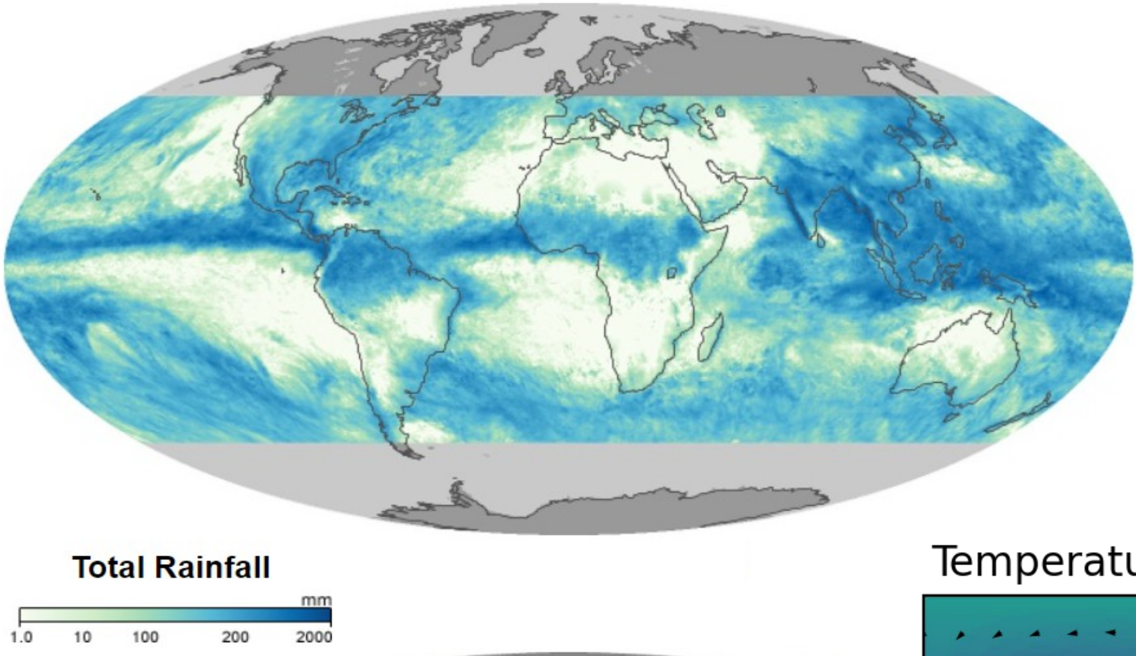
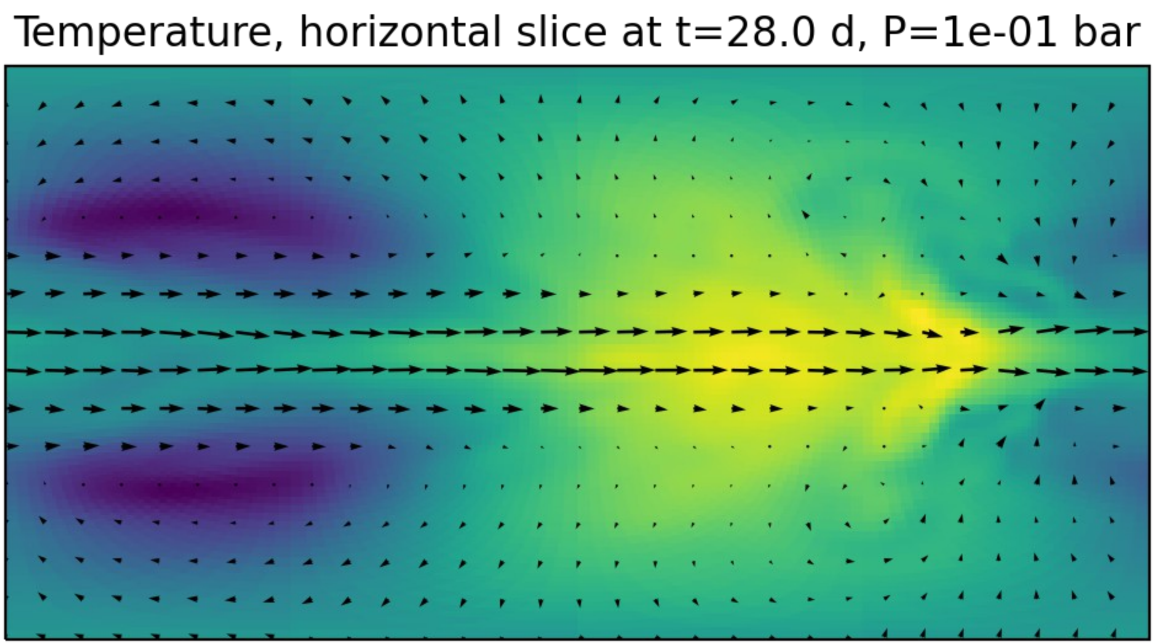
