Introduction to Cosmology

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Based in large part on the script by M. Bartelmann [1]

[1] http://www.ita.uni-heidelberg.de/research/bartelmann/files/cosmology.pdf

Chapter 1

Introduction

1.1 A brief history

If we look at the night sky, the most conspicuous thing to observe is also the one that we always take for granted: that the night sky is black. There are, of course, stars in the sky, but their combined radiation is not enough to read a book by. At first sight this may seem natural, but it isn't. If you stand in a forest, and the forest is large enough, then no matter where you look, your sight is blocked by a tree. If the Universe is infinitely large, then no matter where we look, our sight should be blocked by a star. If the average temperature of these stars is, say, 6000 K, then our entire sky should look like the surface of a star of about 6000 K. In other words, we would fry. Since the sky is dark, something must be wrong with this argument. This is called "Olbers' Paradox". It shows that our Universe cannot be a static, infinitely large and infinitely old Universe. A better model of the Universe must be constructed.

The history of modern cosmology starts with the discovery that the Universe is expanding. The spectra of galaxies were found to be redshifted, and that this redshift increases with the galaxy's distance. The most obvious interpretation was that there exists a linear relation between velocity and distance. Hubble (1929, Astrophysical Journal 74, 43) is customarily credited with discovery, because his observations established this relation beyond reasonable doubt. In fact, the expansion coefficient is called the "Hubble constant". However, the book by Nussbaumer & Bieri (2009, "Discovery of the expanding Universe", Cambridge University Press) points out that a tentative discovery of this velocity-distance relation was already made by Wirtz (1922, Astronomische Nachrichten 215, 349) and Lundmark (1925, Monthly Notices of the Royal Astronomical Society 85, 865). Even earlier, Slipher (1917 Proc. Amer. Phil. Soc., 56, 403) already detected the redshift of spectral lines in "spiral nebulae" before they were proven by Hubble (1925) to be extragalactic objects.

In the same decade, Alexander Friedmann found a solution to the equations of Einstein's theory of General Relativity that describes the universe (1922, Zeitschrift für Physik 10, 377; 1924, Zeitschrift für Physik 21, 326). The spatial geometry of that universe is either flat, or positively or negatively curved. And most importantly: It expands. At first, Einstein did not believe in the expansion predicted by these solutions and dismissed them in an article. He later realized his mistake and rehabilitated Friedman in the same journal. George Lemaître, a Belgian monk and physicist, was apparently unaware of Friedmann's solution, but he was well aware of the observational evidence of receding galaxies. In an article written in the somewhat obscure journal "Annales de la Société Scientifique de Bruxelles" in 1927 (an English version can be found on the web) he presented a similar model as Friedmann's to explain these receding motions. In the 1930s it was shown by Robertson in 1933 and Walker in 1934 that the spatial geometry assumed by Friedmann and Lemaître (flat, or positively/negatively curved) are in fact the only homogeneous and isotropic geometries possible, and thereby showing that the Friedmann-Lemaître equations (with a cosmological constant) give a complete description of the dynamics of the universe. Indeed, these models are still valid and in active use today!

If the universe is expanding, it is logical (though not necesssarily imperative) to assume that at some finite time in the past all the matter in the universe must have converged into a single point, or in other words that the universe was born out of a huge point explosion. Fred Hoyle was not particularly fond of this idea and called this scathingly "the Big Bang" in a radio interview. It has been called the Big Bang ever since.

So, how do we verify if this model is correct, and how do we know how the contents of the Universe developed over time? Fortunately for us, the finite light travel time acts as a kind of "time machine": By looking at the sky with ever bigger and better telescopes we can look at objects ever farther away, and as a consequence, further back in time. In fact, we can look back in time to the point where the Universe was just 3×10^{-5} times the present age, as we shall see later.

