Exercises for
Introduction to Cosmology (WS2011/12)

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(Exercise 7 from Matthias Bartelmann's lecture)

Exercise sheet 6

1. **Growth of structure in pressure-less dark matter**
   Consider the linear perturbation equation for the density contrast of pressureless matter,
   $$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_0\delta \tag{16}$$
   where ρ_0 is the mean background density.

   (a) Transforming the time derivative to a derivative with respect to the scale factor a, show that Eq.(16) can be written as
   $$(a^3 H\delta')' = \frac{3\Omega_{m0}H_0^2}{2Ha^2}\delta \tag{17}$$
   where the prime denotes the derivative with respect to a.

   (b) Show that $\delta_1 = H$ is one solution of Eq. (17) *provided* H^2 is of the form
   $$H^2 = \frac{C}{a^3} + \frac{D}{a^2} + E \tag{18}$$
   where C, D and E are arbitrary constants. Argue why this is important for cosmology.

   (c) Use the *ansatz* $\delta_2 = Hf$ to show that δ_2 is the other solution of Eq. (17), provided
   $$f' = \frac{1}{a^3H^3} \tag{19}$$
   Hint: Underway, use that H is a solution of Eq. (17). This is an example of the so-called d'Alembert reduction). Thus,
   $$\delta_2 = H(a)\int_0^a \frac{d\bar{a}}{\bar{a}^3H^3(\bar{a})} \tag{20}$$
   is the other solution of the linear growth equation.