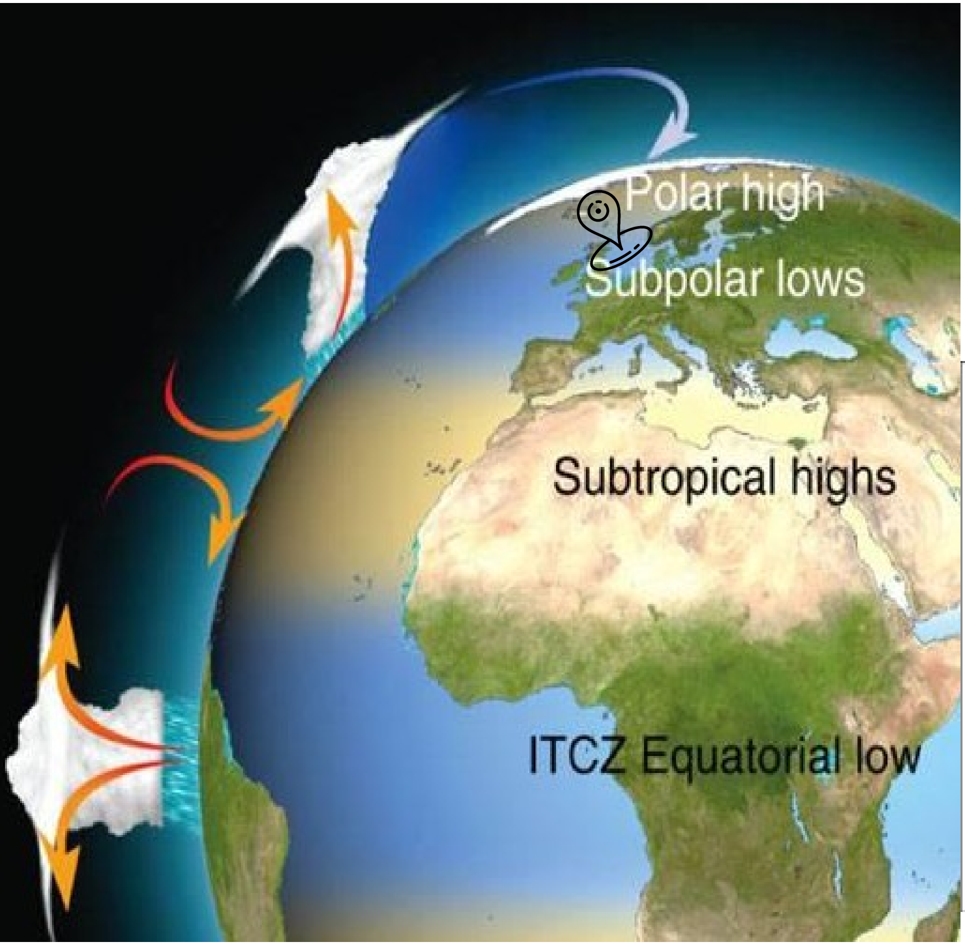
Application of hydrodynamics in 3D modelling of exoplanet atmospheres





