Exercises for
Numerical Fluid Mechanics (WS2012/13)

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Exercise sheet 4 *(duration: 1 week)*

Shock tube test

We now add the energy equation to our solver and apply this to solving a shock tube test.

1. **Adding the energy equation**

   (a) Extend your hydrodynamics program by the energy equation. Also here advect the $\rho e_{\rm tot}$ using the gas velocity at the interfaces, and add the pressure term as a source term. See the lecture script for the recipe. Take $\gamma = 5/3$.

   (b) Apply this now to the exact same problem as before, but now with $e_{\rm therm} = 1$. In other words, the initial condition for $\rho e_{\rm tot} = \rho$, because the velocity is zero.

   (c) Make a plot of the density at $x = 0$ as a function of time between $0 \leq t \leq 60$. Overplot the density at $x = 50$ as a function of time. If all goes well you should notice that the density at $x = 0$ is on average higher than the density at $x = 50$. Why is that?

2. **Shock tube test**
   Set up the following shock tube test: $-50 \leq x \leq 50$. For $x < 0$ we have $\rho = 2$, $e_{\rm therm} = 1$ and $u = 0$. For $x \geq 0$ we have $\rho = 1$, $e_{\rm therm} = 1$ and $u = 0$. We take again $\gamma = 5/3$. As boundary conditions simply copy the values of ρ, ρu and $\rho e_{\rm tot}$ from cell 1 into cell 0 (the left ghost cell) and from cell M into cell $M + 1$ (the right ghost cell).

   (a) Plot the $\rho(x)$ at time $t = 18$.

   (b) Where is the rarefaction wave, where the contact discontinuity and where the shock?

   (c) Plot the $e_{\rm therm}(x)$ at $t = 18$. Explain what you see: what physics causes the increase/decrease of $e_{\rm therm}(x, 18)$ compared to $e_{\rm therm}(x, 0) = 1$?