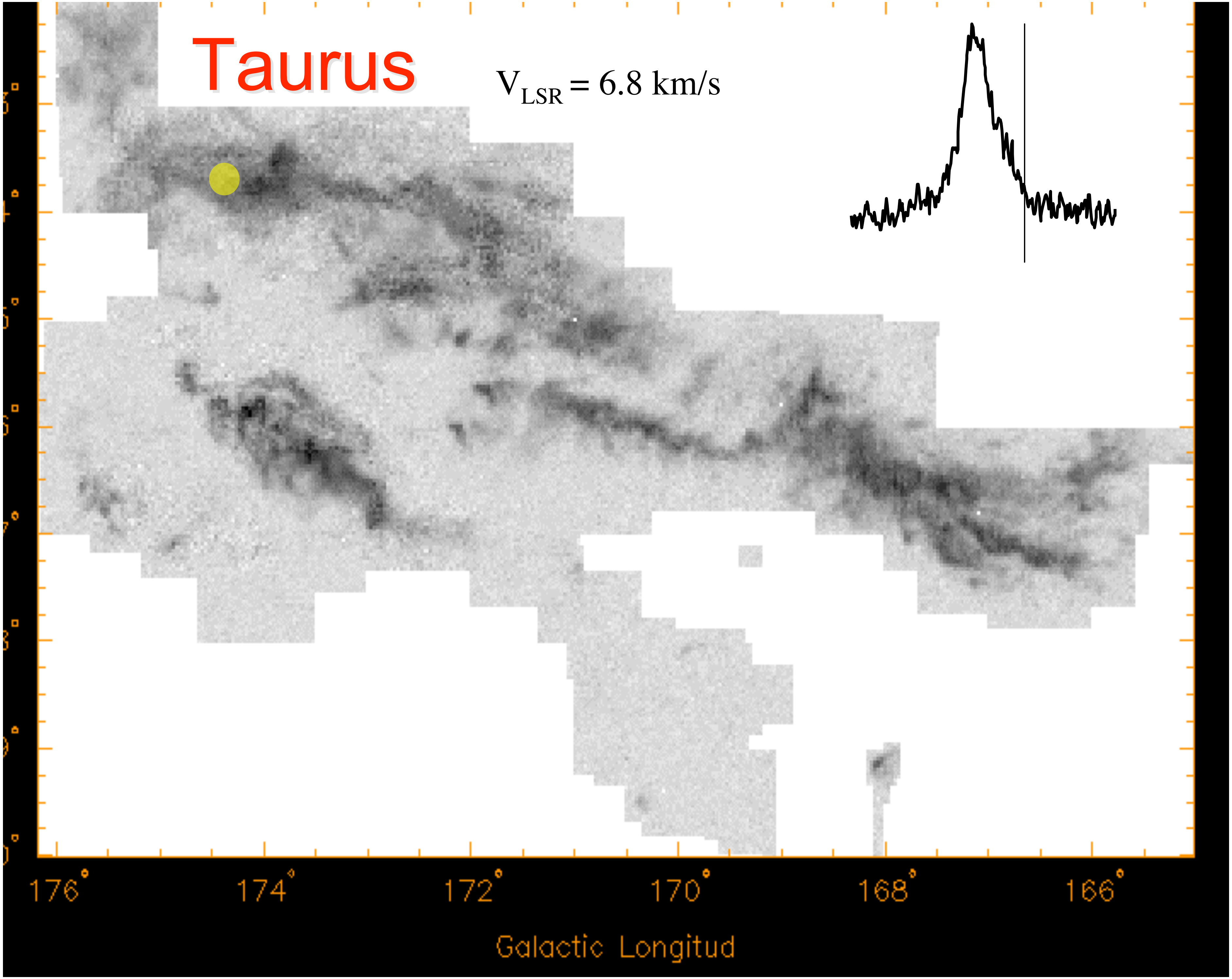
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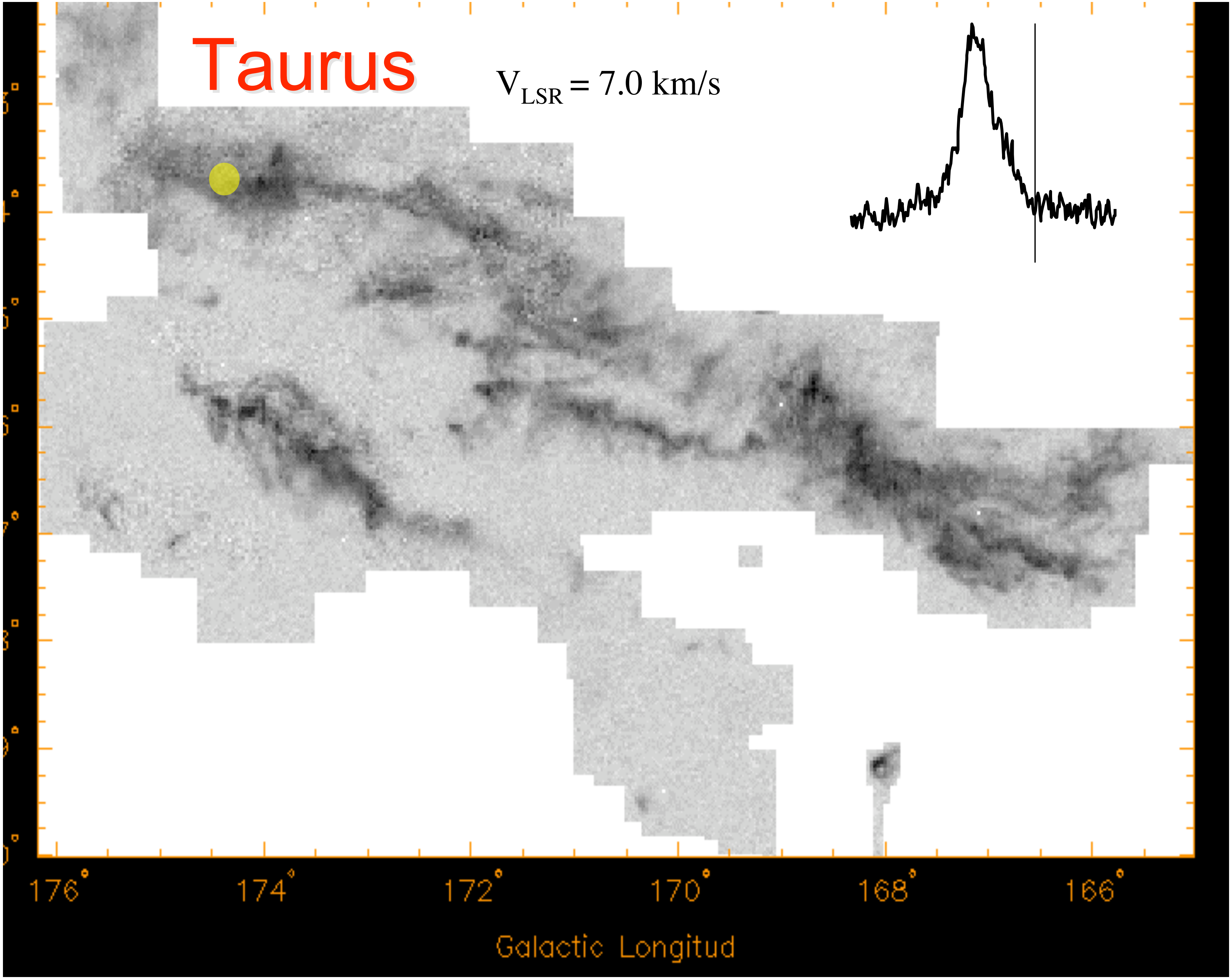

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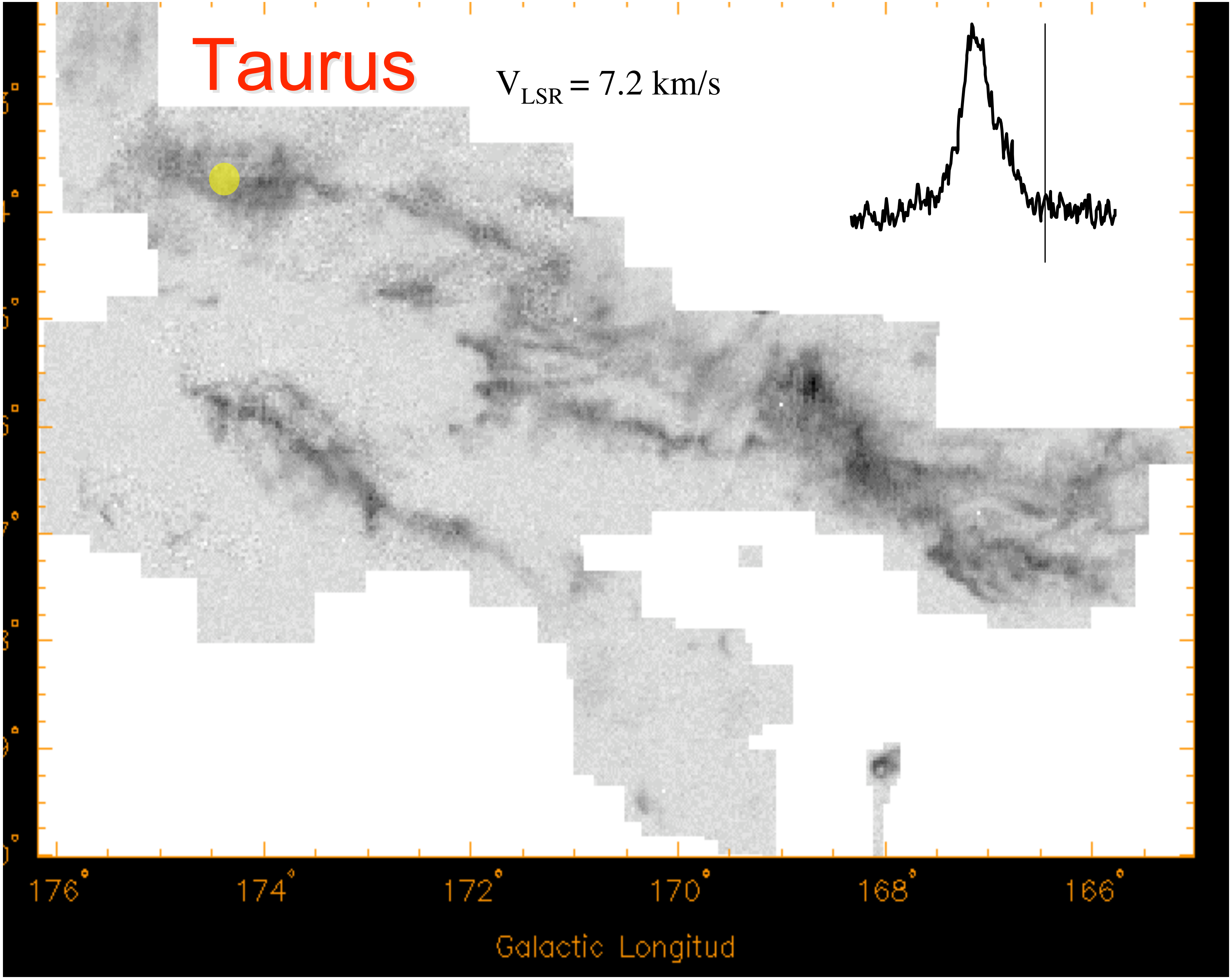

