Exercises accompanying lectures 12 & 13: digital imaging & photometry

May 25, 2011

CCDs

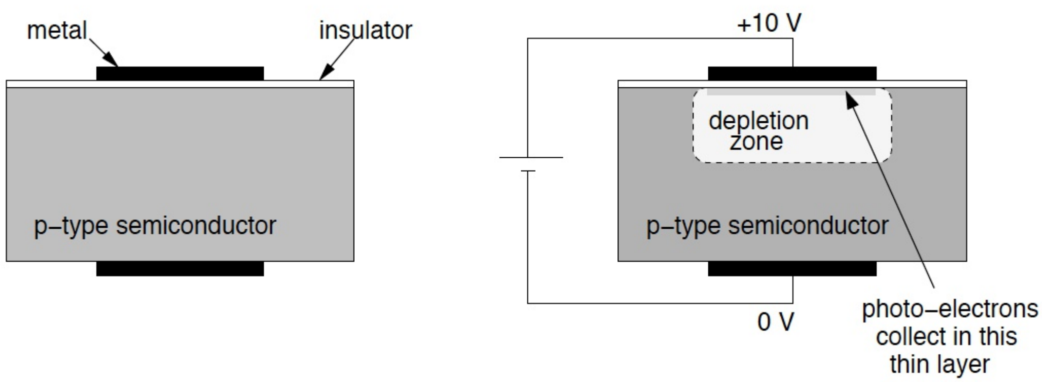

Figure 1: Schematic illustration of a CCD pixel

1. Detectors based on semi-conductors can be used to record photons. Silicon based p-type semiconductors are normally used in CCDs, see figure 1. The dopants each take an e^- from the Silicon lattice and become effectively negative ions, that are "trapped", i.e are not free to move through the material. They leave positively charged "holes" in the Silicon lattice, that are mobile. If we put a positive potential on the top the holes are expelled deep into the semi-conductor, creating a region without mobile charge carriers (depletion zone), while at the

same time storage capacity for electrons is created below the anode (which is isolated from the semi-conductor material by a thin insulator layer, usually SiO_2). Now there is an electric field within the pixel, strongest at the top directly below the anode, decreasing as we go into the pixel and reaching zero at the edge of the depletion zone. When exposed to light, photons may be absorbed, creating an electron-hole pair. The electric field will force the hole to move out of the depletion zone and the and the electron to move just below the anode, where it is held below the insulating layer.

a) [1 point] What is the "band gap", and how does it relate to the spectral range to which a detector is sensitive?

b) [1 point] If an electron-hole pair is created outside the depletion zone, will it be recorded?

c) [1 point] We are exposing, and ever more electrons gather below the anode. What happens to electric field in the pixel, and to the depletion zone?

d) [2 points] We can think of a CCD pixel as being a small parallel plate capacitor, able to hold a finite amount of charge with a capacitance of $\frac{\epsilon A}{d}$, where ϵ is the permittivity of the medium between the plates, d is the distance between the plates, and A their area. The amount of charge on the capacitor depends on the potential difference between the plates: $Q = VC$. Estimate how many electrons a CCD pixel can hold at most if the pixel size is $15\,mu$m and the anode voltage is $+10\,$V. The thickness of the pixel is $10\,\mu$m and that of the isolating SiO_2 layer is $2\,\mu$m. The permittivity of SiO_2 is 3.45×10^{-11} $\mathrm{A^2s^4kg^{-1}m^{-3}}$, that of silicon 2.30×10^{-10} $\mathrm{A^2s^4kg^{-1}m^{-3}}$ at liquid nitrogen temperature.

e) [2 points] Consider a photomultiplier tube operating with a total potential 1000 V, divided over 15 "stages" (dynodes). The "gain" of each stage of this particular device, i.e. the number of secondary electrons released when a primary electron hits the dynode, scales approximately linearly with the kinetic energy of the incoming electrons, as long as the potential difference between successive dynodes ΔV remains below 500 V, and has a value of 6 if $\Delta V = 100\,$V. Assume the efficiency of the photo-cathode layer to be 15% and that of the rest of the device to be 100%. What is the total amplification of the device, i.e. how many electrons will be received at the back end if 1 electron is released from the photocathode layer? How does this number change if we increase the operating voltage to 2000 V? If a source we are observing "injects" 100 photons per second into our photomultiplier tube, what is the *average* current yielded by our device, when operated at 2000 V?