The atmosphere: a heat engine

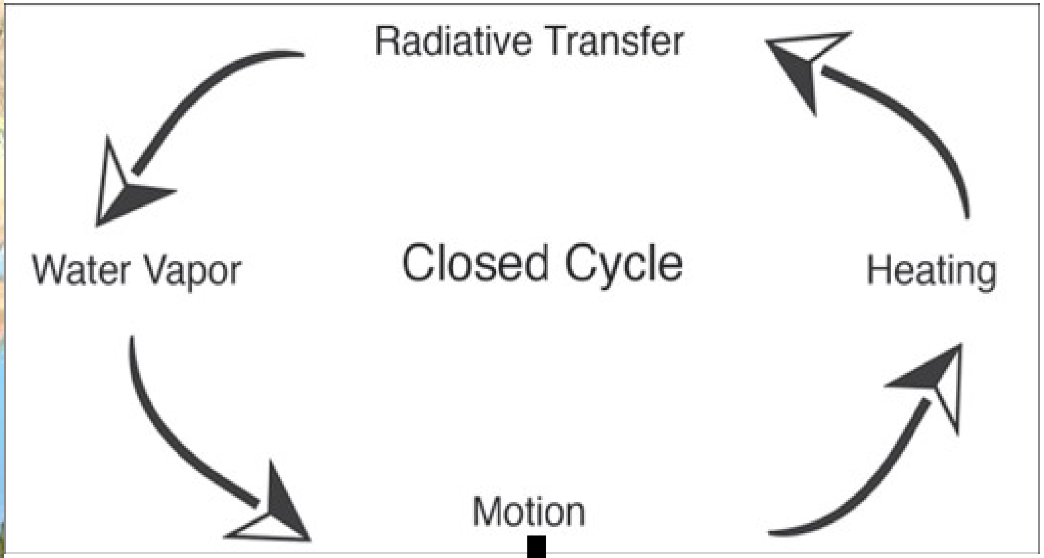


Wikipedia

Rotation

+

Δ T
Pole-Equator



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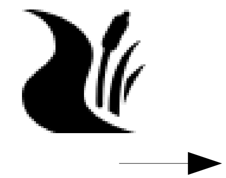
Surface friction

Textbook Marshall & Plumb

Let's follow the flow!

- Lagrangian: We follow the air parcel in space in time with flow u

- Eulerian: We stay fixed in space & time and watch air parcels (Field (x,t)) pass by with u



Local (Eulerian) change

Advection

$$\frac{D\,Field(\vec{x},t)}{Dt} = \frac{\partial\,Field(\vec{x},t)}{\partial t} + (\vec{u}\cdot\nabla)\,Field(\vec{x},t)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \qquad \vec{u} = u_x, u_y, u_z$$

Example of advection: Moist air blowing from the sea towards land

Material Derivative

$$\frac{D\,Field(\vec{x},t)}{Dt} = \frac{\partial\,Field(\vec{x},t)}{\partial t} + (\vec{u}\cdot\nabla)\,Field(\vec{x},t)$$

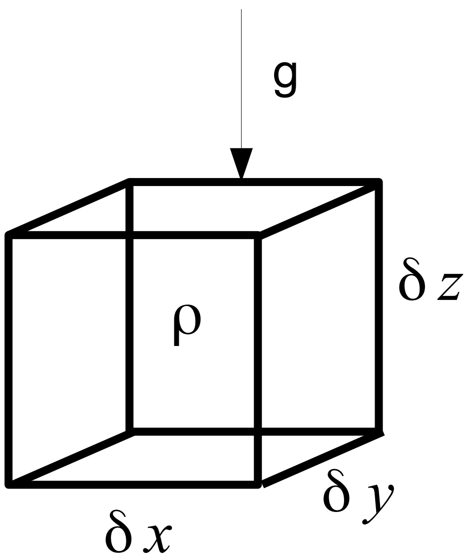
$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \qquad \vec{u} = (u_x, u_y, u_z)$$

$$\boxed{\vec{u}\cdot\nabla = u_x\frac{\partial}{\partial x} + u_y\frac{\partial}{\partial y} + u_z\frac{\partial}{\partial z}}$$

Stuff gets only advected if there is change within the stuff in one direction!

Newton's law in Lagrangian viewpoint = Momentum equations

- Forces on a fluid air package: Newton's law

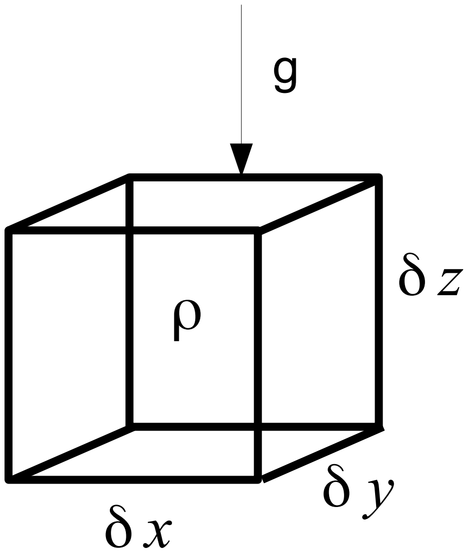


Mass x acceleration = Force

$$\rho \delta x \delta y \delta z \frac{D\vec{u}}{Dt} = \vec{F} \qquad \vec{F}_{\text{Gravity}} = (0, 0, -g \rho \delta x \delta y \delta z)$$

