Exercises for
Radiative Transfer in Astrophysics (SS2012)

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Exercise sheet 11

Strömgren sphere

1. **A simple model of an HII region**
   Let us consider an ionized gas sphere around a hot O6 star with $T_* = 4 \times 10^4$ K producing

$$L_N = 5 \times 10^{48}\,\mathrm{s}^{-1} \tag{9}$$

   UV photons / second with frequencies $h\nu > h\nu_T = 13.6$ eV (i.e. $\nu_T = 3.29 \times 10^{15}$ Hz). Let us assume that the hydrogen gas around the star is atomic, and has a density of $N_H = 10$ cm^{-3}. The equation of ionization balance is:

$$N_{H^0} \int_{\nu_T}^{\infty} \frac{4\pi}{h\nu} J_\nu a_\nu d\nu = N_p N_e \alpha(T) \tag{10}$$

   where J_ν is the mean intensity of the radiation of the star:

$$J_\nu(r) = \frac{L_\nu}{(4\pi)^2 r^2} e^{-\tau_\nu(r)} \tag{11}$$

   with r the distance to the star and $\tau_\nu(r)$ the optical depth toward the star at frequency ν. Let us approximate the integral over frequency as:

$$\int_{\nu_T}^{\infty} \frac{4\pi}{h\nu} J_\nu a_\nu d\nu \simeq \frac{4\pi}{h\nu_T} J a_{\nu_T} \tag{12}$$

   The ionization balance then becomes:

$$N_{H^0} a_{\nu_T} \frac{L_N(r)}{4\pi r^2} = N_p N_e \alpha(T) \tag{13}$$

   where

$$L_N(r) = L_N \exp\left(-\tau_{\nu_T}(r)\right) \tag{14}$$

   Take the following values: $a_{\nu_T} \simeq 6 \times 10^{-18}$ cm^2 and $\alpha(T) \simeq 4 \times 10^{-13}$ cm^3/s.

   Now solve the following problems:

   (a) Write

$$N_{H^0} = \xi N_H \qquad , \qquad N_p = N_e = (1-\xi) N_H \tag{15}$$

   Ignore, for now, the extinction effect (i.e. take $\tau_{\nu_T} = 0$ in the ionization balance equation). Show that ξ at a distance of $r = 5$ parsec from the star is roughly $\xi \simeq 4 \times 10^{-4}$, i.e. nearly complete ionization.

   (b) Make a plot of $\xi(r)$ out to $r = 100$ parsec, again assuming $\tau_{\nu_T} = 0$.

(c) Show that if we include the effect of extinction, the $L_N(r)$ obeys the following equation:

$$\frac{dL_N(r)}{dr} = -4\pi r^2 N_p N_e \alpha(T) \tag{16}$$

Tip: Ionization balance means that all recombinations must be compensated by photo-ionizations, which "eat away" stellar UV photons.

(d) Show that there is a radius r_s, the Strömgren radius, where L_N drops to zero, and which is approximately:

$$r_s = \left(\frac{3}{4\pi}\frac{L_N}{N_H^2\alpha(T)}\right)^{1/3} \tag{17}$$

(e) Numerically integrate Eq. (16) from $r = 0$ to $r = 100$ parsec while solving the local ionization balance $\xi(r)$ at each point. Show that the transition from ionized to neutral indeed takes place where Eq. (17) says it is (roughly at 10 parsec). And show that the transition from ionized to neutral is very abrupt.