Exercises belonging to lecture
Observational Astronomy MKEP5 (SS 2011)

Sheet 9

Please note: As always, all necessary information can be found either in the lecture notes or in the description of the exercises.
Also note: The exercises are often supposed to paint "realistic" situations, so not always all information given necessarily needs to be used!

In these exercises we use CGS units. In these units the Planck constant $h = 6.6262 \times 10^{-27}$ $\mathrm{cm^2 g/s}$, the speed of light is $c = 2.9979246 \times 10^{10}$ cm/s, Boltzmann's constant is $k_B = 1.3807 \times 10^{-16}\ \mathrm{cm^2 g\, s^{-2} K^{-1}}$, AU=$1.4959800 \times 10^{13}$ cm and parsec=3.0857200×10^{18} cm.

1. **A reflection grating**
   Consider a relection grating with groove-spacing d. If we let light with wavelength λ fall vertically onto the grating, we derived in the lecture that the angular locations θ_m of the m-th order is at:
   $$\sin\theta_m = \frac{m\lambda}{d} \tag{1}$$
   This is called the grating equation. In most spectrographs, however, the light falls onto the grating at some angle i, see Fig. 1.

   Let us take the sign-convention of this angle to be the same as for θ_m: positive angle means tilted to the right, so in the figure the angle $i < 0$. This non-zero incidence angle causes the optical path differences to change, so that also the location of the orders changes. In other words: Eq. (1) changes.

   (a) Show that the grating equation then becomes
   $$\sin\theta_m = \frac{m\lambda}{d} - \sin i \tag{2}$$

   (b) Show in the figure where the 0-th order will now be.

   As we know from the lecture, the spectral resolving power R of a grating is given by
   $$R = \frac{\lambda}{\Delta\lambda} = mN \tag{3}$$
   where m is again the order and N is the number of grooves in the grating. Echelle gratings are meant for high resolution spectroscopy, so the objective is to maximize R, meaning we wish to maximize m and N.

   (c) Show that, for a given wavelength λ, there is an upper limit to how high order m you can go for a given groove spacing d.

   (d) How would you choose i and at which angle θ would you put your CCD camera to get maximum spectral resolution (i.e. maximum m)? Hint: Ignore practicalities such as the size of the camera or the width of the beam; think in terms of ideal circumstances.

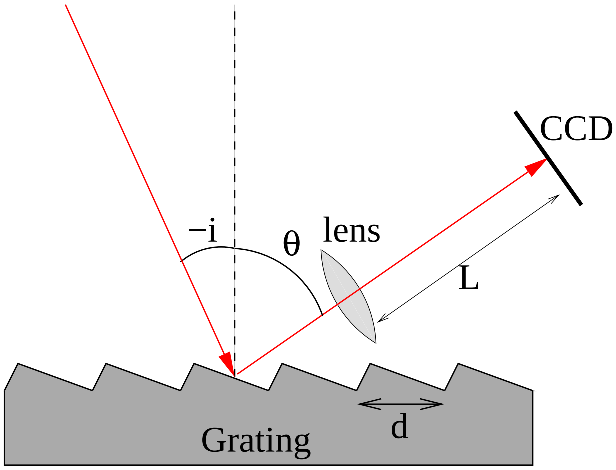

Figure 1: Schematic view of the setup of a grating with groove spacing d with incident radiation at an angle of i, and a CCD chip at angle θ, with a lens of focal length L between the CCD chip and the grating to focus the light onto the CCD.

(e) Suppose we want to measure a spectrum around λ =0.5 μm at very high resolution. We choose to go for the 100-th order ($m = 100$). What is the minimal groove spacing d our grating must have? Hint: Also here: ignore details of the experimental setup; we want to gain understanding, not waste time with details.

In the lecture we derived that the angular spacing between two spectral resolution elements is given by

$$\Delta \sin\theta = \frac{\lambda}{Nd} \tag{4}$$

(f) Show that for $\theta < \pi/2$ and for $\Delta\theta \ll 1$ we can write this approximately as

$$\Delta\theta \simeq \frac{\lambda}{Nd\cos\theta} \tag{5}$$

If we put our CCD at some distance from the grating and we put a focusing lens with focal length L between the grating and the CCD, this converts into a Δx in centimeters on the CCD. Since CCDs have pixels of about 20 μm or so in size, there is a minimum number of grooves N we need in order to put each spectral resolution element onto a single pixel (putting it onto more pixels might be better, but let's do it onto a single pixel for now).

(g) Let us say that we choose our grating spacing $d = 25.98\mu$m, which is such that for incident radiation at $i = 70$ degrees we have our $m = 100$ order at $\theta = 80$ degrees. We put our CCD detector at $\theta = 80$ with a lens of $L = 20$ cm. What is the number N of grooves we need to get one spectral resolution element on one pixel?

(h) How big does this make our grating?

(i) What is then our spectral resolution R?

(j) If we choose N smaller, what is the advantage and what is the disadvantage? Hint: think in terms of pixels per resolution element and in terms of the spectral resolution.

2. **Converting an Echelle spectrograph into a high-resolution long-slit spectrograph**
   Attached to this exercise is a small article by Hubrig et al. on VLT-UVES long slit spectroscopy. UVES is an echelle spectrograph, and Hubrig et al. produced very narrow band filters to convert this echelle spectrograph into a long-slit spectrograph. Explain why these filters were needed.

