Exercises for
Introduction to Cosmology (WS2011/12)

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Exercise sheet 5

1. **Evolution of the visible Universe**
   In the previous exercise we calculated the particle horizon between two points at the redshift of the CMB release. Now let us calculate our own particle horizon.

   (a) [10 pt] What is the redshift corresponding to a point on the horizon?

   (b) [10 pt] What is the comoving distance to our current particle horizon? You can use the information from the script (because direct calculation would require numerical integration, which is not what is requested here).

   Suppose our civilization will survive for many millions of years, so that our grand-grand$\cdots$grand-children can compare their observations of the Universe to ours, and thus *see* how the universe evolves. *Important:* Let us assume that they will keep the definition of comoving distance calibrated to the year 2011.

   (c) [20 pt] By how much is the comoving distance to our particle horizon increasing per second (i.e. how much more of the Universe will we be able to see)? Derive this from the formula of the definition of comoving distance.

   (d) For a galaxy at some redshift $0 < z \ll 1$ (in the year 2011), will its redshift, as measured by our descendants, be smaller, the same or larger than what we measure today (assume zero peculiar velocity)? Answer this in two steps:

      i. [20 pt] Show that the answer depends on $\ddot{a}$ obeying $\ddot{a} < 0$, $\ddot{a} = 0$ or $\ddot{a} > 0$, and associate these with the above three possibilities.

      ii. [20 pt] Show that today $\ddot{a} > 0$. *Hint:* Start from the equation $(\dot{a}/a)^2 = H_0^2(\Omega_{m,0}/a^3 + \Omega_{\Lambda,0})$ which is approximately valid today.

2. **Behaviour of "progress of time"**
   Type Ia supenovae are thought to be standard candles. Their intrinsic lightcurves (luminosity as a function of time) are always the same, apart from a rescaling. To interpret the measured light curve (*currently* measured flux as a function of *our* time) we must be able to translate what 1 second of *our* current time corresponds to in terms of the progress of time when the light was emitted:

$$dt_{\text{source}}(z) = f(z)\, dt_{\text{measured}} \tag{15}$$

   [20 pt] Give a general expression for $f(z)$, valid for all z. *Hint:* No large derivation needed; it is simpler than you might think.