Newton's law in Lagrangian viewpoint = Momentum equations

- Forces on a fluid air package: Newton's law

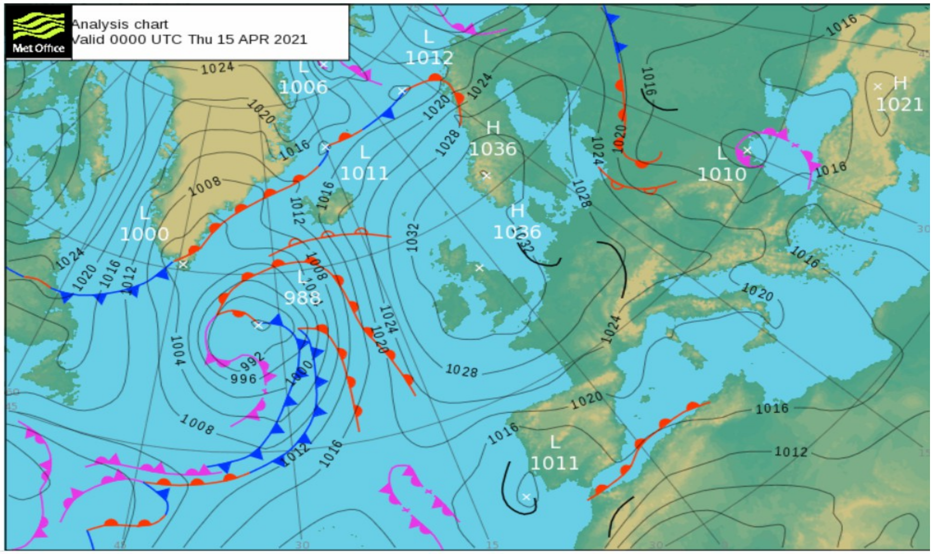


Mass x acceleration = Force

$$\rho\,\delta x\,\delta y\,\delta z \frac{D\vec{u}}{Dt} = \vec{F}$$

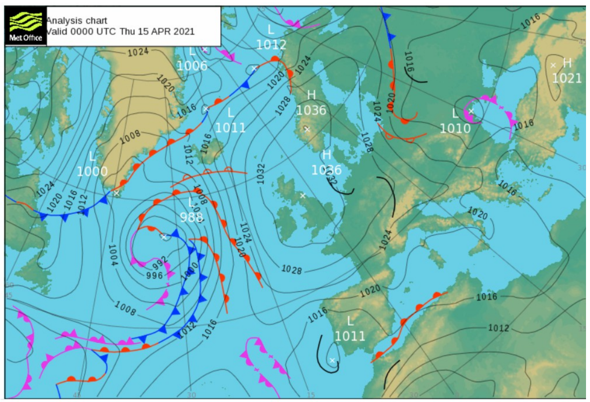
$$\vec{F}_{\text{Gravity}} = (0,0,-g\,\rho\,\delta x\,\delta y\,\delta z)$$

Pressure gradient



Why are meteorologists obsessed with pressure? Pressure gradient force!

- Forces on a fluid air package: Newton's law



Mass x acceleration

$$\rho\,\delta x\,\delta y\,\delta z\,\frac{D\vec{u}}{Dt}=\vec{F}$$

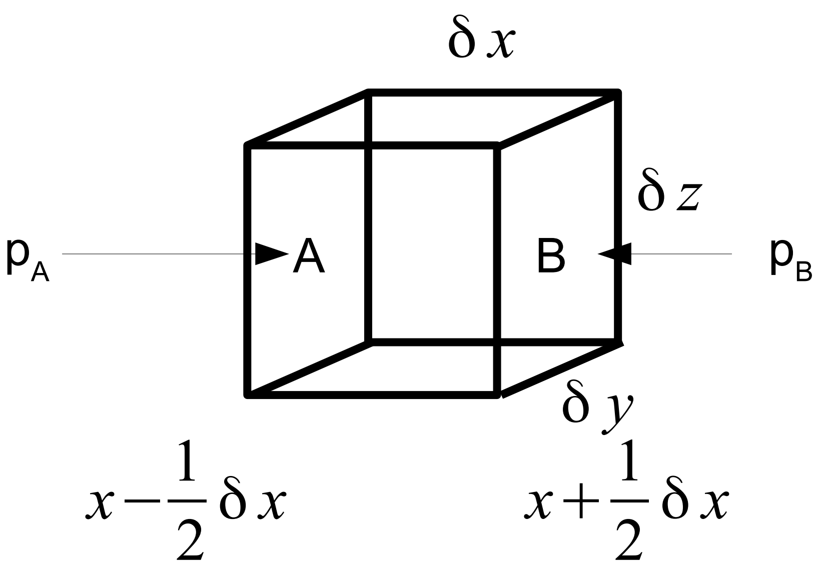
$$\vec{F}_{\text{Gravity}}=(0,0,-g\rho\,\delta x\,\delta y\,\delta z)$$