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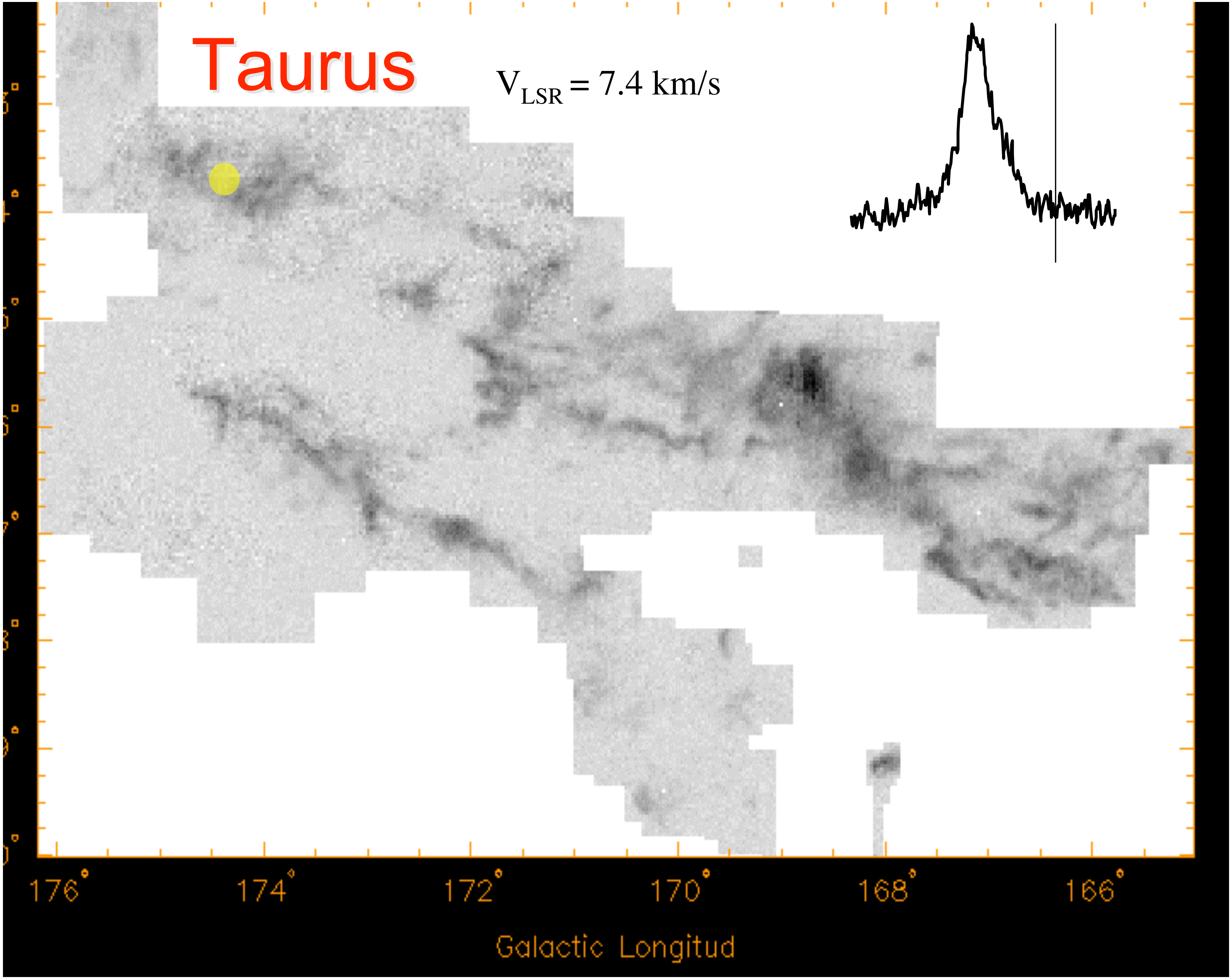
Galactic Longitud



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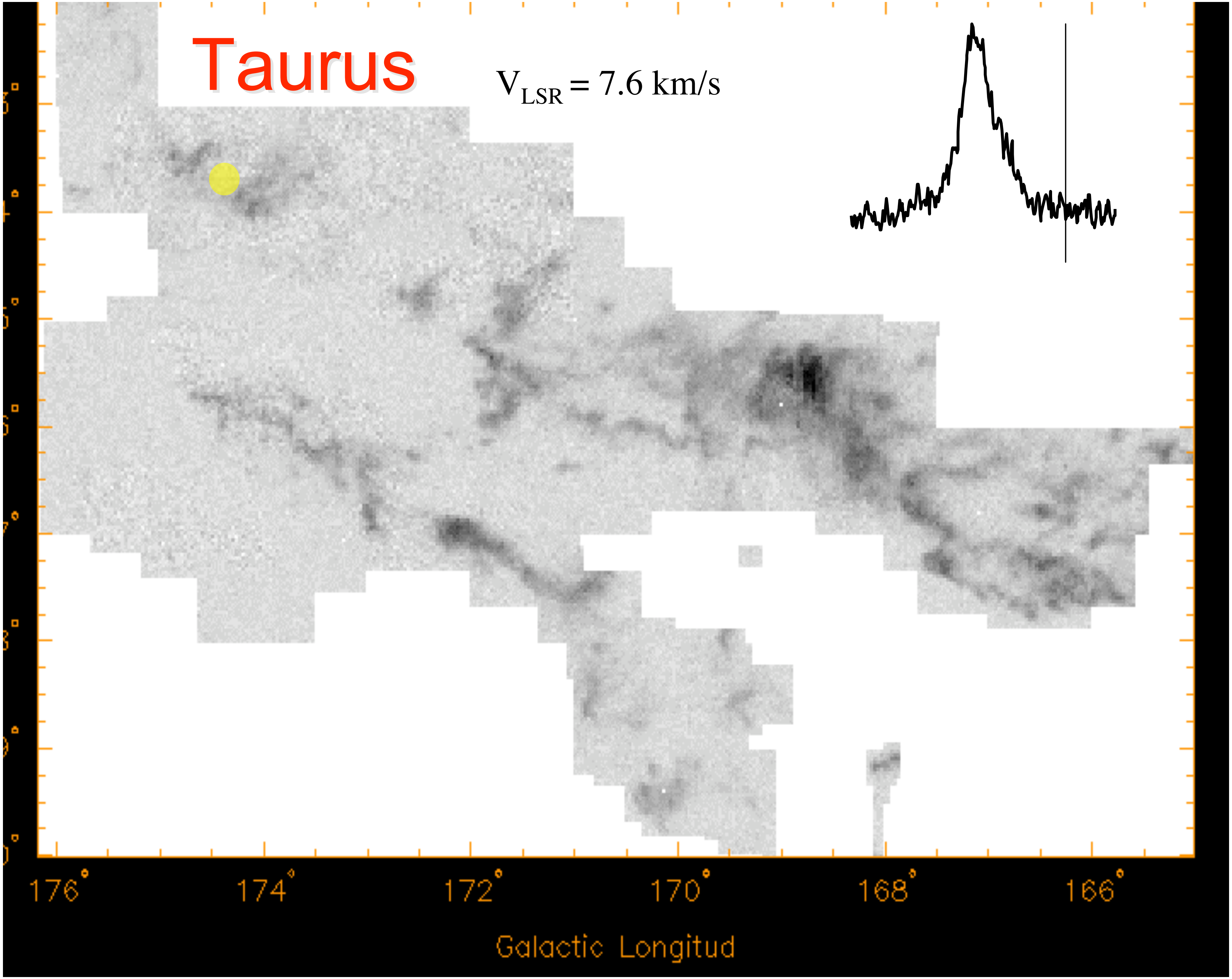

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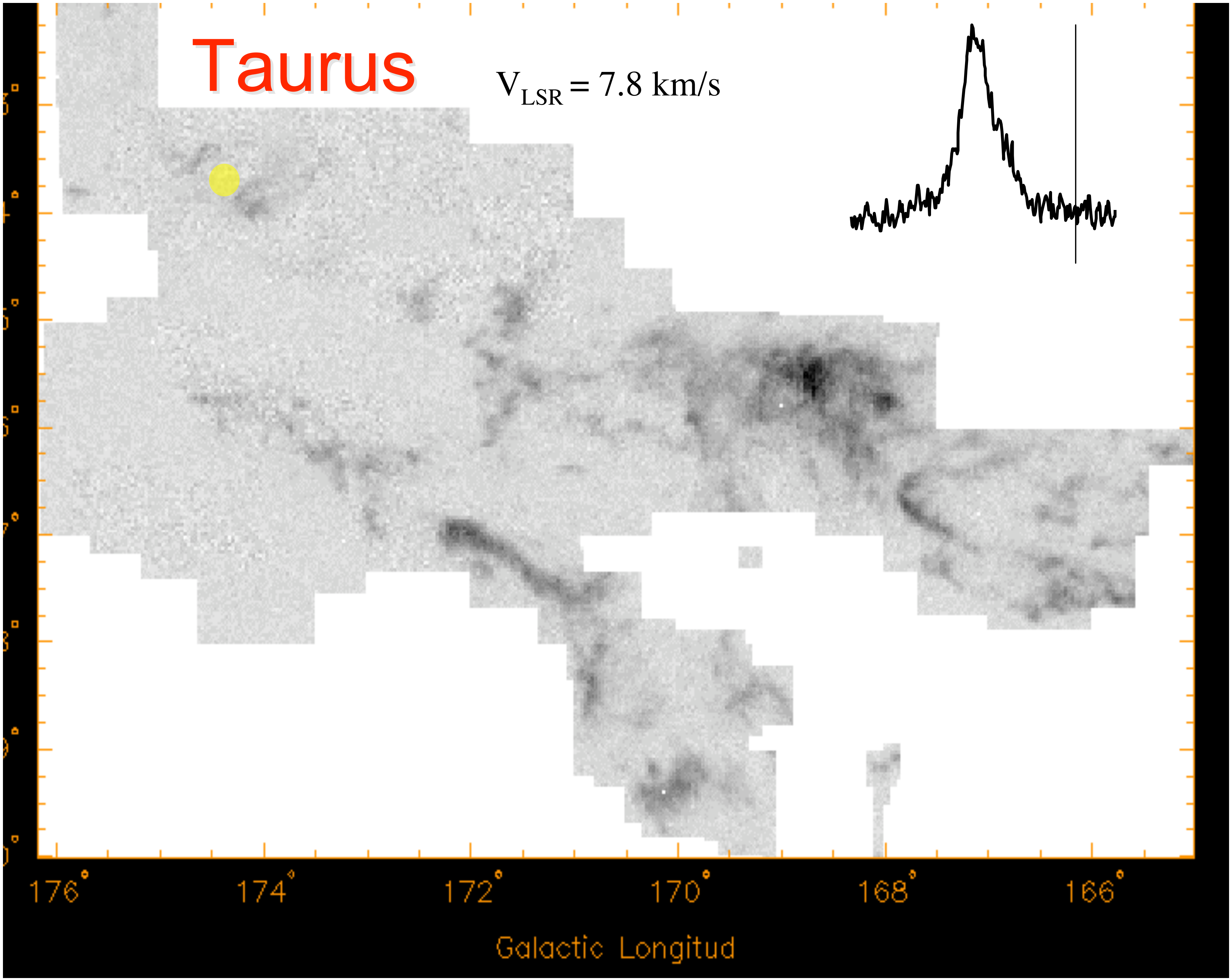

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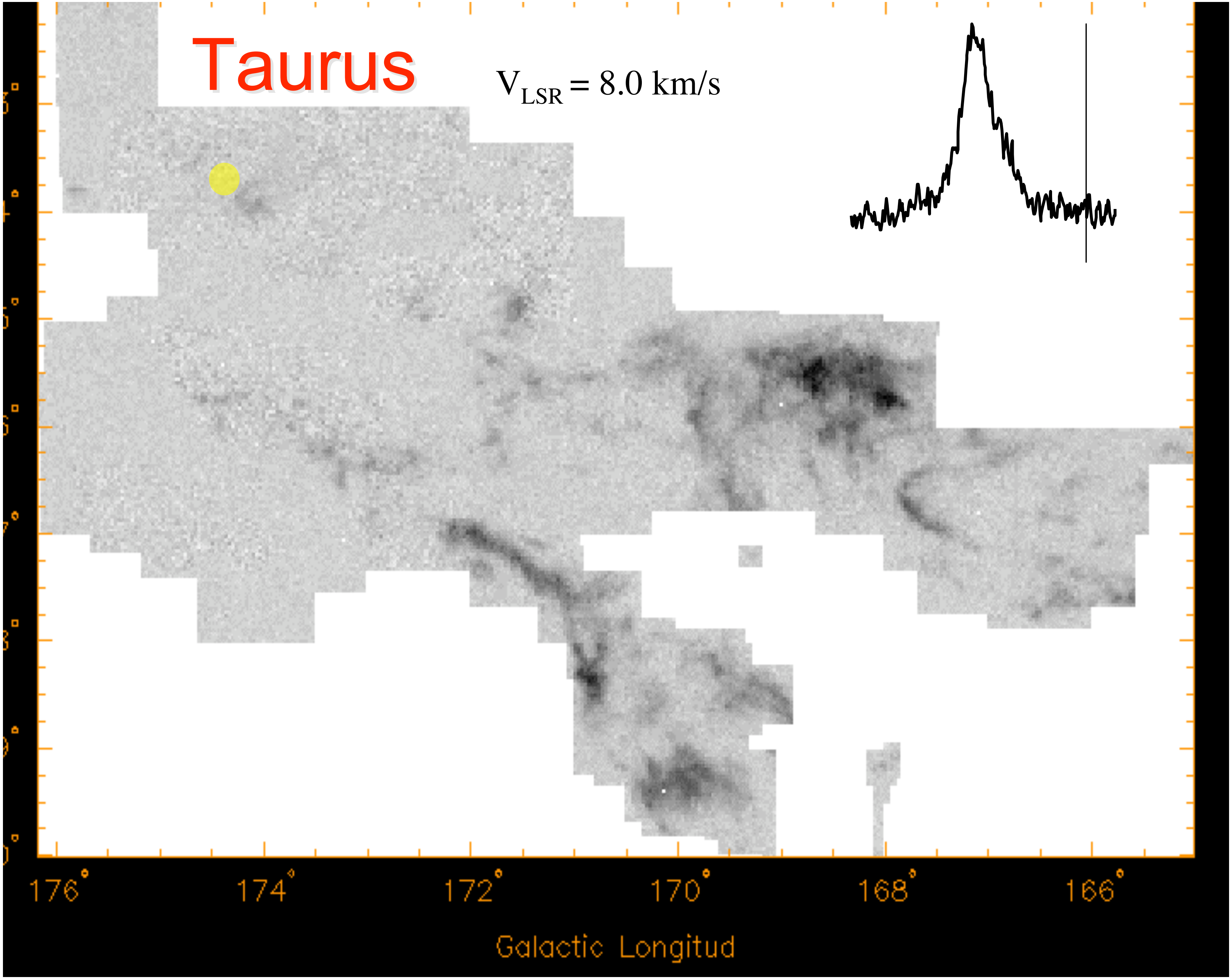

