Exercises for
Introduction to Cosmology (WS2011/12)

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Exercise sheet 8

1. **Integrated Sachs-Wolfe effect**
   The Integrated Sachs-Wolfe effect is the effect of a time-varying gravitational potential on the CMB. It can be used to find evidence for dark energy.

   (a) Show that for a flat *dark matter* dominated Universe ($\Omega_m = 1$ and $\Omega_\Lambda = \Omega_r = \Omega_K = 0$) the perturbations in the potential $\delta\Phi$ stay constant in time, at least during the linear growth phase.

   (b) Argue that if $\Omega_\Lambda \neq 0$, $\Omega_m \neq 0$ (but $\Omega_r = \Omega_K = 0$) the equation for the growth of perturbations in pressureless dark matter is still

   $$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_0\delta \qquad (27)$$

   i.e. why does the Λ force not introduce an extra term in this equation?

   A non-zero Ω_Λ still affects the solutions of Eq. (27) through its effect on $H(t)$ and $\rho_0(t)$. The full solution with $\Omega_\Lambda \neq 0$, $\Omega_m \neq 0$ is a bit complicated, but the trend once $\Omega_\Lambda \gtrsim \Omega_m$ can be estimated by studying the asymptotic behavior for $t \to \infty$, i.e. for $\Omega_\Lambda \to 1$ and $\Omega_m \to 0$.

   (c) Derive this asymptotic behavior of δ. Will δ continue to grow linearly? Or faster/slower? Or will it even decline?

   (d) Show that this now means that $\delta\Phi$ is no longer constant. Will it decline or grow?

   Now suppose a set of CMB photons travel through the gravitational potential of a large DM overdensity at relatively low redshift (say, $z \simeq 1$). Assume that our linear analysis is still valid. At some point later we observe these photons and compare their wavelengths to CMB photons that have *not* passed through such an overdensity.

   (e) Argue qualitatively how the wavelengths of the photons that passed through the overdensity differ from those that did not. This is known as the *Integrated Sachs-Wolfe effect.*

   (f) Why can we use this as a probe of Dark Energy?