Exercises for Introduction to Cosmology (WS2011/12)

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(Exercise 8 from Matthias Bartelmann's lecture)

Exercise sheet 7

1. **Growth of structure in pressure-less dark matter when radiation is not negligible**
   Consider the growth of dark matter perturbations when matetr and radiation have to be taken into account, but curvature and the cosmological constant can be neglected.

   (a) Define the scale factor $x = a/a_{\rm eq}$ and show that the Hubble function can be written in the form

   $$H(x) = \frac{C}{x^2}\sqrt{1+x}, \qquad C = \sqrt{\frac{8\pi G}{3}\rho_{\rm eq}} \tag{21}$$

   where $\rho_{\rm eq}$ is the density of matter at the time of matter-radiation equality.

   (b) Use Eq. (17) to derive

   $$\delta'' + \left(\frac{1}{x} + \frac{1}{2(1+x)}\right)\delta' = \frac{3\delta}{2x(1+x)} \tag{22}$$

   where the prime now denotes d/dx. Hint: For the rhs use $\rho_{\rm eq} = \rho_{m0}a_{\rm eq}^3$.

   (c) Show that one solution of this equation is

   $$\delta_1 = 1 + \frac{3}{2}x \tag{23}$$

   (d) Use the ansatz $\delta_2 = \delta_1 f$, with an as yet unknown function $f(x)$, to show that δ_2 is a solution of Eq. (22), provided $f(x)$ satisfies the equation

   $$\frac{f''}{f'} = -\left(\frac{3}{1+3x/2} + \frac{1}{x} + \frac{1}{2(1+x)}\right) \tag{24}$$

   It turns out that the following two functions are solutions to this equation:

   $$f = \pm\ln\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right) \pm \frac{3\sqrt{1+x}}{1+3x/2} \tag{25}$$

   (e) Argue why the minus sign should be chosen so that the second solution becomes

   $$\delta_2 = \left(1+\frac{3x}{2}\right)\ln\left(\frac{\sqrt{1+x}+1}{\sqrt{1+x}-1}\right) - 3\sqrt{1+x} \tag{26}$$