2. We wish to perform photometry of a faint point source in the Johnson V filter using the Subaru Telescope of the national observatory of Japan. It has a primary mirror diameter of 8.2 m and a secondary mirror of 1.3 m diameter (which is the only structure substantially 'blocking' the primary mirror). It is located on top of Mauna Kea, Hawaii, at an altitude of 4139 m. Assume that the following parameters well describe the instrument performance:

   - optical throughput of entire system 75% (telescope and instrument combined)
   - Quantum Efficiency (QE) of CCD detector 80%
   - Transmission of the V-band filter to be 90% between 485 nm to 580 nm and zero at all other wavelengths (i.e. "box" shape)
   - detector readout "gain" 1.4 electrons (i.e. 1 "count" represents 1.4 detected electrons)
   - square pixels of 0.2×0.2 arcseconds
   - detector readout noise 3 electrons/pixel
   - negligible dark current

   a) [3 points] With an exposure time of 30 minutes, we find that we have gathered 2500 counts from the source. What is, approximately, the apparent spectral flux density of the source, and what is its apparent magnitude (ignoring for now extinction in the Earth atmosphere)? Hint(s): consider the energy of a single photon at the central wavelength, assume the source spectrum to be "isophotonic" over the V-band (i.e. have the same number of photons in each wavelength bin, or if you choose to work in frequency rather than wavelength domain, per frequency bin), and consider the light gathering area of the Subaru telescope and the spectral bandwidth of our filter.

b) [1 point] When keeping 5 significant digits in the above calculation, one finds a slight difference in the magnitudes derived when calculating in the wavelength domain (i.e. expressing intensity in terms of erg/s/cm^2/μm) compared to those derived when working in the frequency domain (with intensities in erg/s/cm^2/Hz). In the former case we obtain V=27.886 mag, in the latter V=27.924 mag. Whence does this small difference arise?

c) [1 point] At its darkest, the sky background at Mauna Kea has a brightness of $\approx$21.9 mag/arcsec^2. Assuming the night sky spectrum to be iso-photonic, how many photons will be recorded on average per pixel during our integration time?

Let us now calculate the SNR of our measurement. Assume that the image of our object is spread over 10 pixels, each catching equal amounts of source flux (obviously this is a rather gross simplification of an actual point spread function). Assume that we make no systematic errors in the subtraction of the sky background.

d) [1 point] which sources of noise do we need to consider?

e) [2 points] What is the SNR on our detection? If you did not find the answer to question c), assume that the average number of photons from the sky background is 30,000 per pixel for the whole integration (thirty-thousand).

Our observation is done and we have made a significant detection of our very faint source. We have, however, not yet performed a photometric calibration of our source. To this purpose, we will observe a standard star of known brightness ($V = 18.0$ mag) immediately after our science observation, at (nearly) identical airmass of 1.25. We expose for 30 seconds, in a setup identical to that of our science observation, find that we detect a total of 457,500 photons.

f) [1 point] Given the system parameters of our telescope, and making the same assumptions as for the previous calculations (iso-photonic spectrum etc.), how many photons would we expect to detect from this standard star (ignoring atmospheric extinction)?

g) [1 point] We detect somewhat fewer photons than expected, and the difference is due to atmospheric extinction. What is the atmospheric extinction coefficient in the V-band for Mauna Kea, at the moment we were observing, in units of mag/airmass?

h) [2 points] What is the final, calibrated magnitude and its $1\,\sigma$ uncertainty of our science target?

Note that in practice, the individual efficiencies of the sub-components of our measurement system are usually not considered. Instead, the sensitivity of the whole system is determined empirically, simultaneously with the measurement of the atmospheric extinction coefficient, by observing a set of standard stars at a number of different airmasses.

i) [1 point] Since in this simple example we assume the effect of the atmosphere on the science and calibration observation to be identical, we can also calibrate the magnitude of our science target directly by comparing the observed "raw" photon detection rates of science target and calibrator. How, and what is the numerical result?