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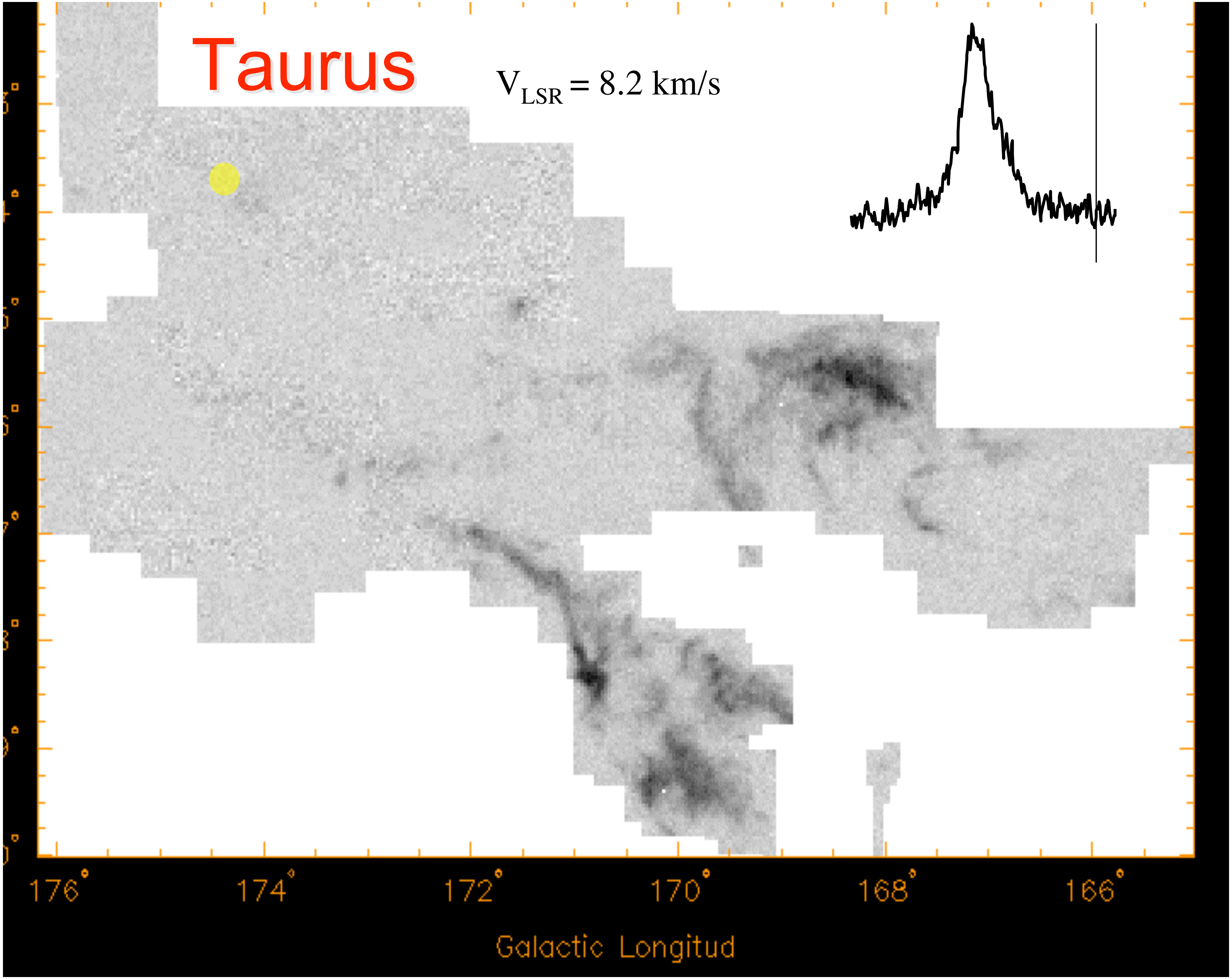

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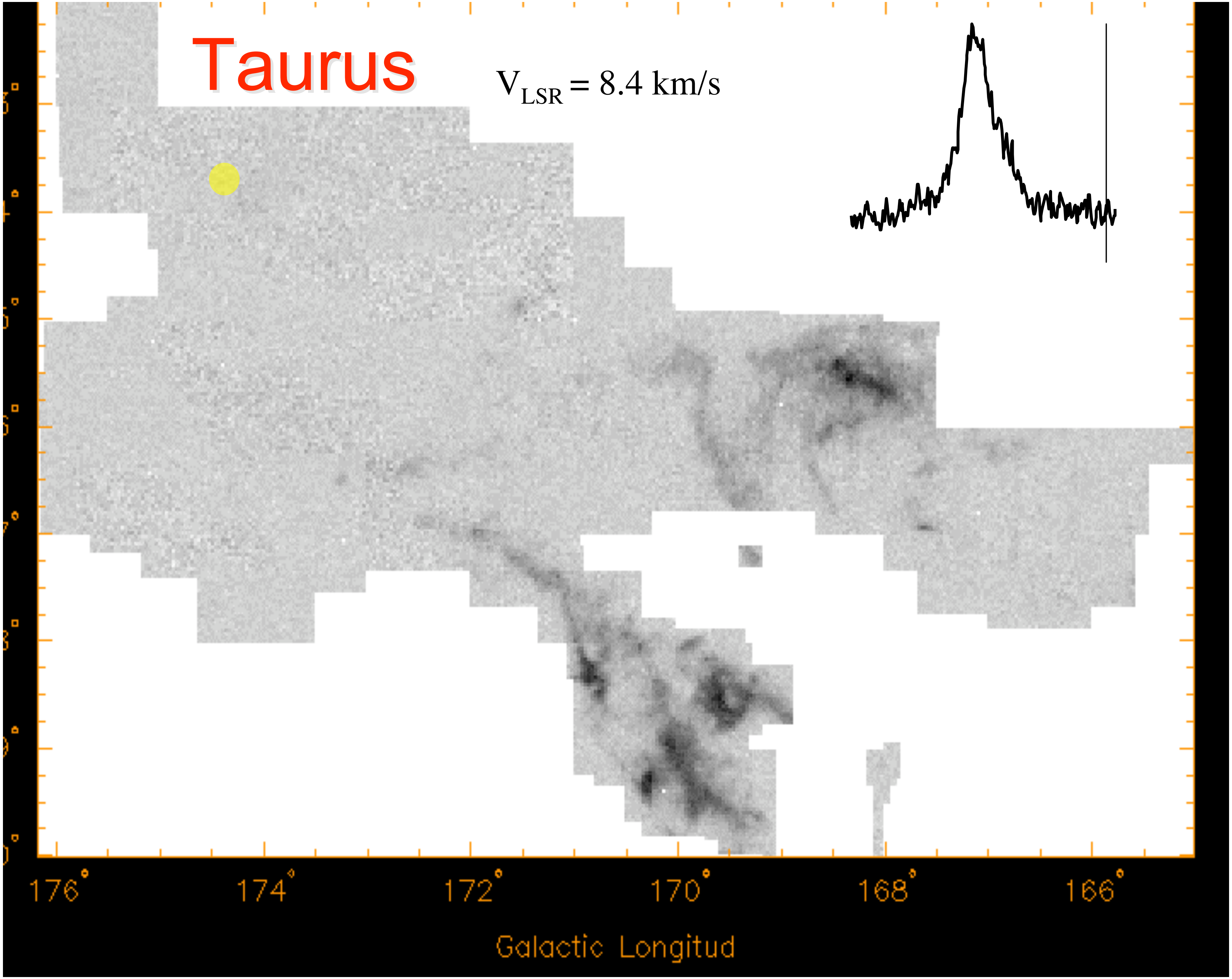
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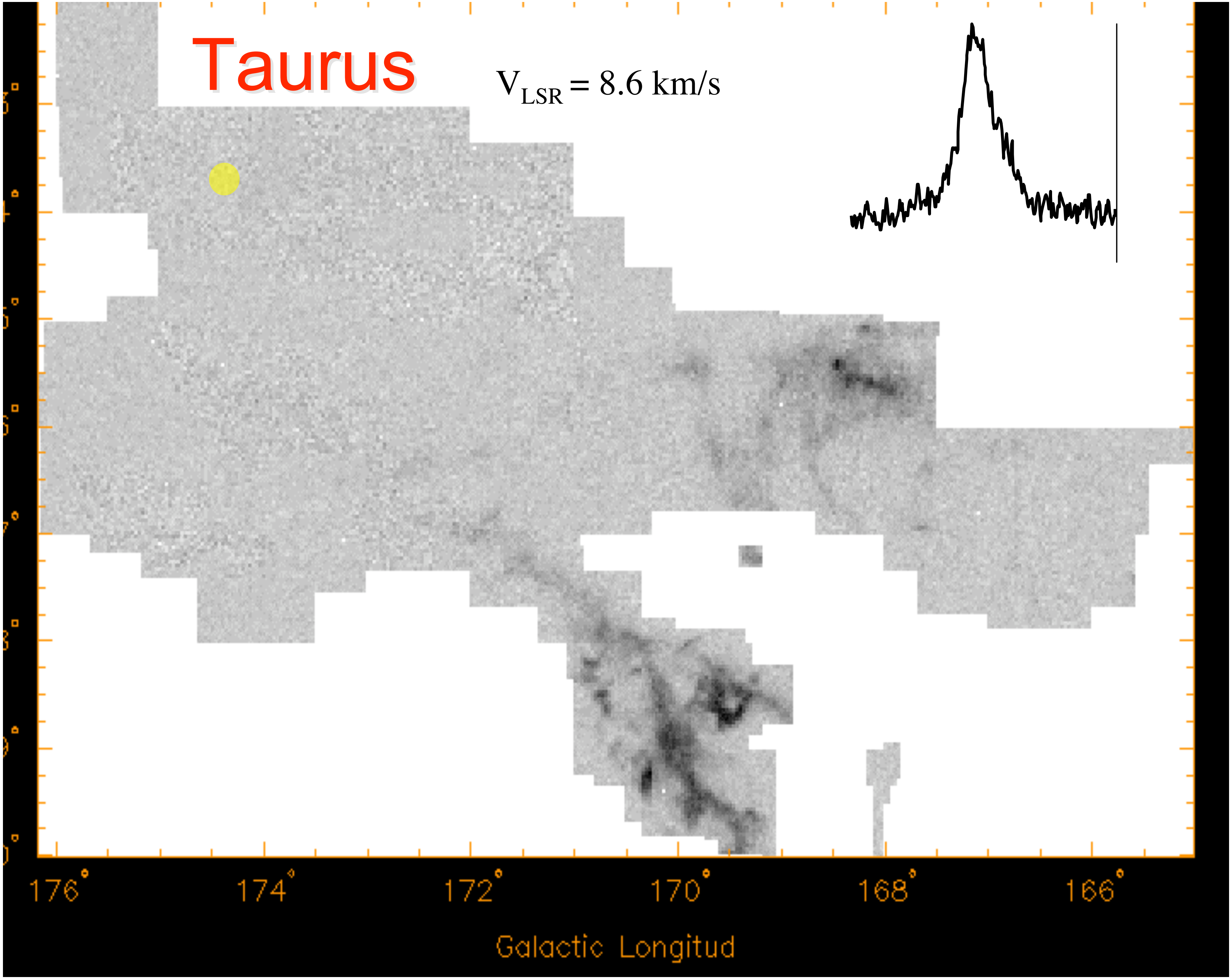