Going back in time there must therefore be a point in time when the universe was extremely dense. Using the laws of adiabatic compression, we must conclude that the universe must also have been extremely hot in these early days of its existence. It turns out that in about the first 20 minutes of the Universe's existence the temperatures and densities were so high that neutrons and protons were formed and paired up to form (in addition to Hydrogen of course) Deuterium, Helium and a bit of Lithium. The predictions from these primordial cosmic abundances by this theory of "Big Bang Nucleosynthesis" are very well confirmed by observations. All heavier elements that are around today are all made by stars.

Apart from these elemental abundances, is there some other evidence of the Big Bang? We know that hot dense matter tends to produce a lot of thermal radiation, like the surface of the sun. When the universe became tenuous and cool enough for electrons to recombine with the H and He atoms, the universe become transparent (roughly half a million years after the Big Bang). This radiation would continue to flow through space. Gamov, Alpher and Herman realized in the 1940s that this radiation should still be all around us today, albeit redshifted to very low temperatures: just a few Kelvin. In the 1960s the group around Dicke at Princeton University essentially rediscovered that such low temperature background radiation must exist, and started creating the necessary equipment to detect it. Unfortunately for them, another team accidently stumbled on this radiation before Dicke's team found it. Arno Penzias and Robert Wilson were technicians of Bell Labs who were testing new radio equipment. They were trying to get rid of a perpetual noise, but whatever they tried, they could not get rid of it. They happened to work at Crawford Hill in New Jersey, which is just a few miles from Princeton, so they called Dicke to ask for advice. When Dicke put up the phone he is famously quoted as saying "Boys, we have been scooped". Penzias and Wilson won the Nobel Prize for their discovery of the cosmic microwave background radiation (often abbreviated as CMB). This radiation is still today the most distant signal that we can observe with telescopes, dating from redshift $z \simeq 1100$, when the universe was only half a million years old. Compare that to the current age of the universe, 13.7 ± 0.13 *billion* years, this means that the CMB is really a relic from the infant universe.

Interestingly, the CMB is very homogeneous. The temperature is 2.725 Kelvin, no matter into which direction you look. The *current* universe is, however, highly structured, at least on scales of galaxy clusters and smaller. So how did the universe get so structured? And what seeded these structures? If the universe was born perfectly homogeneously, then no structures could ever form. So it was a huge discovery when the COBE Satellite detected tiny perturbations on the CMB in the early 1990s, making George Smoot and John Mather Nobel laureats in 2006 (the latter for the proof that the CMB is a near-perfect blackbody curve). These anisotropies were tiny: on scales of a few μK! But they are the signatures of the initial seeds that created the rich structure of the cosmos we have today. Later measurements with the BOOMERanG balloon observatory and the Wilkinson Microwave Anisotropy Probe (WMAP) space telescope have since dramatically improved the measurements of these anisotropies, allowing detailed analysis of their spatial power spectra and thereby narrowing down many of the previously unknown characteristics of the universe. Currently the Planck space telescope is redoing this at even higher precision.

Now, what caused these anisotropies in the first place? For that, we have to go to *much* earlier times: to times around 10^{-36} seconds after the Big Bang. Before that time it is believed that several of the fundamental forces of Nature were unified into a single "Grand Unified Theory". Then, according to current theories, a period of extremely rapid exponential growth followed, known as "inflation". This period lasted from roughly 10^{-36} seconds until about 10^{-32} seconds after the Big Bang during which the volume of the Universe expanded by about a factor of 10^{80}. This inflation period had several effects: (a) regions in the universe became mutually "disconnected", (b) spatial curvature got flattened away, (c) inhomogeneities were mostly washed out. But during this period quantum fluctuations in the material density also got "frozen in" as the universe expanded so rapidly. Tiny fluctuations at the quantum level could thus cause large scale effects. Also, because of the many orders of magnitude expansion, while quantum fluctuations kept occurring, the macroscopic perturbations they caused should be on all scales: The earlier fluctuations having expanded further and being responsible for the larger scale perturbations. The power spectrum of these perturbations therefore are expected to be scale-free (Harrison-Zel'dovich-Peebles power spectrum). If this theory is correct, then we also expect the anisotropies of the CMB to be Gaussian: the δT should have a probability distribution that is a Gauss curve. This is a prediction from quantum field theory. So far the CMB appears to be consistent with Gaussianity. The search for non-Gaussianity is on-going.