$$F_{A,B}=p_{A,B}\,\delta y\,\delta z$$

$$F_A=p(x-\delta x)\,\delta y\,\delta z$$

$$F_B=-p(x+\delta x)\,\delta y\,\delta z$$

$$F_x=[p(x-\delta x)-p(x+\delta x)]\,\delta y\,\delta z$$



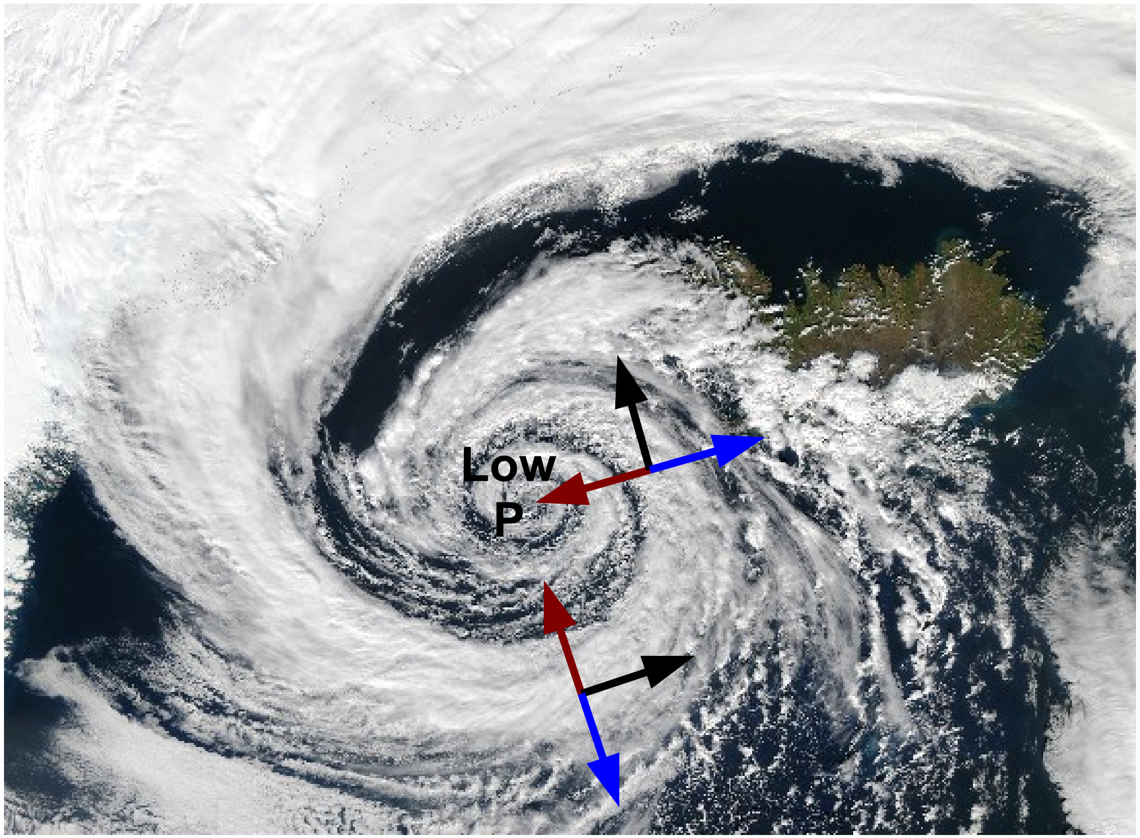
Taylor expansion

$$p_{A,B}=p(x)\mp\frac{\delta x}{2}\frac{\partial p}{\partial x}$$

$$\vec{F}_{\text{Pressure}}=\left(-\frac{\partial p}{\partial x}\delta x\,\delta y\,\delta z,0,0\right)$$

Add Rotation = Coriolis force

Source:
NASA,
Aqua MODIS
04/09/2003



$$-\frac{1}{\rho}\nabla_h p$$

$$-2\left(\vec{\Omega}\times\vec{v}\right)_h$$

$$\vec{v}_h$$

The center of a low-pressure system:The winds spiral towards the center in **counter-clockwise**

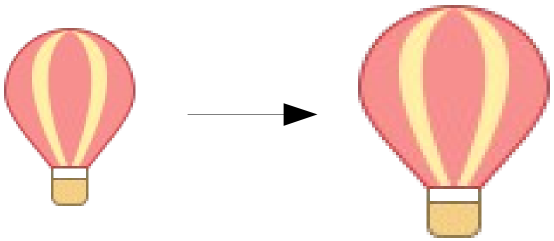
Basic equations that make stuff move in a planetary atmosphere

- Equation of motion

$$\frac{D\vec{u}}{Dt}+(0,0,g)+\frac{1}{\rho}\nabla p+2\Omega\times\vec{u}=\vec{F}_{\text{friction}}$$