3. **Millimeter-wave line observations of a protoplanetary disk**
   One can also do spectroscopy with microwave telescopes ("millimeter wave radio telescopes"). Suppose we wish to observe the emission line from the J=2→1 rotational transition of the CO molecule at λ =1.3004037 millimeter originating from a protoplanetary disk around a T Tauri star. The star has an effective temperature $T_* = 4000$ Kelvin and radius $R_* = 2R_\odot = 1.4 \times 10^{11}$ cm. This star+disk system is at a distance of 140 parsec (the distance to the Taurus molecular cloud complex, one of the main large star forming regions in our viscinity). Suppose the disk has a radius of 70 AU and is entirely optically thick at line-center everywhere in the disk, and the disk has a sharp outer edge. Suppose also that the disk is face-on, so that doppler shifts due to the rotation of the disk are zero. Also suppose, for simplicity, that the disk is everywhere exactly at a temperature of $T = 50$ Kelvin, and that the line is in local thermodynamic equilibrium (LTE).

   (a) Show that the observed flux F_ν in units of erg sec_{-1} cm^{-2} Hz^{-1} from the *star* at λ =1.3 millimeter equals 2.13×10^{-29} erg sec^{-1} cm^{-2} Hz^{-1}, assuming a simple blackbody (=Planck) spectrum for the star. Hint: the flux from a blackbody emitting surface at temperature T is $F_\nu = \pi B_\nu(T)$.

   (b) Show that the observed flux the *disk* at line center is 1.4×10^{-23} erg sec^{-1} cm^{-2} Hz^{-1}. Although the disk has a central hole (where the star is located) this hole is so small compared to the overall disk size, that one can safely ignore this. Hint 1: Here it is easiest to use the intensity at the disk surface (rather than the flux) and use the solid angle of the disk on the sky. Hint 2: The solid angle on the sky of an object with projected surface area S is $\Delta\Omega = S/d^2$ where d is the distance of the object to the observer (=us). Hint 3: An optically thick thermal line means that the line is saturated (at which value?), so there is no need for details of the line emission/absorption mechanism!

   These results show that the stellar flux at λ =1.3 millimeter is absolutely negligible compared to that of the disk! This is one of the main reasons why observations of protoplanetary disks are often done at long wavelengths. From now on we will neglect the star emission altogether.

   We now plan to observe this object with the IRAM 30-meter telescope[1]. As our receiver instrument we use the HERA 1mm array receiver[2].

   (c) Calculate the angular resolution $\Delta\theta$ in arcseconds for this telescope at λ =1.3 mm. You can check if you have the right answer by comparing the value to that wat is given on the HERA website.

[1] http://www.iram.es/IRAMES/index.htm
[2] http://www.iram.es/IRAMES/mainWiki/HeraforAstronomers

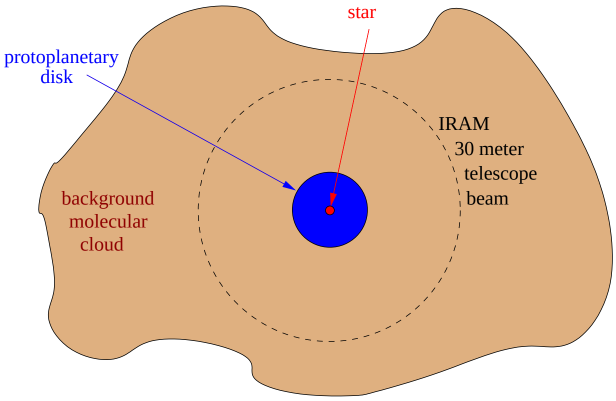

Figure 2: Pictographic representation of exercise 3. Not to scale.

(d) Calculate the diameter (!) of the disk on the sky in arcseconds, and show that the IRAM 30 m telescope does not have enough angular resolution to spatially resolve the disk.

This lack of angular resolving power of the IRAM 30 m telescope could be a problem, in particular if the disk is surrounded by background emission. This is the effect of *beam dilution* that we will discuss here. See Fig. 2.

Suppose that several parsec *behind* the disk there is a large molecular cloud with a temperature of 20 K. The molecular cloud is optically thick at 1.3 mm (as well as at all the wavelength near 1.3 mm). The cloud is much larger than the resolution of the IRAM telescope (so the molecular cloud "fills the beam" of the telescope).

(e) What is the *intensity* I_ν (in units of erg sec_{-1} cm^{-2} Hz^{-1} ster^{-1}) of the background molecular cloud at $\lambda = 1.3$ mm?

(f) Convert this into an observed *flux* F_ν (in units of erg sec_{-1} cm^{-2} Hz^{-1}), by *approximating* the "beam" (i.e. the part on the sky the telescope instrument will collect light from and convert into a measured signal) of the telescope by a disc on the sky with angular diameter equal to the $\Delta\theta$ you calculated for the IRAM telescope at 1.3 mm above[3]. Hint: Remember that the flux and intensity are related via a solid angle.

(g) Compare the observed flux from the disk to the flux from the background, and show that the background flux dominates the observed signal.

(h) We can get better source/background contrast if we go to a telescope with a better spatial resolution. Argue why.

Since there are no movable radio telescopes that are large enough to resolve such a disk, we have to make use of radio *interferometry* to artificially narrow the beam (i.e. increase the spatial resolution) to improve contrast of the disk emission compared to the background. This is the topic of the next lecture.

[3]In reality this "beam" will be an Airy pattern on the sky, but for the current purpose a "disk-like" beam with uniform sensitivity is sufficient.