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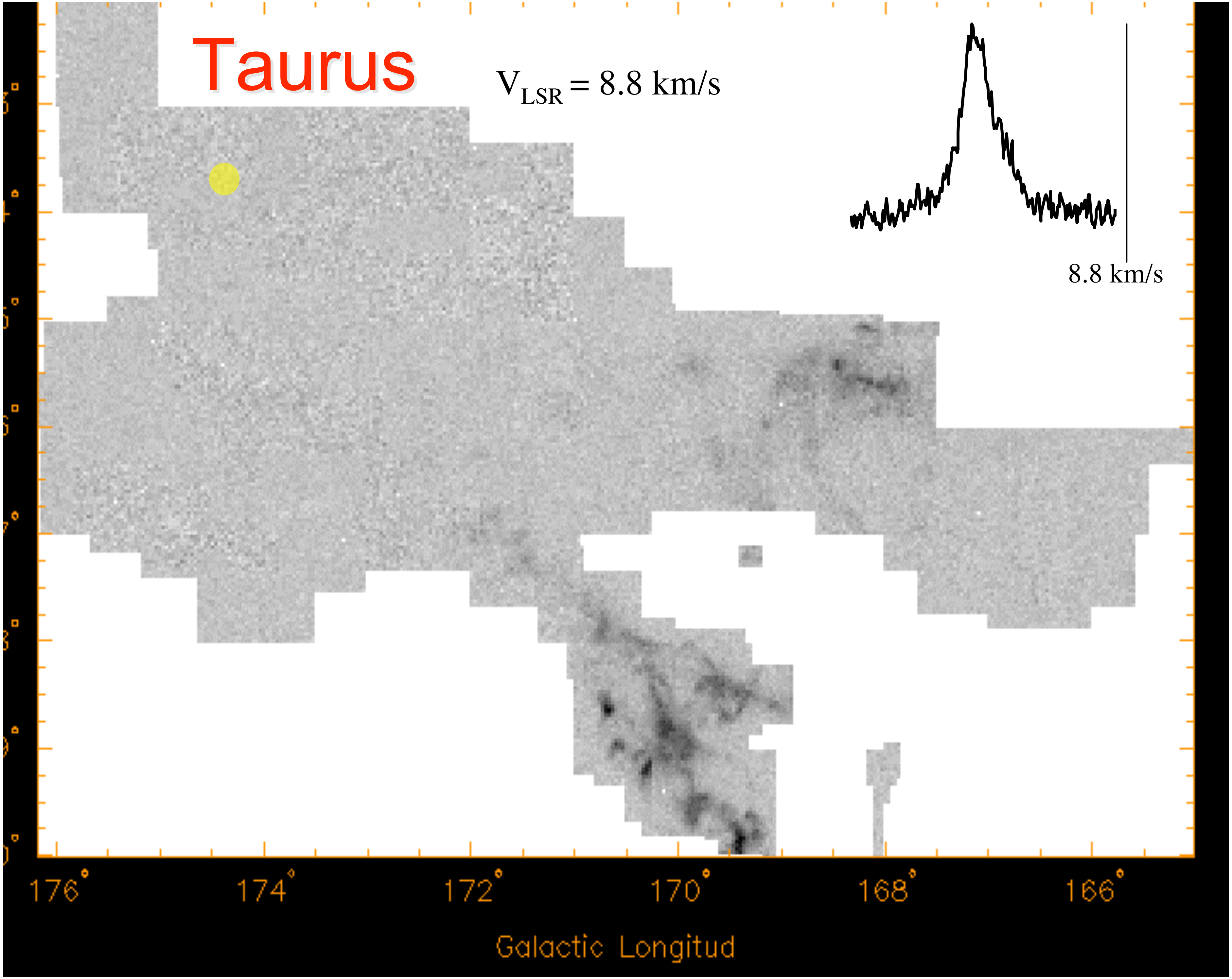
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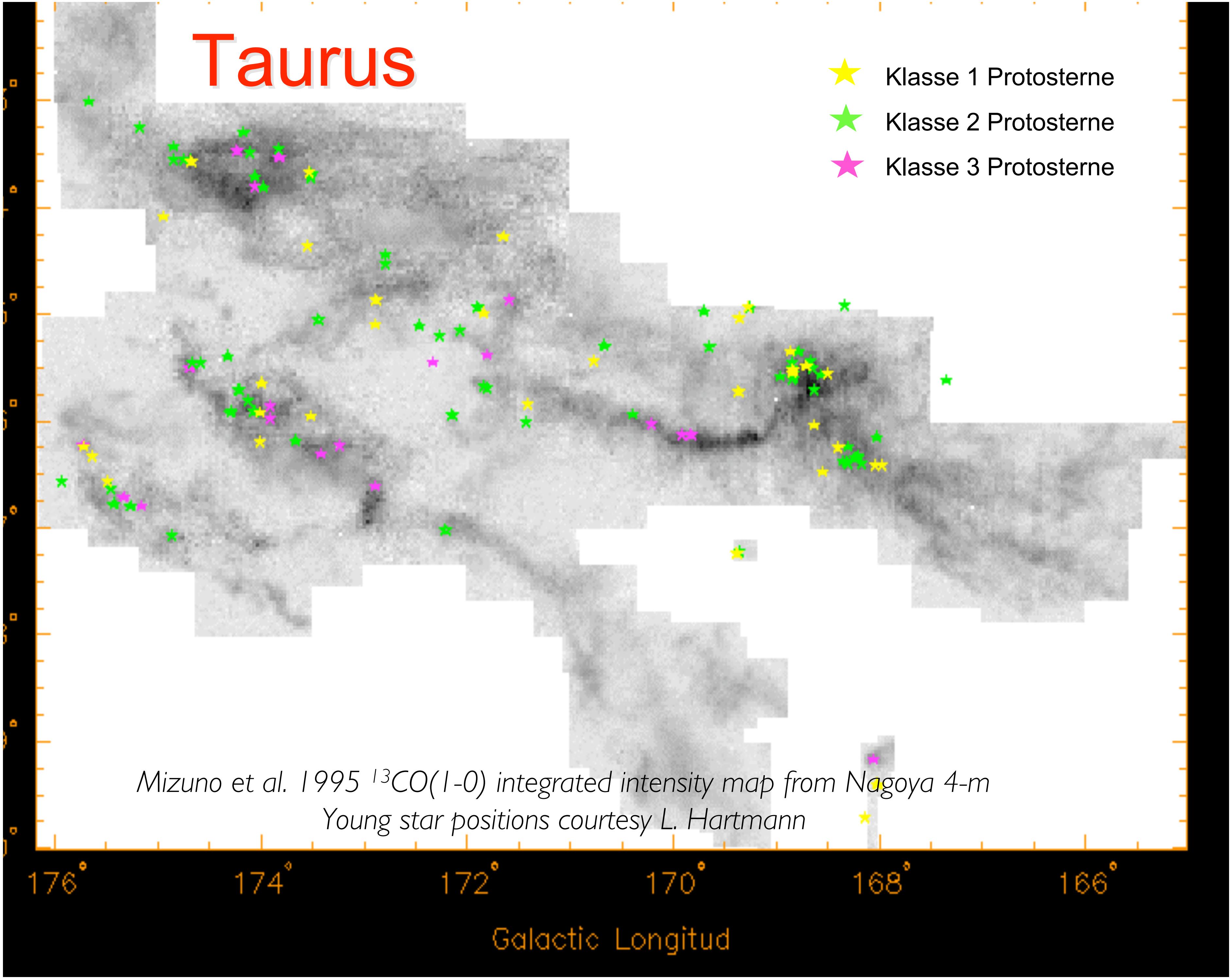

Taurus
Klasse 1 Protosterne
Klasse 2 Protosterne
Klasse 3 Protosterne
Mizuno et al. 1995 ¹³CO(1-0) integrated intensity map from Nagoya 4-m
Young star positions courtesy L. Hartmann
176°
174°
172°
170°
168°
166°
Galactic Longitud

Hydrodynamics

- gases and fluids are *large* ensembles of interacting particles
- $\longrightarrow$ state of system is described by location in $6N$ dimensional phase space $f^{(N)}(\vec{q}_1 ... \vec{q}_N, \vec{p}_1 ... \vec{p}_N) d\vec{q}_1 ... d\vec{q}_N d\vec{p}_1 ... d\vec{p}_N$
- time evolution governed by 'equation of motion' for $6N$-dim probability distribution function $f^{(N)}$
- $f^{(N)} \rightarrow f^{(n)}$ by integrating over all but n coordinates $\longrightarrow$ BBGKY hierarchy of equations of motion (after Born, Bogoliubov, Green, Kirkwood and Yvon)
- physical observables are typically associated with 1- or 2-body probability density $f^{(1)}$ or $f^{(2)}$
- at lowest level of hierarchy: 1-body distribution function describes the probability of finding a particle at time t in the volume element $d\vec{q}$ at $\vec{q}$ with momenta in the range $d\vec{p}$ at $\vec{p}$.
- **Boltzmann equation** – equation of motion for $f^{(1)}$

$$\begin{aligned}\frac{df}{dt} &\equiv \frac{\partial f}{\partial t} + \dot{\vec{q}} \cdot \vec{\nabla}_{q} f + \dot{\vec{p}} \cdot \vec{\nabla}_{p} f \\ &= \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_{q} f + \vec{F} \cdot \vec{\nabla}_{p} f = f_{\mathbf{c}}\end{aligned}$$

- **Boltzmann equation**

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \dot{\vec{q}} \cdot \vec{\nabla}_{q} f + \dot{\vec{p}} \cdot \vec{\nabla}_{p} f$$
$$= \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_{q} f + \vec{F} \cdot \vec{\nabla}_{p} f = f_{\mathbf{c}}.$$

→ first line: transformation from comoving to spatially fixed coordinate system.

→ second line: velocity $\vec{v} = \dot{\vec{q}}$ and force $\vec{F} = \dot{\vec{p}}$

→ all higher order terms are 'hidden' in the collision term $f_{\mathbf{c}}$

- observable quantities are typically (velocity) moments of the Boltzmann equation, e.g.

→ density:

$$\rho = \int m\, f(\vec{q}, \vec{p}, t) d\vec{p}$$

→ momentum:

$$\rho\vec{v} = \int m\vec{v}\, f(\vec{q}, \vec{p}, t) d\vec{p}$$

→ kinetic energy density:

$$\rho\vec{v}^{\,2} = \int m\vec{v}^{\,2}\, f(\vec{q}, \vec{p}, t) d\vec{p}$$

- in general: the i-th velocity moment $\langle \xi_i \rangle$ (of $\xi_i = m\vec{v}^{\,i}$) is

$$\langle \xi_i \rangle = \frac{1}{n} \int \xi_i \, f(\vec{q}, \vec{p}, t) d\vec{p}$$

with the mean particle number density n defined as

$$n = \int f(\vec{q}, \vec{p}, t) \, d\vec{p}$$

- the equation of motion for $\langle \xi_i \rangle$ is

$$\int \xi_i \left\{ \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_{\mathrm{q}} f + \vec{F} \cdot \vec{\nabla}_{\mathrm{p}} f \right\} d\vec{p} = \int \xi_i \left\{ f_{\mathbf{c}} \right\} \, d\vec{p} \, ,$$

which after some complicated rearrangement becomes

$$\frac{\partial}{\partial t} n \langle \xi_i \rangle + \vec{\nabla}_{\mathrm{q}} \left(n \langle \xi_i \vec{v} \rangle \right) + n \vec{F} \langle \vec{\nabla}_{\mathrm{p}} \xi_i \rangle = \int \xi_i f_{\mathbf{c}} \, d\vec{p}$$

(Maxwell-Boltzmann transport equation for $\langle \xi_i \rangle$)

- if the RHS is zero, then ξ_i is a conserved quantity. This is only the case for first three moments, mass $\xi_0 = m$, momentum $\vec{\xi}_1 = m\vec{v}$, and kinetic energy $\xi_2 = m\vec{v}^{\,2}/2$.
- MB equations build a hierarically nested set of equations, as $\langle \xi_i \rangle$ depends on $\langle \xi_{i+1} \rangle$ via $\vec{\nabla}_{\rm q}\left(n\langle \xi_i \vec{v} \rangle\right)$ and because the collision term cannot be reduced to depend on ξ_i only.