- Thermodynamics (heating rate)

$$\frac{DQ}{Dt}=c_p\frac{DT}{Dt}-\frac{1}{\rho}\frac{Dp}{Dt}$$



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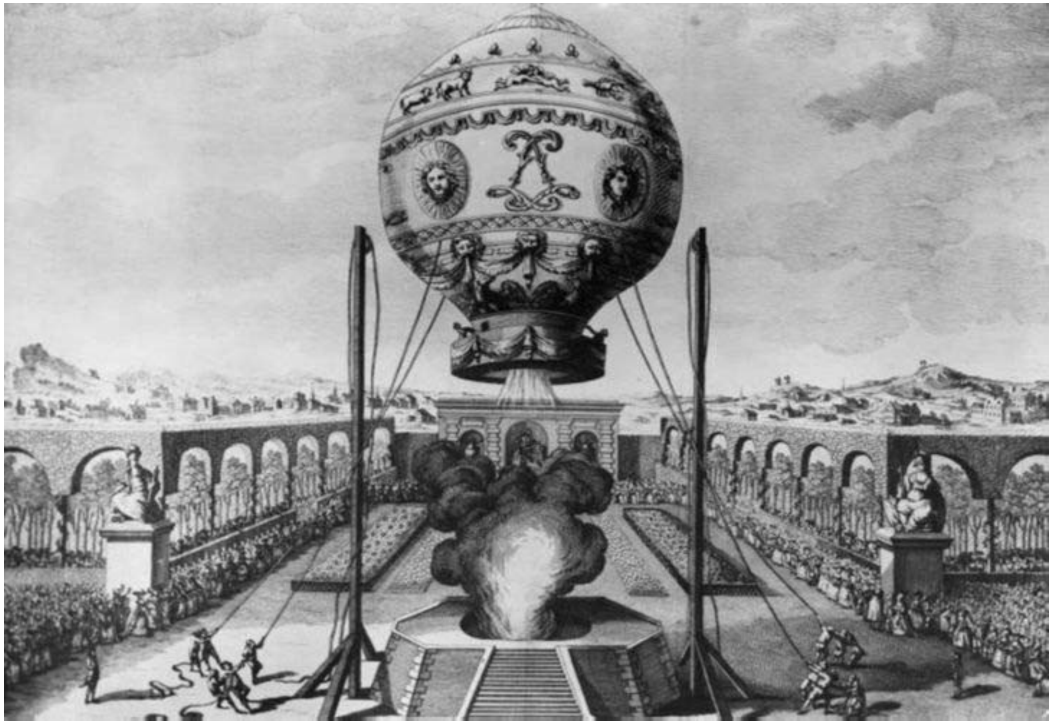
Conservation laws & Structure of the atmosphere

