Radiative Transfer in Astrophysics

Theory, Numerical Methods and Applications

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Chapter 1

Introduction

1.1 Motivation of this lecture

Astrophysics is a branch of physics in which extraterrestrial objects and astronomical phenomena are being studied in the context of the laws of physics. The goal is to try to understand our cosmic surroundings in terms of basic physical principles, and thus put our own world into context. In contrast to many other areas of physics, however, our objects of interest are so remote that, in most cases, we will never be able to make in-situ measurements. Nearly all information that we obtain from these objects has reached us in the form of electromagnetic radiation. And in many cases this radiation contains this information in a cryptic way. For instance, the fact that the Sun contains the element Helium has been inferred already in the 19th century through spectroscopy: A strong yellowish component of light was discovered (a spectral line of Helium) that indicated the presence of an element that, at the time, was not known to exist on Earth.

In a way one can regard astrophysics as a kind of experimental physics in which the experiments were not designed by us, but by Nature, and which we can only observe from a distance.

One of the most central challenges of astrophysicists is therefore to decode the "encrypted" information we receive from our experimental setup through radiation. If we measure some spectral feature with some strength, what does it say about our star of interest? If we see a dark cloud obscuring stars in the background, what can we learn about the dust content of the cloud? If we see Doppler-shifted spectral lines, what does this tell about the dynamics of the system? If we see a strong infrared line of ionized atomic carbon, what does this mean for the cooling rate of the gas?

In almost all these cases we need to invoke our understanding of the physical processes that *produce* and *modify* this radiation inside our object of interest before we can understand what the radiation is trying to tell us. This topic is called "radiative transfer" or "radiation transport", and is the topic of this lecture.

Radiative transfer is often so complex that we need to resort to numerical modeling, in most cases involving the use of complex radiative transfer codes. However, there is always fundamental physics involved, and it is important to always understand these processes, even when using a black-box computer program for radiative transfer modeling.

Because of its complexity and diversity, and because radiative transfer is often more a *tool* to remotely measure certain quantities rather than the actual physical process of interest itself, radiative transfer is often treated as a side topic in astrophysics: a necessary evil, something that is better avoided as long as one can afford to do so. Yet, it is one of the key issues of astrophysics, for three reasons:

1. Diagnostic radiative transfer is the link between observations and theory,
2. radiative transfer is usually the main cooling process of astrophysical objects, and in many cases also the main heating process,
3. much of the chemistry and ionization physics of gaseous astrophysical objects is driven by radiation.

1.2 The radiative transfer approximation to light propagation

Electromagnetic radiation is a solution to the Maxwell equations in which energy is transported by a transverse wave in the electric and magnetic fields. How such waves propagate through, and interact with, matter, is in principle a problem that needs to be tackled from the perspective of the Maxwell equations. This is a very complicated problem. We shall see examples of this problem, and methods of solution, when we discuss the theory of light scattering off small particles (Chapter 6). For light propagation through macroscopic media this is, however, usually too complicated to be feasible. Fortunately, however, in most cases this problem can be considerably simplified.

For instance, if the object through which the light propagates is much larger than the wavelength, we can use the approximation of *geometric optics*, in which light can be considered as propagating along rays. The wavelike nature is then only treated to the extent that it induces light bending on surfaces of changing index of refraction. For all other purposes light can be regarded as photons propagating along rays. Computer programs for 3-D visualization by *ray-tracing* make use of this approximation. An example of such software is POV-Ray[1], which is an open source program available for many platforms.

One further step of simplification can be made if light propagates through low density environments in which the index of refraction can be safely approximated to be $n = 1$ everywhere. This is the approximation at the basis of *radiative transfer*, the topic of this lecture. This approximation is fairly good in e.g. planetary or stellar atmospheres, circumstellar or interstellar clouds, and many other applications. With this approximation, light travels along straight lines. Every now and then a photon may experience a scattering event that changes its direction of propagation, but between scattering events light strictly propagates along straight rays. Photons may also get absorbed, which ends their journey along the ray. And new photons can be created. As we shall see later, the formal radiative transfer equation is nothing more than injecting photons into a ray, and removing photons from that same ray.

1.3 Why radiative transfer is hard

One might be tempted to think that the radiative transfer approximation makes the problem of radiative transfer almost trivial. In some way it indeed is. If we focus on just the injection into, and the removal of photons from a single ray, and if we know exactly the rates of injection and removal, then radiative transfer is truly not difficult. To do this accurately with a computer program for any kind of ray through the medium may still require some tricky programming (and we shall encounter some interesting possible pitfalls in this lecture), but we will not be faced with a fundamental difficulty.

However, the problem of scattering complicates things hugely. Photons can then move along erratic paths. Or in other words: photons can switch from one ray to another through a scattering event, and which ray it switches to is to a large extent randomly

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[1] `http://www.povray.org/`

determined. One can no longer deterministically trace a photon back to its origin because of the random nature of the scattering events.

Also the fact that radiation can affect the state of the medium itself can cause major complexities. It means that we do not know in advance the injection and removal rates of photons into/from the ray, since they depend on the local state of the matter, which in turn depends on the radiation field - which we do not know in advance (we want to calculate it!). We are then faced with a typical chicken-or-egg dilemma. To know the state of the matter, we need to know the radiation field. To know the radiation field, we need to know the injection/removal rates. To know the injection/removal rates, we need to know the state of the matter.

As we shall see in this lecture, the problem of scattering and the problem of radiation-affecting-the-medium are mathematically the same type of problem. Indeed, sometimes the term "scattering" is used by radiative transfer experts when they in fact refer to how radiation can affect the occupancies of certain quantum states of atoms or molecules. The core of the difficulty lies in the fact that in most radiative transfer problems one cannot regard a single ray in isolation. The rays that cross each other can affect each other. We must then solve the radiative transfer equation *along all rays simultaneously*. This is one of the main reasons why the problem of radiative transfer is an immensely complex problem. Another reason is that the coupling of the radiative transfer equation to the state of the matter means that different regions inside an object can "communicate" with each other over long distances. For instance, if one side of an interstellar cloud is hot and the other side is cold, then the hot side can radiatively cool by emitting radiation. Part of that radiation can be captured by the cold side of the cloud, heating that side of the cloud up. As it heats up, it starts sending photons back to the hot side, partly counter-acting its radiative cooling, etc. etc.. This means that we must solve the state of the matter *at all locations in the cloud simultaneously*.

These problems of radiative transfer can be pretty challenging. Several "naive" methods of solution have turned out to be problematic: they are either too slowly converging or they give unreliable answers.

Fortunately, over the last few decades, several powerful numerical techniques have been developed to solve these problems reliably and (comparatively) rapidly. In this lecture we will learn these techniques, learn their strengths and weaknesses, and we will experiment with them in the exercise classes.

1.4 Conventions

Radiative transfer is a discipline that is practiced in many fields of physics and engineering. Therefore there are many different conventions, notations and different nomenclature. We follow the conventions and nomenclature of radiative transfer in stellar atmospheres. This is because in astronomy this branch of radiative transfer has presumably the longest history and is the farthest developed - although in 3-D radiative transfer most developments appear to take place elsewhere these days. At any rate, I will try to regularly connect to other conventions where I can, and where I am aware of them.