Exercises for
Numerical Fluid Mechanics (WS2012/13)

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Exercise sheet 1

Numerical integration of ODEs

In numerical fluid dynamics the goal is to numerically integrate a set of partial differential equations (PDEs) in time. Let us first, however, see how we numerically integrate a set of *ordinary* differential equations (ODEs) in time.

1. **First order Euler integration**
   Consider the following simple ODE:

$$\frac{dy}{dt} = -Ay + B \tag{1}$$

   At $t = 0$ we have $y = C$. The A, B and C are constants.

   (a) Give the analytical solution

   (b) Write and test a program to integrate this numerically with first order forward Euler:

$$y^{n+1} = y^n - A\Delta t\, y^n + B\Delta t \tag{2}$$

   (here the upper index is the time index).

   (c) If we want to find the solution over the domain $0 \leq t \leq T$, and we have a given time step Δt, how many time steps N do we need?

   (d) Plot the results for $A = 1$, $B = 1$ and $C = 0$ between $0 \leq t \leq 10$, using different time step sizes (e.g. $\Delta t = 0.1$, $\Delta t = 0.3$, $\Delta t = 1$, $\Delta t = 3$). Overplot also the analytical solution.

   (e) For which Δt does the algorithm become unstable? Generalize this to arbitrary A, B and C.

   (f) For which Δt do you get results within 1% of the analytical answer? Again generalize your answer to arbitrary A, B and C.

2. **Second order midpoint method**
   Consider the same equation as above, but now let us integrate this a bit better: using a second order time integration:

$$y^{\mathrm{mid}} = y^n - \tfrac{1}{2}A\Delta t\, y^n + \tfrac{1}{2}B\Delta t \tag{3}$$

$$y^{n+1} = y^n - A\Delta t\, y^{\mathrm{mid}} + B\Delta t \tag{4}$$

   Now compare the results to the first order integration:

   (a) For which Δt does the method become unstable?

   (b) For which Δt do you get results within 1% of the analytical answer?

For all exercises, please always do the following:

- Make an electronic document (DOC or PDF) which includes your text concerning the exercises, as well as figures belonging to it.
- Upload your document *and your computer program* to the Moodle.