Hydrostatic equilibrium

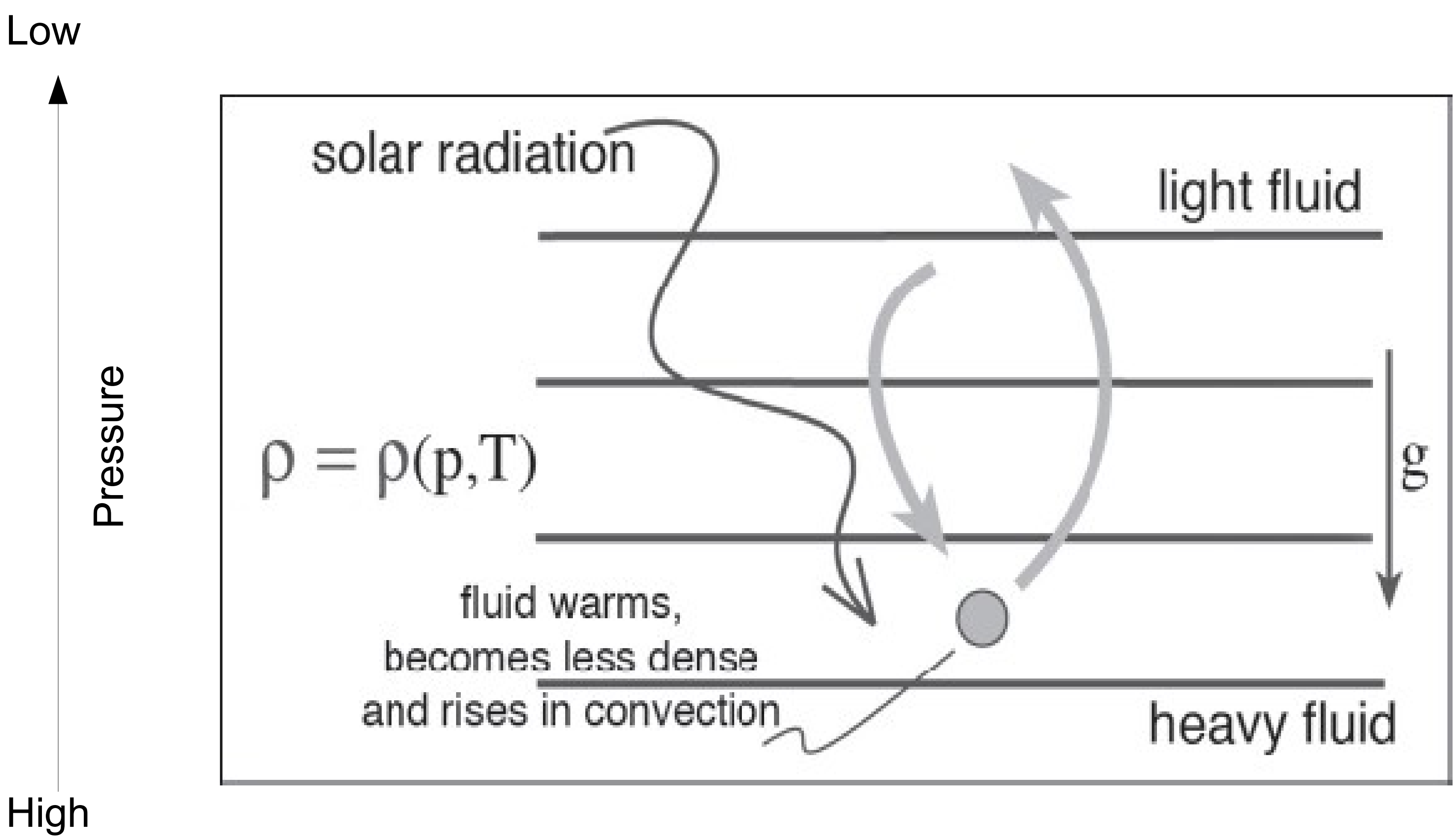


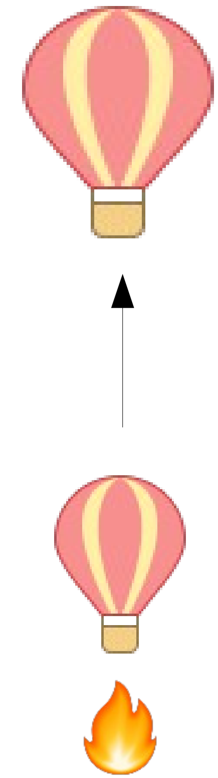
Hot air balloon of the Montgolfier brothers (1783)

$$\frac{\partial p}{\partial z} = -\rho g$$

Vertical structure of the atmosphere is generally in equilibrium between gravity and buoyancy (the latter is determined by **temperature**)

Why does stuff move in an atmosphere?





Ideal gas $\rho = \rho(p) = \frac{p}{T}\frac{R}{M}$

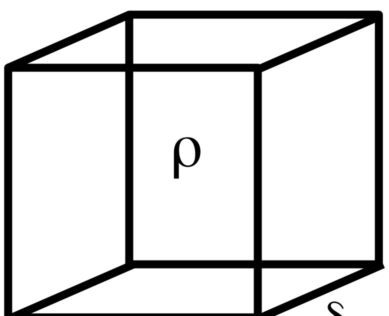
8.314 J/(K mol)

28.9645 g/mol
~ 0.80 x 28 (N_2)+0.2 x 32 (O_2)

Textbook Marshall & Plumb

Mass continuity in pressure coordinates

- Mass continuity (follow $\frac{D\rho}{Dt}$)



$$\text{Mass} = \rho\, \delta x\, \delta y\, \delta z$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\delta z}{\delta p} = \frac{\partial z}{\partial p}$$

$$\text{Mass} = \rho - \left(\frac{1}{g\rho}\right) \delta x\, \delta y\, \delta p$$
$$= \frac{1}{g} \delta x\, \delta y\, \delta p$$

Mass continuity: $\nabla_p \cdot \vec{u}_p = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_p}{\partial p} = 0$

To first order g= const

Summary of Hydrostatic primitive equations

Momentum equation

$$\frac{D\vec{u}}{Dt} + (0,0,g) + \frac{1}{\rho}\nabla p + 2\Omega \times \vec{u} = \vec{F}_{\text{friction}}$$

$$(\ldots, \partial_i u_j u_i, \ldots.) \quad \text{2nd order differential eq.}$$

Heating

$$\frac{DQ}{Dt} = c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$$

(Cartesian)

Mass continuity

$$\nabla_p \cdot \vec{u}_p = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_p}{\partial p} = 0$$

(Pressure)