  $\longrightarrow$ need for a closure equation

  $\longrightarrow$ in hydrodynamics this is typically the equation of state.

assumptions

- **continuum limit:**
  - $\rightarrow$ distribution function f must be a 'smoothly' varying function on the scales of interest $\longrightarrow$ local average possible
  - $\rightarrow$ stated differently: the averaging scale (i.e. scale of interest) must be larger than the mean free path of individual particles
  - $\rightarrow$ stated differently: microscopic behavior of particles can be neglected
  - $\rightarrow$ concept of fluid element must be meaningful
- **only 'short range forces':**
  - $\rightarrow$ forces between particles are short range or saturate $\longrightarrow$ collective effects can be neglected
  - $\rightarrow$ stated differently: correlation length of particles in the system is finite (and smaller than the scales of interest)

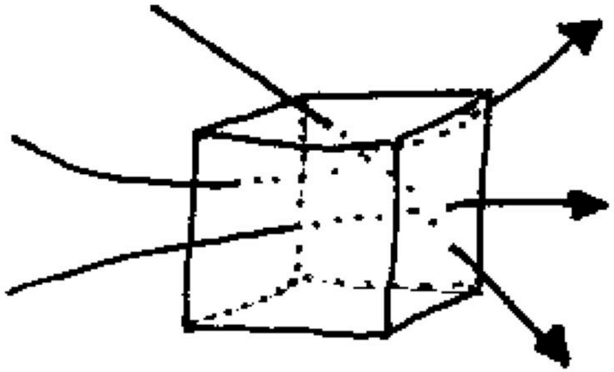
limitations

- shocks (scales of interest become smaller than mean free path)
- phase transitions (correlation length may become infinite)
- description of self-gravitating systems
- description of fully fractal systems

the equations of hydrodynamics

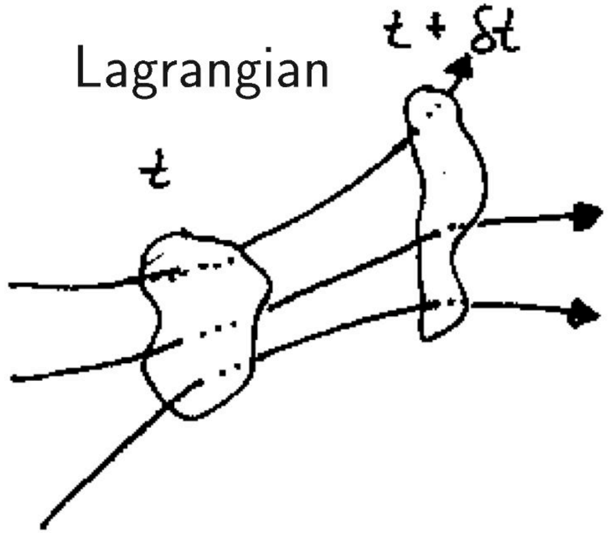
- hydrodynamics ≡ book keeping problem
  One must keep track of the ‘change’ of a fluid element due to various physical processes acting on it. How do its ‘properties’ evolve under the influence of compression, heat sources, cooling, etc.?
- Eulerian vs. Lagrangian point of view

Eulerian



consider spatially fixed volume element

Lagrangian



following motion of fluid element

- hydrodynamic equations = set of equations for the five conserved quantities $(\rho, \rho\vec{v}, \rho\vec{v}^{\,2}/2)$ plus closure equation (plus transport equations for ‘external’ forces if present, e.g. gravity, magnetic field, heat sources, etc.)

- equations of hydrodynamics

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla}\rho = -\rho \vec{\nabla} \cdot \vec{v}$$ (continuity equation)

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p - \vec{\nabla}\phi + \eta\vec{\nabla}^2\vec{v} + \left(\zeta + \frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla} \cdot \vec{v})$$

(Navier-Stokes equation)

$$\frac{d\epsilon}{dt} = \frac{\partial \epsilon}{\partial t} + \vec{v} \cdot \vec{\nabla}\epsilon = T\frac{ds}{dt} - \frac{p}{\rho}\vec{\nabla} \cdot \vec{v}$$ (energy equation)

$$\vec{\nabla}^2\phi = 4\pi G\rho$$ (Poisson's equation)

$$p = \mathcal{R}\rho T$$ (equation of state)

$$\vec{F}_B = -\vec{\nabla}\,\frac{\vec{B}^2}{8\pi} + \frac{1}{4\pi}(\vec{B}\cdot\vec{\nabla})\vec{B} \quad \text{(magnetic force)}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) \quad \text{(Lorentz equation)}$$

ρ = density, $\vec{v}$ = velocity, p = pressure, ϕ = gravitational potential, ζ and η viscosity coefficients, $\epsilon = \rho\vec{v}^{\,2}/2$ = kinetic energy density, T = temperature, s = entropy, $\mathcal{R}$ = gas constant, $\vec{B}$ = magnetic field (cgs units)

- mass transport – continuity equation

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho = -\rho \vec{\nabla} \cdot \vec{v}$$

(conservation of mass)

- transport equation for momentum – Navier Stokes equation

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p - \vec{\nabla}\phi + \eta\vec{\nabla}^2\vec{v} + \left(\zeta + \frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})$$

momentum change due to

→ pressure gradients: $(-\rho^{-1}\,\vec{\nabla}p)$

→ (self) gravity: $-\vec{\nabla}\phi$

→ viscous forces (internal friction, contains $\mathrm{div}(\partial v_i/\partial x_j)$ terms):
$\eta\vec{\nabla}^2\vec{v} + \left(\zeta + \frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})$

(conservation of momentum, general form of momentum transport: $\partial_t(\rho v_i) = -\partial_j \Pi_{ij}$)

◦ transport equation for internal energy

$$\frac{d\epsilon}{dt} = \frac{\partial\epsilon}{\partial t} + \vec{v}\cdot\vec{\nabla}\epsilon = T\,\frac{ds}{dt} - \frac{p}{\rho}\,\vec{\nabla}\cdot\vec{v}$$

→ follows from the thermodynamic relation $d\epsilon = T\,ds - p\,dV = T\,ds + p/\rho^2 d\rho$ which described changes in ϵ due to entropy changed and to volume changes (compression, expansion)

→ for adiabatic gas the first term vanishes ($s =$constant)

→ heating sources, cooling processes can be incorporated in ds

(conservation of energy)

- closure equation – equation of state
  - → general form of equation of state $p = p(T, \rho, ...)$
  - → ideal gas: $p = \mathcal{R}\rho T$
  - → special case – isothermal gas: $p = c_\mathrm{s}^2 T$ (as $\mathcal{R}T = c_\mathrm{s}^2$)

<u>Note:</u>

- in reality, computing the EOS is VERY complex!
- depends on detailed *balance* between *heating* and *cooling*
- these depend on *chemical composition* (which atomic and molecular species, dust)
- and on the ability to radiate away „cooling lines“ and black body radiation
  --> problem of *radiation transfer*

virial theorem

Derivation of virial theorem from momentum equation:

- consider pressure gradients, gravity, magnetic fields,
- neglect viscous forces

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p - \rho \vec{\nabla}\phi - \vec{\nabla}\left(\frac{B^2}{8\pi}\right) + \frac{1}{4\pi}(\vec{B}\cdot\vec{\nabla})\vec{B}$$

pressure term — gravity — magnetic "pressure" — magnetic tension

- in component form:

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} - \rho \frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i}\left(\frac{B^2}{8\pi}\right) + \frac{1}{4\pi} B_j \frac{\partial}{\partial x_j} B_i$$

- multiply by x_i and integrate over volume:
- consider term by term:

① $\int \rho x_i \frac{dv_i}{dt} dV = \int \rho x_i \frac{d^2 x_i}{dt^2} dV$

integrate by parts

$$= \int \rho \frac{d}{dt}\left(x_i \frac{dx_i}{dt}\right) dV - \int \rho \frac{dx_i}{dt}\frac{dx_i}{dt} dV$$

$$= \int \rho \frac{d^2}{dt}\left(\frac{x_i x_i}{2}\right) dV - \int \rho v_i v_i dV$$

$$= \frac{1}{2}\frac{d^2}{dt^2}\int \rho x_i x_i dV - 2\cdot \int \frac{1}{2}\rho v_i v_i dV$$

$$= \frac{1}{2}\ddot{I} - 2T$$

where $I \equiv \int \rho r^2 dV$ is called moment of inertia

[but not quite, because no axis defined.]

and $T \equiv \frac{1}{2}\int \rho v^2 dV$ is the kinetic energy

[note, does not contain random = thermal motions]

(II) $-\int x_i \frac{\partial p}{\partial x_i} dV = -\int \underbrace{\frac{\partial}{\partial x_i}(x_i p)}_{\text{div}(\vec{x}p) \to \text{Gauß}} dV + \int p \frac{\partial x_i}{\partial x_i} dV$

$$= -\oint x_i p \cdot dS_i + 3\int p\, dV$$

$$= -\oint p \vec{r} \cdot d\vec{S} + 2U \quad = -2T_S + 2U$$

where U = thermal energy $= \frac{3}{2}\int p\, dV$

and T_S = surface term of kinetic energy $= \frac{1}{2}\oint p \vec{r}\, d\vec{S}$

(III) $\int \rho x_i \frac{\partial \phi}{\partial x_i} dV = \int \rho \vec{r} \cdot \vec{F}_g \cdot dV$ $\qquad \vec{F}_g = -\vec{\nabla}\phi$ = grav. force

= total potential energy $= W$

↳ neglecting magnetic fields for the moment, we get:

$$\frac{1}{2}\ddot{I} = 2\cdot(T-T_s) + 2U + W$$

scalar virial theorem

- virial theorem describes fundamental relation between morphological parameters (I = tensor of inertia, W = tensor of potential energy) and kinematic quantities (tensor of kinetic energy $K = T + U - T_s$)

- in equilibrium: $\ddot{I} = 0$!

  ↳ $$\boxed{2K + W = 0}$$

  $K = T + U$, neglecting surface effects T_s

  ↳ the system is called <u>virialized.</u>

- total energy: $$\boxed{E_{tot} = K + W}$$

  in virial equilibrium it follows

  $$\boxed{E_{tot} = K + W = \frac{W}{2} = -K}$$

  virialized systems are always bound with binding energy equal to $-K$.