Going back to the era after the production of the CMB, around half a million years after the Big Bang. The first small perturbations had already reached the few percent level and continued to grow by gravitational contraction. Most of the matter is in the form of "Dark Matter", i.e. presumably weakly interacting massive particles that form a kind of collisionless self-gravitating "fluid". In fact, only roughly 1/6 of the matter is thought to be baryonic matter (stars, planets, gas, dust etc), while 5/6 of the matter in the universe is thought to be in the form of this collisionless (cold) dark matter (often written as CDM). The formation of structure is therefore mostly a matter of the non-linear evolution of a self-gravitating collisionless fluid in which the initial seed perturbations are imprinted. Large scale simulations such as the Millennium Simulation (Springel, 2005, Nature 435, 629) then follow the non-linear evolution in detail. What comes out is the formation of a "cosmic web" with huge voids, walls, filaments and massive galaxy clusters. However, initially these structure do not produce any light because no stars have yet formed. The period between the CMB and the first stars is often called the "Dark Ages". The first stars are thought to have formed roughly a few hundred million years after the Big Bang. These stars produced so many UV photons that they started to reionize the gas in the universe. One of the hot topics today is to search for indications of this reionization process happening, as it would give evidence for early star formation as well as the structure of the early universe and the end of the Dark Ages. In particular the LOFAR radio array currently under construction, and the future SKA array, will search for neutral hydrogen from the Dark Ages through measurements of the redshifted 21 cm hydrogen fine structure line.

It will be very difficult, if not impossible, to directly observe the first stars themselves during their formation. But there are many indirect pieces of evidence, such as very low metallicity stars that are still present today. But the search for the highest redshift galaxies is of course still very much ongoing.

Time after BB	z	T	Event
$< 10^{-43}$ sec			Planck era
10^{-43}-10^{-35} sec			GUT era
10^{-35}-10^{-32} sec			Inflation
10^{-32}-10^{-12} sec			Electroweak epoch
10^{-12}-10^{-6} sec			Quark epoch
10^{-6}-10^{-1} sec			Hadron epoch
0.2 sec	1.15×10^{10}	3.2×10^{10} K	Freeze-out of $\nu + \bar{\nu} \leftrightarrow e^+ + e^-$
2 sec	3.3×10^9	9.3×10^9 K	Freeze-out of n+$\nu_e \leftrightarrow$p+e$^-$
40 sec	8×10^8	2×10^9 K	Freeze-out of $e^+ + e^- \leftrightarrow 2\gamma$
3 - 5 min	$\sim 3 \times 10^8$	$\sim 8 \times 10^8$ K	Nucleosynthesis
60×10^3 year	3233	8590 K	Matter starts dominating
$\sim 300 \times 10^3$ year	1100±80	~3100 K	Recombination (Origin of CMB)
$0.3 \cdots 400$ Myr			"Dark Ages"
~ 0.4 Gyr	~ 11	~ 30 K	Re-ionization by first stars
~ 0.65 Gyr	~ 8	~ 25 K	Farthest known galaxies (Sep 2011)
9.17 Gyr	0.43	3.9 K	Formation of Earth
9.6 Gyr	0.39	3.8 K	Dark energy starts dominating
13.74 Gyr	0	2.725 K	Today

Table 1.1: Time line of the Universe, computed according to the Friedmann model using the parameters from Jarosik et al. (2011), see table 4.1. Information about the very early Universe can be found at e.g. `http://en.wikipedia.org/wiki/Timeline_of_the_Big_Bang`.

