Chapter 8

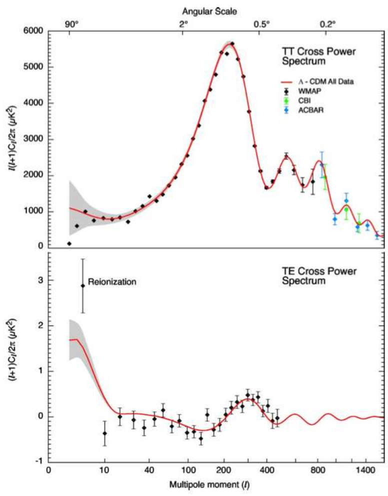
Observing the CMB

By observing the CMB sky at high angular resolution it has become possible to derive several of the most important parameters of the Universe. There is a degeneration in the derived parameters, so it is important to supplement the CMB data with other observational constraints. And, of course, the analysis requires a good model of the expansion of the Universe and the processes shaping the CMB.

From the fact that non-linear structures exist today in the Universe, the linear growth theory predicts that density perturbations at $z = 1100$ (the time of CMB release) must have been of the order of

$$\delta(a_{\text{CMB}}) = \frac{\delta(a = 1)}{D_+(a_{\text{CMB}})} \gtrsim 10^{-3} \tag{8.1}$$

Currently we know that non-linear structures in the Universe already existed at redshifts $z = 10$ or even higher, so this imples that $\delta(a_{\text{CMB}}) \gtrsim 10^{-2}$ at least at small scales.



After the CMB was found in 1965, fluctuations were sought at the relative level of 10^{-3}, but they were not found. Eventually they were found at a level of 10^{-5}. The reason is that already before the CMB release the DM perturbations started growing independently. While the radiation-Baryon fluid oscillated and therefore didn't grow in amplitude, the DM perturbations continued to grow. Before the DM dominated the mass (i.e. $z \gtrsim 3300$) this growth was slow (logarithmic), while once DM dominate the mass the growth was linear. Since DM has no coupling to the electromagnetic spectrum, nor to the Baryons, this growth happened without pumping the perturbations in the CMB to equal levels. In fact, this can be seen as a proof of the existence of such a DM as a non-interacting form of matter.

In this chapter we will investigate how the CMB perturbations at a level of 10^{-5} in fact do appear and which effects shape their power spectrum.

8.1 Analysis of the CMB sky with spherical harmonics

Since we observe the CMB on the sky, which is a sphere, we have to use spherical harmonics instead of plane waves to do a "Fourier analysis". We observe the temperature fluctuation as a function of angular position on the sky $\delta T(\vec{\theta})$. The decomposition in spherical harmonics is then:

$$\delta T(\vec{\theta}) = \sum_{lm} a_{lm} Y_l^m(\vec{\theta}) \tag{8.2}$$

in which coefficients a_{lm} are complex. The spherical harmonics form an orthonormal basis:

$$\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \, Y_{l_1}^{m_1 *}(\theta,\varphi) Y_{l_2}^{m_2}(\theta,\varphi) \delta_{l_1 l_2} \delta_{m_1 m_2} \tag{8.3}$$

and the coefficients a_{lm} are given by

$$a_{lm} = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \, \delta T(\theta,\varphi) Y_l^{m*}(\vec{\theta}) \tag{8.4}$$

The power spectrum is then defined by

$$C_l = \left\langle |a_{lm}^2| \right\rangle \tag{8.5}$$

Note that C_l only depends on l, because the index m stands for directional dependence. Since the CMB is isotropic, this directional dependence should vanish.

Note, however, that because the sun and the Earth are moving relative to the CMB radiation, we observe a dipole on the sky. This is simply the doppler shift due to our motion. We remove this dipole before we analyse the CMB.

A more meaningful quantity would, however, be $l(l+1)C_l$, because it gives the total power in the multipole l. This is shown in the figure.

8.2 The last scattering surface

As we saw in Chapter 7, the opacity that makes the early Universe opaque is electron scattering. Electron scattering is not an *emission* process, and thus is not expected to thermalize the radiation field. It only changes the direction of photons (more on that in Section 8.3.4). When the Universe recombines around $z \simeq 1100 \cdots 1300$ the free electrons needed for electron scattering get depleted, and each photon thus experiences a "last scattering". As we shall see, this happens at slightly different z depending on how deep this event happens inside a gravitational potential well. On the sky we can translate this in a (slightly) varying comoving distance. This defines a surface around us on the sky at $D_{\mathrm{com}} \simeq 1.43 \times 10^{10}$ parsec with slight dimples in it.