Virial Theorem applied to MC dynamics

- derivation & definition via effective pressure (= energy density; unit = erg/cm^3)

  * trace of tensor of inertia: $I = \int r^2 dm$
  * evaluate $\ddot{I} = \frac{d^2 I}{dt^2}$ from equation of motion
  * $$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \frac{1}{4\pi} (\vec{\nabla} \times \vec{B}) \times \vec{B} + \rho \vec{g}$$

    $p = p_{th} + p_{turb}$; magn. field ; gravity
  * $$\longrightarrow \boxed{\ddot{I} = 2\left[T - T_S\right] + M + W}$$

    $2[T - T_S]$: E_{kin} ; M: E_{magn} ; W: E_{pot}

- kinetic energy = from thermal (random) motion + bulk (turbulent) motion

* $$\boxed{\mathcal{T}} = \int_{\text{cloud volume}} \left(\frac{3}{2} P_{th} + \frac{1}{2}\rho v^2\right) dV \stackrel{!}{=} \boxed{\frac{3}{2}\bar{P}\, V_{cl}}$$

  mean pressure ($\frac{3}{2} P_{th} = \frac{3}{2}\rho c_s^2$) — cloud volume

* relate $\bar{P}$ to (observable) velocity dispersion along LOS

$$\sigma_{1D}^2 \equiv \frac{1}{M}\int \left(c^2 + \frac{1}{3}v^2\right) dM = \frac{\bar{P}}{\bar{\rho}}$$

* surface term (often neglected, but may be important in interstellar turbulence)

  here we assume cloud is embedded in ambient medium with constant pressure P_s

$$\boxed{\mathcal{T}_S} = \frac{1}{2}\int_{\text{surface}} P_s\, \vec{r}\, d\vec{s} = \boxed{\frac{3}{2} P_s V_{cl}}$$

* $$\boxed{2(\mathcal{T} - \mathcal{T}_S) = 3(\bar{P} - P_s)\, V_{cl}}$$

- magnetic energy:

$$* \quad \mathcal{M} = \frac{1}{8\pi} \int_{\text{volume}} B^2 dV + \frac{1}{4\pi} \int_{\text{surface}} \vec{r} \cdot \left(\vec{B} \cdot \vec{B} - \frac{1}{2} B^2 \mathbb{1} \right) \cdot d\vec{S}$$

* if stresses in ambient medium small → assume field is force free there, then

$$\boxed{\mathcal{M} = \frac{1}{8\pi} \int_{\text{volume}} \left(B^2 - B_0^2 \right) dV}$$

B: fieldstrength in cloud

B_0: –"– far from cloud

NB:
$1 \text{ gauss} = 1 \sqrt{\frac{g}{cm\, s^2}}$
$= 10^{-4}$ Tesla

- gravitational energy:

* $$\mathcal{W} = \int_{\text{Volume}} \rho \, \vec{r} \cdot \vec{g} \, dV = -\frac{3}{5} \xi \left(\frac{GM^2}{R} \right)$$ with $\xi \approx 1$

* now: define "gravitational" pressure:

$$\mathcal{W} \equiv -3 \, p_g \cdot V_{cl}$$

* intuitively, p_g is the mean weight of the material in the cloud

$$p_g = \frac{3\pi \xi}{20} G \Sigma^2 \longmapsto 1{,}4 \; 10^5 \left(\frac{\bar{N}_H}{10^{22} \text{cm}^{-2}} \right)^2 \; \text{K} \cdot \text{cm}^{-3}$$

with $\Sigma = \frac{M}{\pi R^2} = \mu_H \bar{N}_H$ and evaluated for cloud with $n_H(R) \propto 1/R$.

- in steady state (virial equilibrium): $\ddot{I} = 0$

↳ $$\bar{P} = P_s + P_g \cdot \left(1 - \frac{\mathcal{M}}{|\mathcal{W}|}\right)$$

i.e.: mean pressure inside cloud is surface pressure plus weight of material inside cloud reduced by magnetic stresses (reduction factor $\mathcal{M}/|\mathcal{W}|$)

- total energy:

$$E = \mathcal{T} + \mathcal{M} + \mathcal{W}$$
$$= \frac{3}{2}\left(P_s + P_g\left[1 - \frac{\mathcal{M}}{|\mathcal{W}|}\right]\right) \cdot V_{cl}$$

$E < 0$ bound

$E > 0$ unbound

- are MC's bound?

* in solar vicinity $p \approx 2{,}8 \cdot 10^4$ K cm^{-3}

  ↳ $0{,}7 \cdot 10^4$ K cm^{-3} due to cosmic rays
  $0{,}3 \cdot 10^4$ K cm^{-3} due to B-field

  ↳ $p_S \approx 1{,}8 \cdot 10^4$ K cm^{-3}

* what is minimum p_g?
  visual extinction in order to get H_2
  $A_V \gtrsim 2$

  ↳ $p_g \gtrsim 2 \cdot 10^4$ K cm^{-3}

* ⇒ MC's are <u>at least</u> marginally bound.

  Typically: $A_V \gg 2$

  (typical values: $p_g \approx 2 \cdot 10^5$ K cm^{-3}

- filling factor: MC's are highly clumped

* mean densities: $n_H \approx 10^3 \dots 1{,}2\ 10^4\ cm^{-3}$ ⇒ say $n_H \approx 3000\ cm^{-3}$
* with $M \propto \langle n_H \rangle R^3$
* and $\langle N_H \rangle \propto \langle n_H \rangle \cdot R$

} we have $\langle n_H \rangle = 84\ cm^{-3} \left(\frac{M}{10^6 M_\odot}\right)^{-1/2} \left(\frac{\langle N_H \rangle}{1{,}5 \cdot 10^{22} cm^{-2}}\right)$

↳ filling factor: $$f \equiv \frac{\langle n_H \rangle}{n_H} \approx 0{,}084 \cdot \left(\frac{M}{10^6 M_\odot}\right)^{-1/2} \left(\frac{n_H}{10^3 cm^{-3}}\right)^{-1/2} \left(\frac{\langle N_H \rangle}{1{,}5 \cdot 10^{22} cm^{-2}}\right)$$

PS: Clouds $M \leq 10^4 M_\odot$ must have $n_H > 10^3\ cm^{-3}$ to have the column densities found by Solomon et al.

* difference between mass weighted and volume weighted gas distribution

- Magnetic Field vs. Gravity

↳ derivation of magnetically critical mass

* total energy: $E = T + \mathcal{M} + W$

$$= \frac{3}{2}\left[P_s + P_g\left(1 - \frac{\mathcal{M}}{|W|}\right)\right]$$

* collapse possible if $E < 0$, and assuming (incorrectly, though!) $P_s \equiv 0$ ⟶ Collapse if $1 - \frac{\mathcal{M}}{|W|} < 0$!

* recall: $\mathcal{M} = \frac{1}{8\pi} \int_{\text{volume}} (B^2 - B_0^2)\, dV$

$$= \frac{1}{8\pi} \int \bar{B}^2 dV = \left(\frac{\zeta}{3}\right) \bar{B}^2 R^3$$

$\zeta \approx 1$

Magnetic flux: $\Phi = \int_{\text{fläche}} 2\pi R B dR = \pi R^2 \cdot \bar{B}$

↳ $$\mathcal{M} = \left(\frac{\zeta}{3\pi^2}\right) \frac{\Phi^2}{R}$$

$\Rightarrow$ $\mathcal{M} = \left(\frac{\xi}{3\pi^2}\right) \frac{\Phi^2}{R}$

also: $W = \int_{\text{Volume}} \rho\, \vec{r} \cdot \vec{g}\, dV = -\frac{3}{5}\, \xi \left(\frac{GM^2}{R}\right)$ $\xi \approx 1$

* $\Rightarrow$ critical value: $\mathcal{M}\big|_{crit} = |W|$

$$\frac{\xi}{3\pi^2} \cdot \frac{\Phi^2}{R} = \frac{3}{5}\, \xi\, \frac{GM^2}{R}$$

note the same radial dependency!

$\Rightarrow$ $M_{crit} = \left(\frac{5\xi}{9\pi^2\xi}\right)^{1/2} \cdot \frac{\Phi}{G^{1/2}}$

$C_\Phi \approx \frac{1}{2\pi}$

in the absence of ambipolar diffusion a mag. subcritical cloud will remain so forever.

$$M_{crit} \approx \left(4 \cdot 10^6\, M_\odot\right) \cdot \left(\frac{n(H_2)}{1\,\text{cm}^{-3}}\right)^2 \cdot \left(\frac{B}{3\,\mu G}\right)^3$$

* critical mass-to-flux ratio:

$$\left(\frac{M}{\Phi}\right)_{crit} \stackrel{!}{=} \left(\frac{5\,\zeta}{9\pi^2\,\xi}\right) G^{-1/2} = 0{,}16 \ldots 0{,}18 \cdot G^{-1/2}$$

depending on geometry and further details

$1\,G = 1 \cdot g^{1/2}\,cm^{-1/2}\,s^{-1}$ [B]

$1\,esu = 1 \cdot g^{1/2}\,cm^{3/2}\,s^{-1}$ [q]

Grav. const. $G = 6{,}67 \cdot 10^{-8}\ cm^3\,g^{-1}\,s^{-2}$

Observations:

typical molecular cloud cores are magnetically supercritical with

$$\left(\frac{M}{\Phi}\right) \approx 2 \ldots 3 \times \left(\frac{M}{\Phi}\right)_{crit}$$

(Crutcher 1999, Bourke et al. 2001)

B versus ***N(H₂)*** from *Zeeman measurements*.

(from Bourke et al. 2001)

→ cloud cores are

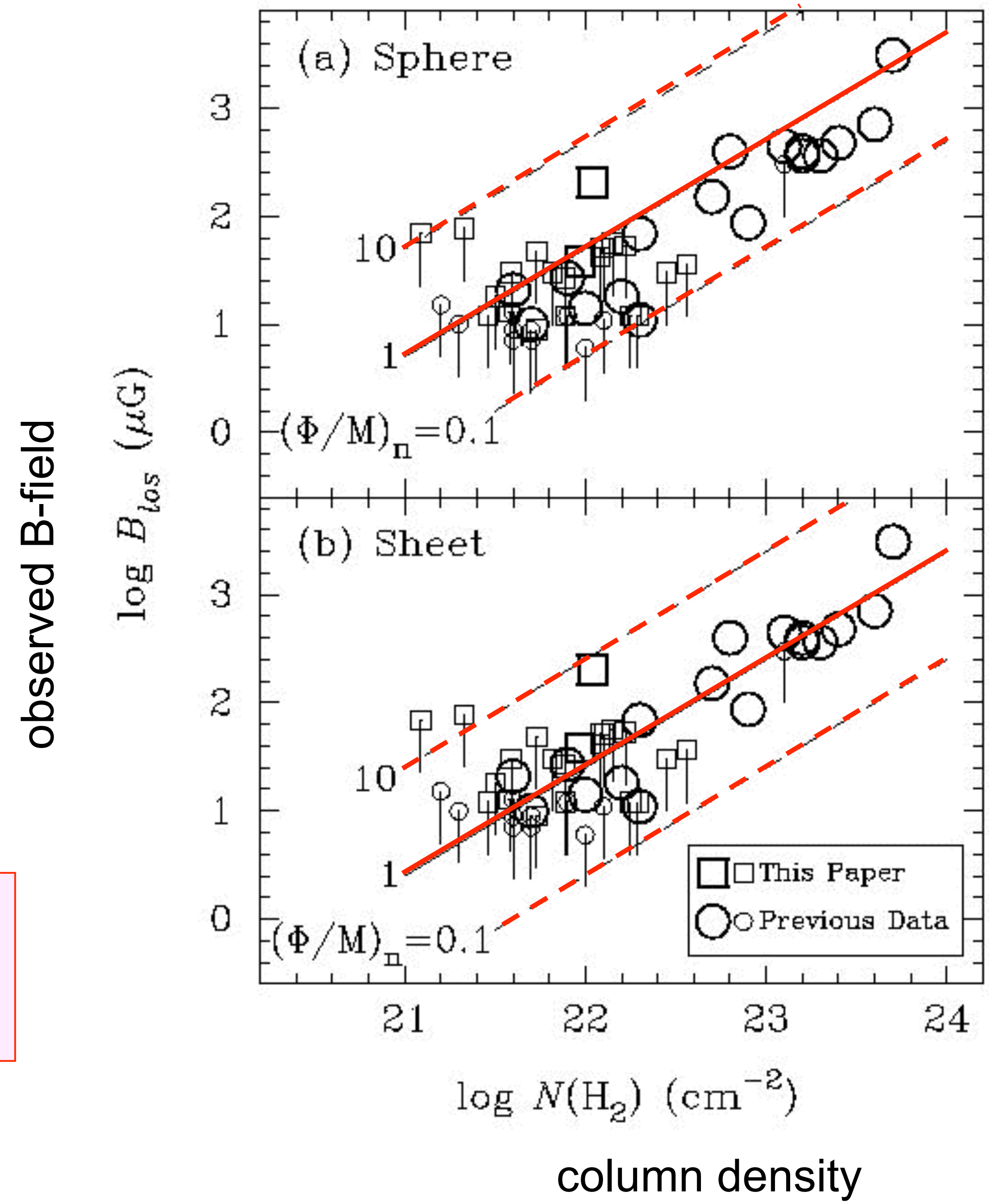
marginally

magnetically

supercritical***!!!***

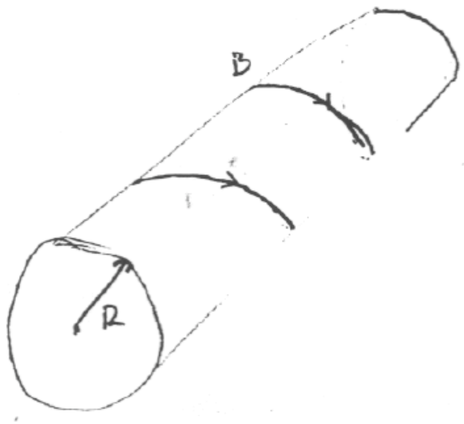
(Φ/M)$_n$ > 1 no collapse

(Φ/M)$_n$ < 1 collapse



NON-ideal MHD: diffusion between neutral gas and charged particles ⟶ <u>ambipolar diffusion</u>