In 1998 a new chapter in cosmology was written with the discovery that the Universe is currently expanding at an accelerated rate. By observing and studying type Ia supernovae at high redshifts it was found that the expansion is faster than expected. Apparently the Universe must be permeated with a mysterious quantity called "dark energy". On Tuesday, October 4, 2011 it was announced that Saul Perlmutter, Brian Schmidt and Adam Riess won the Nobel Prize of Physics 2011 for this discovery. The implication of this discovery is that our Universe will not recollapse, but will expand forever. The current standard model of cosmology, which includes this dark energy, is called the "Λ-CDM model".

1.2 Some fundamental concepts

Before we are going to start with developing a model of the Universe we must first introduce a few concepts.

1.2.1 Redshift

If we make a spectrum of a galaxy and find that the location of the spectral lines has been shifted by

$$z = \frac{\lambda_{\rm observed} - \lambda_{\rm real}}{\lambda_{\rm real}} \tag{1.1}$$

then we call z the *redshift* of this galaxy. The ratio of the energy of the photon as it is observed $\nu_{\rm observed}$ to when it was emitted $h\nu_{\rm real}$ is

$$\frac{h\nu_{\rm observed}}{h\nu_{\rm real}} = \frac{1}{1+z} \tag{1.2}$$

You can see why the radiation is redshifted in two ways. The easiest is to realize that if you send a photon from a moving object, it will be Doppler-shifted with respect to a non-moving object. This explains redshift for the near Universe, where $z \ll 1$, but for galaxies at $z \gtrsim 1$ this classical concept is not self-consistent. A special-relativisitic treatment would be necessary. However, a much easier way to understand it is if one regards the expansion of the Universe as a real expansion of space itself. If a galaxy emits an electromagnetic wave with wavelength λ_{real} propagating through space, but by the time it arrives in our telescope space has been stretched by a factor A then the wavelength is also stretched by this factor: $\lambda_{obs} = A\lambda_{real}$.

1.2.2 The Hubble constant

If we can find a reliable way of determining the distance to the galaxies we observe (and this is not an easy task!), then we can make a direct connection between the redshift and the distance (at least for the near Universe for which $z \ll 1$). This turns out to be linear, and can thus be written as:

$$D = \frac{c}{H_0} z \tag{1.3}$$

where H_0 is called the *Hubble constant*. The Hubble constant is a way to express the expansion rate of the Universe. If we write the redshift z as a velocity $z = v/c$ where v is the velocity by which the galaxy we observe is moving away from us, then

$$H_0 = \frac{v}{D} \tag{1.4}$$

i.e. the Hubble constant is the velocity of recession per unit distance. Currently this value is thought to be $H_0 = 70\,\mathrm{km/s/Megaparsec}$ with reasonable confidence. The unit of H_0 is 1/second, so it gives already a rough estimate of the age of the universe: $1/H_0 = 13.9\,\mathrm{Gyr}$. This simple estimate is called the *Hubble time*. If you multiply this with the light speed c you get a distance called the *Hubble distance* (also called *Hubble radius* which is $r_H = c/H_0 = 4.2\,\mathrm{Gpc}$.

The word "Hubble constant" is a bit misleading. The "constant" refers to the fact that the observed ratio v/D appears not to vary with D. But if we follow the evolution of the Universe over its entire life time, then $H(t)$ is not constant at all. That's why for the present-day Hubble constant we write H_0 with index 0. We will do this for many other variables as well.

It is customary to introduce a constant h defined as

$$H_0 = 100\,h\,\mathrm{km/s/Mpc} \tag{1.5}$$

With the current estimate being $h = 0.7$.

1.2.3 Scale factor $a(t)$

If we wish to describe the Universe how it was long ago, then it is convenient to introduce a parameter $a(t)$ which is the scale parameter of the Universe. At the present time we define it as $a(t_0) = 1$. At earlier times it is $a < 1$ and at the Big Bang it is $a(0) = 0$. If the current distance between us and some object is D, then at some earlier time t this distance was $a(t)D$, i.e. smaller than the present-day distance. The expansion of the Universe can therefore be conveniently described by $a(t)$. The main task of a model of the Universe is to find an expression for the expansion function $a(t)$.