8.3 The effects shaping the CMB power spectrum

As we already mentioned in Chapter 7, the temperature of the CMB radiation drops as $T \propto a^{-1}$ before *and* after the last scattering surface. The presence or absense of the Baryons therefore does not appear to change the temperature. Perturbations in the density of Baryons would therefore also not do this. So one may wonder why there are *any* temperature perturbations observable at all. Indeed, the reasons for the temperature fluctuations are a bit subtle. Let us discuss them in this section.

8.3.1 Sachs-Wolfe Effect

Suppose we are looking at a point on the last scattering surface that happens to be in a gravitational potential well (compared to the average potential). The temperature fluctuations due to the so-called Sachs-Wolfe effect (do not confuse this with the *integrated* Sachs-Wolfe effect) are due to two competing effects: (1) the redshift experienced by the photon as it climbes out of the potential well toward us and (2) the delay in the release of the radiation, leading to less cosmological redshift compared to the average CMB radiation.

The first contribution leads to a redshift of the order of:

$$\frac{\delta T_1}{T} = \frac{\delta\Phi}{c^2} \tag{8.6}$$

Verification: Since in our notation a potential well corresponds to $\delta\Phi < 0$, we indeed get redshift: $\delta T_1 < 0$.

The second contribution is more tricky, and a proper treatment would require a general relativistic approach. Loosely it works as follows. The CMB radiation is set free when the ionization parameter x has dropped to less than one percent, which happens when the temperature drops below about 3000 K. This is only a function of temperature, not of density. Now, because of general relativity, the proper time goes slower inside the potential well than outside. The cooling of the gas in this potential well thus also goes slower, and it therefore reaches 3000 K at a later time relative to the average Universe. The time delay (in terms of global time t) is:

$$\frac{\delta t}{t} = -\frac{\delta\Phi}{c^2} \tag{8.7}$$

This means that 3000 K is reached at a slightly larger (global) scale parameter $a+\delta a > a$. Since in the Einstein-de-Sitter Universe we have $a \propto t^{2/3}$ we can write

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t} = -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{8.8}$$

Now, from that point $a = (a_{\rm cmb} + \delta a)$ until today $a = 1$ the redshift due to expansion is less by:

$$\frac{\delta z}{z} = -\frac{\delta a}{a} \tag{8.9}$$

which leads to a positive contribution to the temperature fluctuation δT that we observe today:

$$\frac{\delta T_2}{T} = -\frac{\delta z}{z} = \frac{\delta a}{a} = -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{8.10}$$

The total is the sum of both contributions:

$$\frac{\delta T}{T} = \frac{\delta T_1}{T} + \frac{\delta T_2}{T} = \frac{1}{3}\frac{\delta\Phi}{c^2} \tag{8.11}$$

This means that CMB radiation from a potential well leads to redshifted CMB radiation, albeit at a redshift that is only 1/3 as much as one would naively expect. This is known as the Sachs-Wolfe effect.

The power in the CMB power spectrum at large scales (small l) is caused by this effect.

8.3.2 Baryonic acoustic oscillations

We have already seen that perturbations on small scales oscillate (cf. Chapter 5). Let us revisit the equation for the evolution of δ for a photon gas, Eq. (5.79), and write it in the form:

$$\frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} = \frac{c^2}{3a^2}\nabla^2\delta + \frac{32}{3}\pi G\rho_0\delta \tag{8.12}$$

Now let us introduce the *conformal time* τ, defined by

$$\tau = \int_0^t \frac{dt'}{a(t')} \tag{8.13}$$

Note that for the standard model, for $t \to 0$ we have $\tau \to 0$, i.e. the integral does *not* diverge near $t \to 0$ in spite of the fact that $a(t) \to 0$ for $t \to 0$.