- only charged particles couple directly to the B field
- neutrals "see" the field only via collisions with electrons & ions!
- example:



- estimate timescale for drift across cylinder with radius R with typical bend of $\vec{B}$-field of order R.
- then Lorentz force $F_L \approx \frac{B^2}{4\pi R}$

from momentum equation: Balance between Lorentz force and i-n-drag:

$$\frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B} = \alpha\rho_i\rho_n\underbrace{(\vec{v}_i - \vec{v}_n)}_{\vec{v}_D}$$

coupling coefficient $\alpha = \frac{\langle\sigma v\rangle}{m_i + m_n} \approx 9.2\cdot 10^{13}$

$$\Rightarrow \tau_{AD} = \frac{R}{v_D} = \left[\frac{4\pi\alpha\rho_i\rho_n R}{(\vec{\nabla}\times\vec{B})\times\vec{B}}\right] \approx \frac{4\pi\alpha\rho_i\rho_n R^2}{B^2}$$

$$\tau_{AD} = \frac{R}{v_D} = \left[\frac{4\pi\alpha\rho_i\rho_n R}{(\vec{\nabla}\times\vec{B})\times\vec{B}}\right] \approx \frac{4\pi\alpha\rho_i\rho_n R^2}{B^2}$$

$$\boxed{\tau_{AD} \approx 25\,\mathrm{Myr} \left(\frac{B}{3\mu G}\right)^{-2} \left(\frac{n(H_2)}{10^2\,\mathrm{cm}^{-3}}\right)^2 \left(\frac{R}{1\,\mathrm{pc}}\right)^2 \left(\frac{x}{10^{-6}}\right)}$$

compare with gravitational free-fall time:

$$\boxed{\tau_{ff} = \left(\frac{3\pi}{32\,G}\right)^{1/2} \rho^{-1/2}}$$

define: $\gamma = \frac{\tau_{AD}}{\tau_{ff}}$ $\Rightarrow$ typically $\gamma \approx 10$ in MC's !

Jeans condition

Gravitational instability:

Jeans criterion

- often the first approach to determine stability properties is to analyze the <u>linearized</u> set of equations and derive a <u>dispersion relation</u> for the perturbation assumed.

- Linearized eqn.'s for isothermal self-gravitating fluid:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \vec{\nabla} \vec{v}_1 = 0 \qquad \text{continuity}$$

$$\frac{\partial \vec{v}_1}{\partial t} = -\vec{\nabla} c_s^2 \frac{\rho_1}{\rho_0} - \vec{\nabla}\phi_1 \qquad \text{momentum}$$

$$\vec{\nabla}^2 \phi_1 = 4\pi G \rho_1 \qquad \text{Poisson}$$

▲ $\vec{\nabla} c_s^2 \frac{\rho_1}{\rho_0} = \frac{1}{\rho_0} \vec{\nabla} p_1$ with $p_1 = c_s^2 \rho_1$ from EOS.

▲ neglecting viscous effects $(\eta = \xi = 0)$

▲ equilibrium characterized by $\rho_0 = \text{const.}$ and $\vec{v}_0 = 0$

▲ Jeans swindle: Poisson's eqn considers only <u>perturbed</u> <u>potential</u> ($\rightarrow$ set $\phi_0 = 0$)

- with $\frac{\partial}{\partial t}$ [continuity] + $\vec{\nabla}$ [momentum] it follows:

$$\frac{\partial^2 \rho_1}{\partial t^2} - c_s^2 \vec{\nabla}^2 \rho_1 - 4\pi G \rho_0 \rho_1 = 0$$

↳ wave equation for $\rho_1(\vec{x}, t)$

- analyze in Fourier space:

$$\rho_1(\vec{x}, t) = \int d^3k \, A(\vec{k}) \, e^{i[\vec{k}\vec{x} - \omega(k)t]}$$

$\frac{\partial}{\partial t} \mapsto i\omega \cdot$

$\vec{\nabla} \mapsto i\vec{k} \cdot$

- dispersion relation:

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

- ▲ if density ρ_0 is small ⟼ disp. rel. of sound waves $\omega^2 = c_s^2 k^2$
- ▲ or small wavelength $\lambda = \frac{2\pi}{k}$
- ▲ self-gravity acts "strongest" on large scales (small k)
  [gravity is long-range force]
- ▲ λ increases / k decreases / ρ_0 grows: frequency decreases and eventually $\omega^2 < 0$!

  ↳ time evolution $\propto \exp(\pm \alpha t)$. (if $\alpha^2 = -\omega^2$)

  Exponentially unstable.

- $\longmapsto$ Gravitational collapse for wave numbers

$$k^2 < k_J^2 \equiv \frac{4\pi G \rho_0}{c_s^2}$$

▼ k_J : Jeans wavenumber

▼ λ_J = Jeans wavelength $= \frac{2\pi}{k_J} = \left(\frac{\pi c_s^2}{G\rho_0}\right)^{1/2}$

▼ M_J = Jeans mass $= \frac{4\pi}{3}\rho_0\left(\frac{\lambda_J}{2}\right)^3 = \frac{\pi}{6}\rho_0\left(\frac{\pi c_s^2}{G\rho_0}\right)^{3/2}$

for a spherical perturbation with $\varnothing = \lambda_J$

$\hookrightarrow$

$$M_J = \frac{\pi^{5/2}}{6}\left(\frac{R}{G}\right)^{3/2} \cdot \rho_0^{-1/2}\, T^{3/2}$$

$$c_s^2 = RT$$

$$= \frac{\pi^{5/2}}{6} G^{-3/2} \cdot \rho_0^{-1/2} c_s^3$$

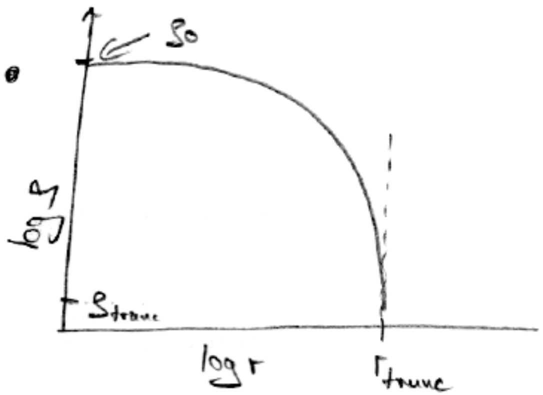
- Energy of sound wave $E_{sound} > 0$; gravitational Energy < 0

$\hookrightarrow$ Instability sets in, when net energy is negative, i.e. when λ exceeds λ_J.

Bonnor-Ebert spheres

(4) Isothermal equilibria of (pressure bounded) self-grav. spheres

- force balance ⟶ static sln ⟶ $\boxed{\vec{\nabla} p = -\rho \vec{\nabla} \Phi}$
- ideal gas: $p = c_s^2 \rho$ with isothermal sound speed $c_s^2 = \frac{R}{\mu} T$
- spherical symmetry: equ. of motion $\frac{c_s^2}{\rho} \frac{d\rho}{dr} = -\frac{d\Phi}{dr}$
- ↦ integration ⟶ $\boxed{\rho = \rho_0 \exp(-\Phi/c_s^2)}$ hydrostatic equ.
- include Poisson's equ.: $\boxed{\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = 4\pi G \rho = 4\pi G \rho_0 \exp\left(-\frac{\Phi}{c_s^2}\right)}$ *

  Lane-Emden equation
- regular sln's from $\Phi = 0$ & $\frac{d\Phi}{dr} = 0$ at $r = 0$
- 

if $\frac{\rho_0}{\rho_{trunc}} > 14$ only unstable equilibria possible!

- Singular isothermal sphere if $\rho_0/\rho_{trunc} \to \infty$
(or equivalently, if outer edge $r_{trunc} \to \infty$; or $p_{ext} \to 0$)

↳ SIS: $\rho = \frac{c_s^2}{4\pi G r^2}$ & $\frac{d\phi}{dr} = \frac{2c_s^2}{r}$

Shu (1977) assumes SIS & $v=0$; but to reach SIS, system evolves through a sequence of unstable equilibria
↳ collapse sets in much earlier → SIS with $v=0$ will never be reached.

- solving *): define $\xi = \frac{r}{c_s}\sqrt{4\pi G \rho_0}$ $\quad \rho_0$ = core density

↳ $$\frac{d}{d\xi}\left(\xi^2 \frac{d\psi}{d\xi}\right) = \xi^2 e^{-\psi} \quad **)$$ $\quad \psi = \ln \rho/\rho_0$

sln of **) with $\Psi(0) = 0$ & $\frac{d\Psi(0)}{d\xi} = 0$ are finite at the center $\xi = 0$.

↳ further change of variables:

$$y_1 = \xi^2 \frac{d\Psi}{d\xi}$$

$$y_2 = \Psi$$

↳ coupled set of 1. order ODE's:

$$\frac{dy_1}{d\xi} = \frac{y_0}{\xi^2}$$

$$\frac{dy_0}{d\xi} = \xi^2 \exp(-y_1)$$

boundary conditions $y_0(0) = 0$ & $y_1(0) = 0$.

↳ ∃ family of solutions characterized by parameter

$$\xi_{max} = \frac{r_{trunc}}{c_s}\sqrt{4\pi G \rho_0}$$

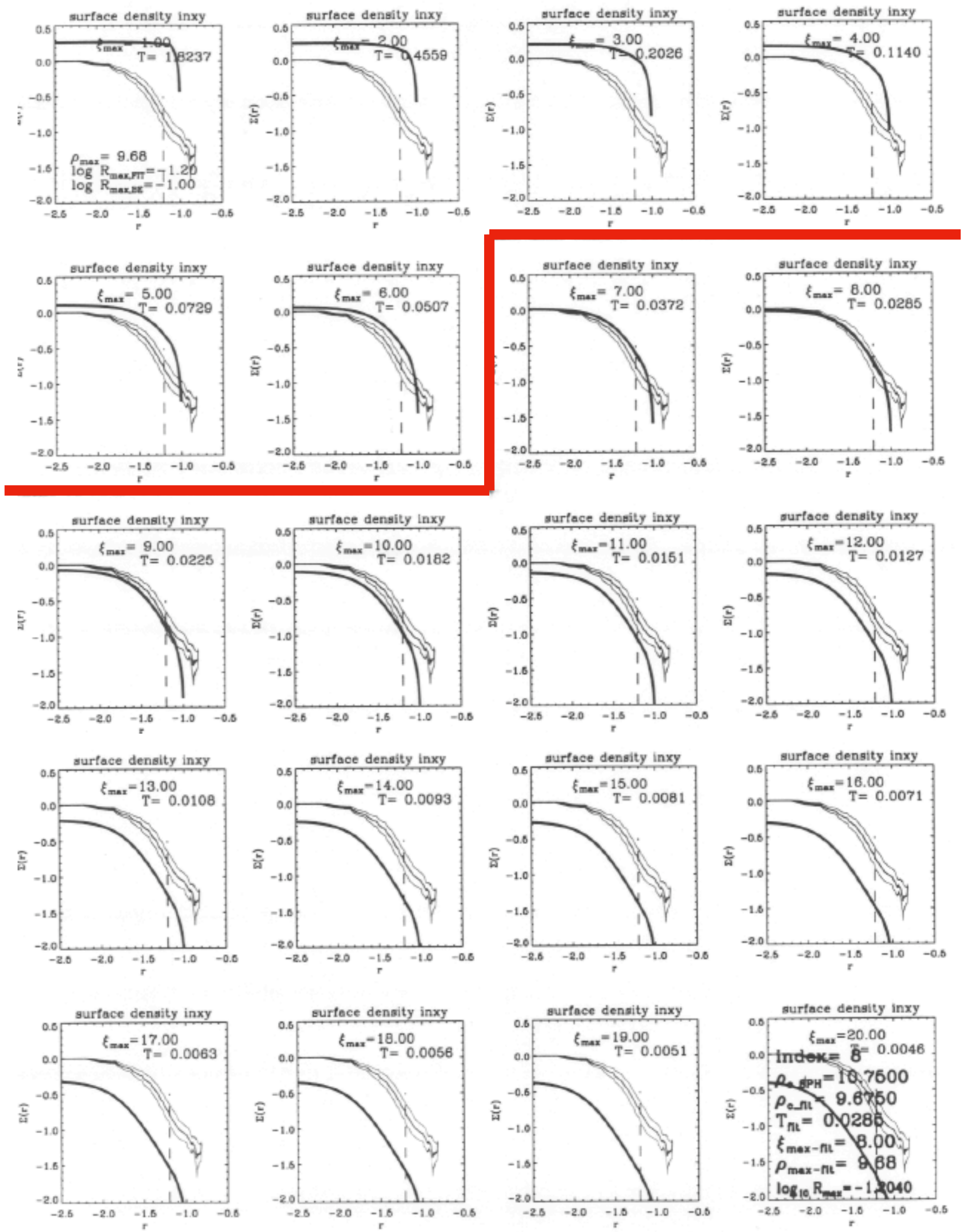
ξ_{max} = value of ξ at outer boundary r_{trunc}