We can now write the second derivative $d^2\delta/dt^2$ as

$$\frac{d^2\delta}{dt^2} = \frac{1}{a^2}\frac{d^2\delta}{d\tau^2} + H\frac{d\delta}{dt} \tag{8.14}$$

so Eq. (8.12) becomes

$$\frac{d^2\delta}{d\tau^2} + 3Ha\frac{d\delta}{d\tau} = \frac{c^2}{3}\nabla^2\delta + \frac{32}{3}\pi a^2 G\rho_0\delta \tag{8.15}$$

In Fourier space, where ω this time belongs to the *conformal* time τ, we thus obtain the dispersion relation:

$$\omega^2 - 3Hai = \frac{c^2}{3}k^2 - \frac{32}{3}\pi a^2 G\rho_0 \tag{8.16}$$

If we assume that we have large enough k and ω that we can ignore both the gravitational term on the right and the term proportional to H on the left, then we arrive at:

$$\omega^2 = \frac{c^2}{3}k^2 \tag{8.17}$$

This means that we have solutions of the form

$$\delta(x,\tau) = \delta_0 \cos(kx + \varphi)\cos\left(\frac{c}{\sqrt{3}}k[\tau - \tau_{\rm start}(k)]\right) \tag{8.18}$$

for $\tau \geq \tau_{\rm start}(k)$, with k and φ are arbitrary. The $\tau_{\rm start}$ is the conformal time at which this mode enters the horizon and thus starts oscillating.

Equation 8.18 is a standing wave with an interesting property: The phase of the time-oscillation is fixed by $(c/\sqrt{3})k[\tau - \tau_{\rm start}(k)]$. At time $\tau - \tau_{\rm start}(k)$ the phase is 0, and at any later time we *know* what the phase is: It is not a random variable. This means that for every wave number k we know what the phase of the oscillating standing wave is at the time of the CMB release. For some modes this phase may be $\pi/2$, in which case the density fluctuation has disappeared by the time of CMB release, but the motion is maximum. For others the density fluctuation may be near maximum (phase 0 or π). This gives a distinct wavy pattern in the power spectrum of the CMB, as can be seen in the figure at scales below about 1 degrees.

8.3.3 Silk damping

As we saw from the exercises, the CMB is released at an ionization parameter $x \simeq 0.01$ at redshift $z \simeq 1100$. CMB release implies that the mean free path of the photons is roughly equal to the scale of the particle horizon, or in other words, roughly equal to the scale of the largest scale baryonic acoustic oscillations represented by the first bump in the CMB power spectrum. At only a bit higher redshift ($z \simeq 1400$) the ionization parameter was near unity, meaning that the mean free path at that time was about 100 times smaller than that scale. So for wave modes k 100 times the k of the largest baryonic acoustic oscillation the radiation decouples already from the mode, since photons can travel freely over one wavelength. Those waves therefore decouple from the radiation pressure and do not behave as oscillations. They strongly damp out.

This damping already happens at scales about ten times larger than that, because even if a wavelength covers 10 mean free paths, due to diffusion (a random walk of the photon) a lot of radiation can diffuse from the peak of the wave to the valley and thus strongly damp the wave.

Indeed, in the power spectrum we see at scales below about 0.2 degrees the oscillations are damped. This is called Silk damping.

8.3.4 Polarized light: Scattering

Electron scattering has the effect of polarizing the radiation. If the radiation field at the last scattering surface is perfectly locally isotropic, all these polarizations cancel out and we expect to observe no polarization. However, if the local radiation field is *not* isotropic, then the resulting scattered radiation will display polarization. Also the power spectrum of this polarization (lower panel in the figure) can give information about the CMB release and can be used to test the model of how this takes place.

8.4 Deriving the flatness of the Universe

The location of the baryonic acoustic oscillation peaks in the power spectrum can be used to derive the fact that the Universe is flat. The idea is that if the Universe were to be closed, then the *spatial* scales corresponding to a given *angular* scale of our CMB sky would be smaller than for a flat Universe. Likewise, for an open Universe, the angular scale would correspond to a much larger spatial scale. Since the baryonic acoustic oscillation peaks are fixed to a *spatial* scale, their identification in the *angular* power spectrum gives us a means to measure the curvature of the Universe. The result is that the peaks are at the locations consistent with a flat universe.

8.5 Sunyaev-Zel'dovich effect

Clusters of galaxies contain hot virialized plasma. CMB photons entering such a cluster could experience inverse Compton scattering against the hot electrons. The effect is that CMB photons get boosted to shorter wavelengths. In a map of the sky at the location of the cluster you therefore see a "hole" in the CMB at long wavelengths and a "bump" in the CMB at shorter wavelengths. This effect is called the Sunyaev-Zel'dovich effect, or the "SZ-effect". It was first detected already in the 80s, but with very recent new telescopes, including Planck, surveys of the sky focusing on the SZ effect are likely to be discovering many new clusters.