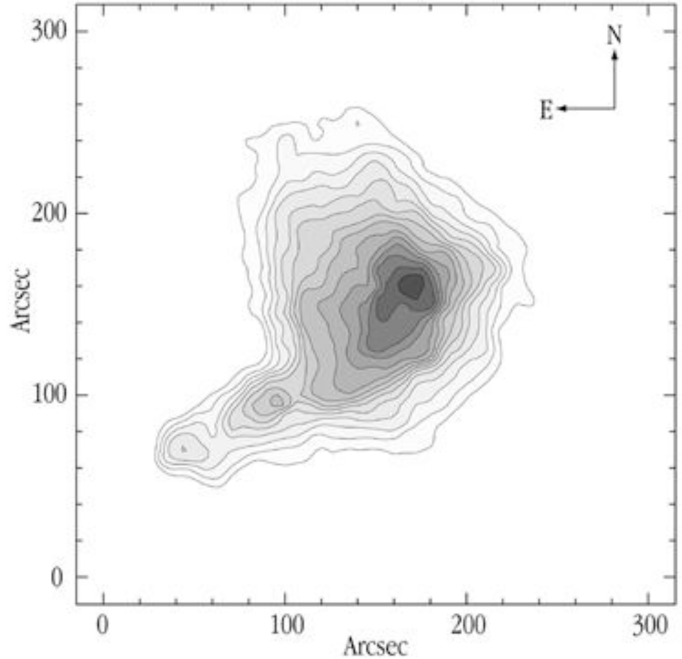
$\xi_{max} > 6{,}5$ unstable equilibria : collaps

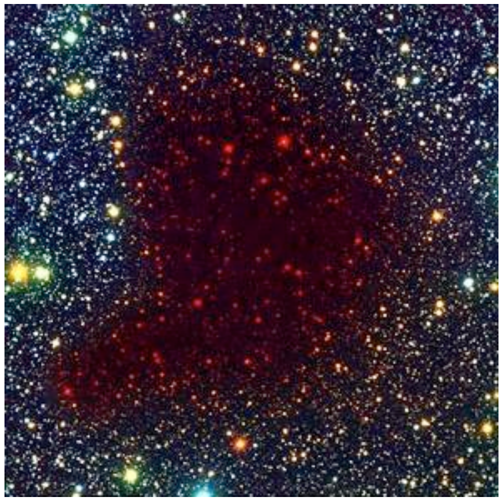
$\xi_{max} < 6{,}5$ stable s/u

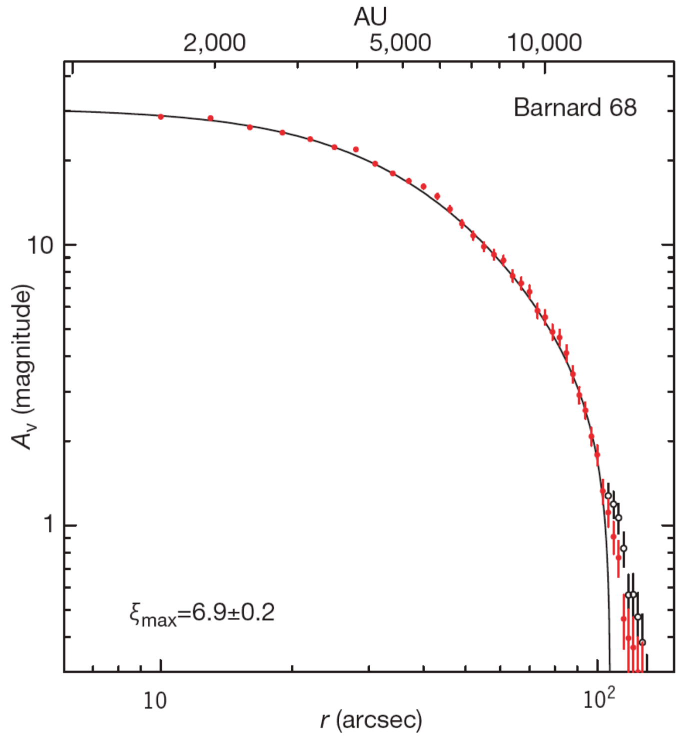
ZAH Zentrum für Astronomie Heidelberg ARI•ITA•LSW stable

unstable



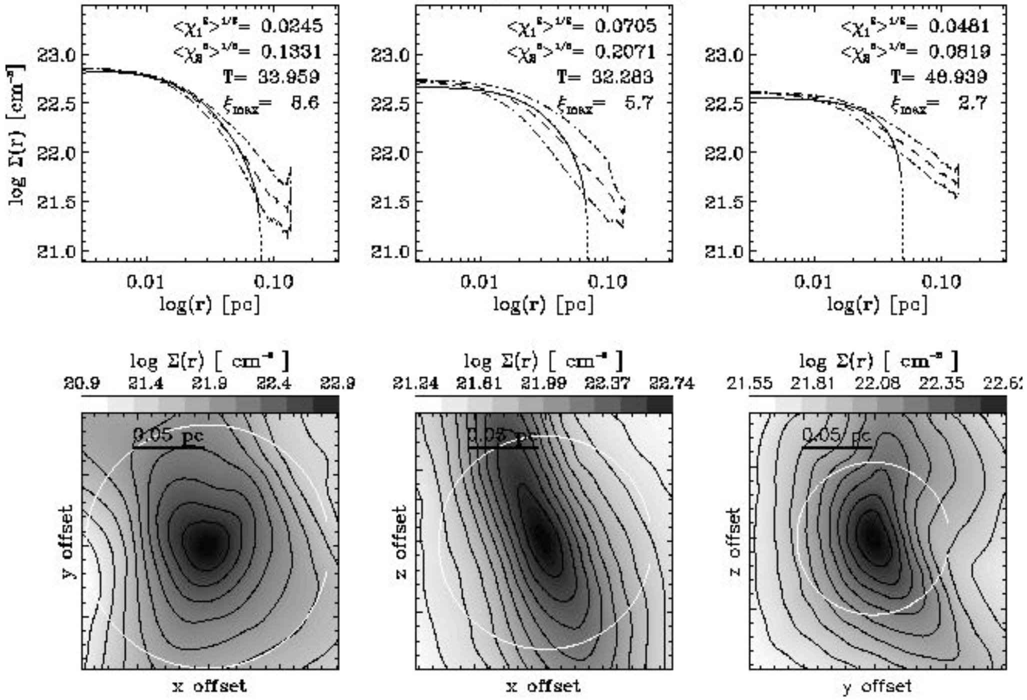


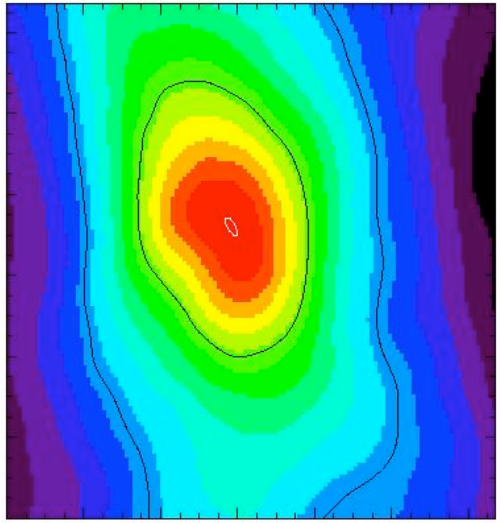




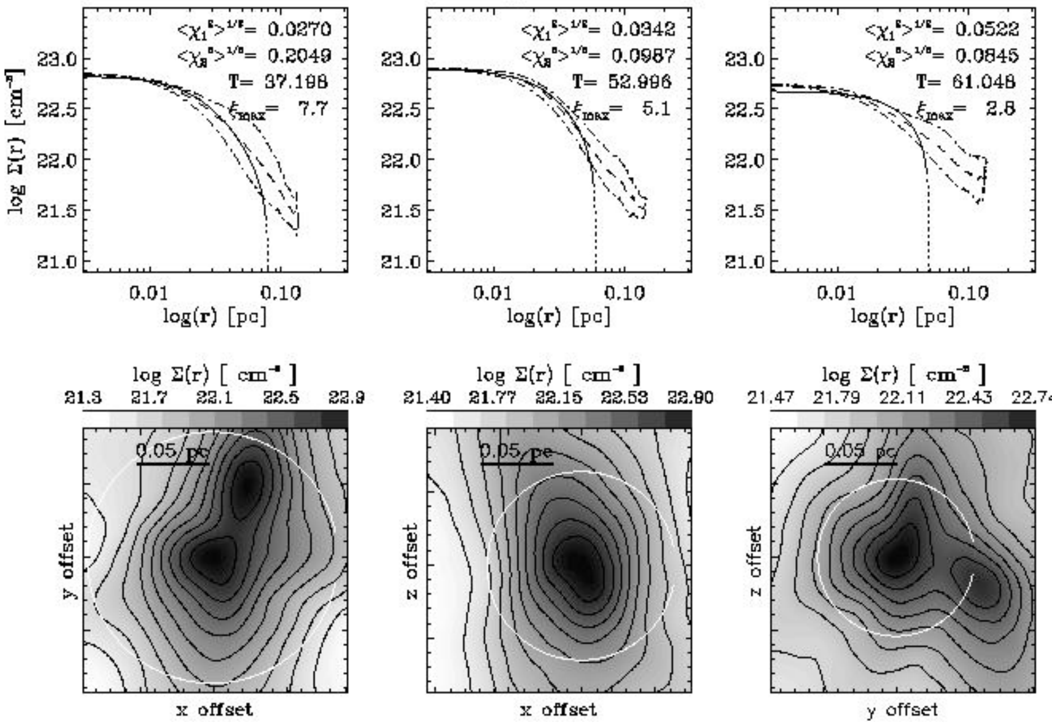
Alves, Lada, Lada (2001)

GC clump 26 time t_1





GC clump 04 time t_0



recommended literature

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- Star Formation:
  - Stahler, S., & Palla, F., 2004, "The Formation of Stars" (Weinheim: Wiley-VCH)
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  - Mannings, V., Boss, A. P., & Russell, S. S. 2000, "Protostars and Planets IV" (University of Arizona Press, Tucson)
- General:
  - Binney, J., & Tremaine, S. 1987, "Galactic Dynamics" (Princeton University Press)
  - Shu, F. 1991, "The Physics of Astrophysics I: Radiation" (University Science Books, Mill Valley)
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  - Landau, L. D., & Lifschitz, E. M. 1986, "Hydrodynamik" (Akademie Verlag, Berlin)
- Review articles:
  - Mac Low, M.-M., Klessen, R.S., 2004, "The control of star formation by supersonic turbulence", Rev. Mod. Phys., 76, 125 - 194.
  - Bromm, V., Larson, R.B., 2004, "The first stars", ARAA, 42, 79 - 118
  - Larson, R.B., 2003, "The physics of star formation", Rep. Prog. Phys, 66, 1651 - 1697
  - Shu, F.H, Adams, F.C., Lizano, S., 1987, "Star formation in molecular clouds: Observations and theory", ARAA, 25, 23 - 81