Hydrostatic equilibrum

$$\frac{\partial p}{\partial z} = -\rho g$$

(Cartesian)

Equation of state

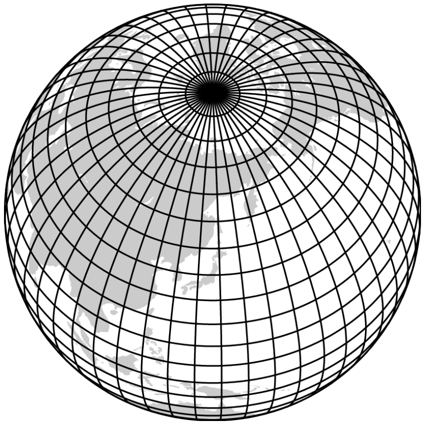
$$\rho = \rho(p, T) = \frac{p}{T}\frac{R}{M}$$

(Pressure)

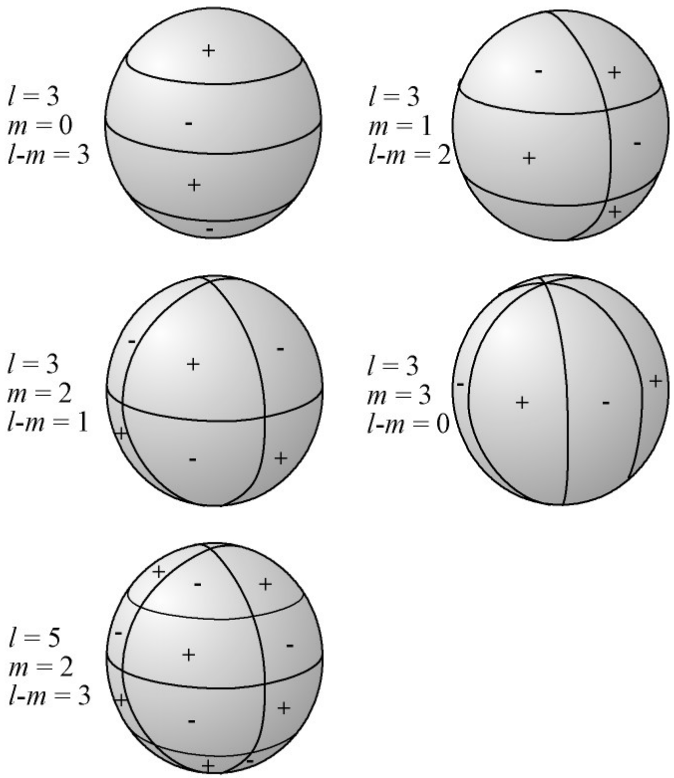
Numerical implementation

- Time stepping
  - Implicit
  - Explicit: Adam-Bashforth 2n order
- Change of stuff over distance/volume
  - Spectral vs grid
    - Finite volume
      - Courant-Friedrichs-Lewy condition < 0.6 (MITgcm specific)

Squaring the circle



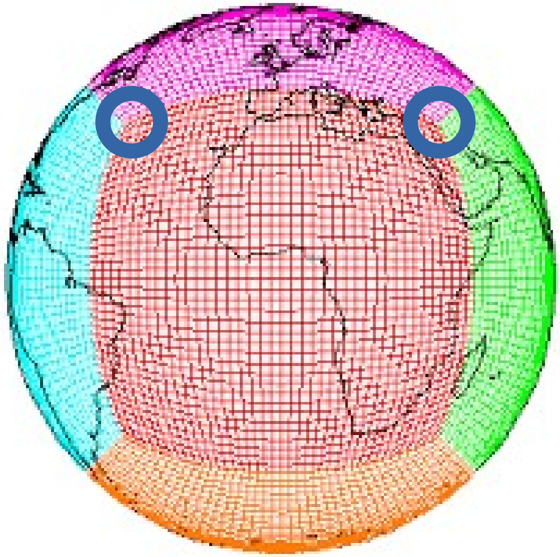
- Spherical harmonics:



Christophe Dang Ngoc Chan
Wikimedia

$$Y_\ell^m(\theta,\phi) := (-1)^m \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\, P_\ell^m(\cos\theta)\, e^{im\phi}.$$

- Gridding:
  - Lat-lon (polar problem)
  - Square grids: 6 Tiles



  - Hexagons

CFL condition

$$C = u\frac{\Delta t}{\Delta x} \leq C_{max}$$ (typically 1, here 0.6)

u: Max of wind flow

Δt: Time step

Δx: Cell size (square)

CAUTION: Inherent diffusive terms in finite volume

Exercises

BUILD YOUR OWN EARTH

Summary of Hydrostatic primitive equations

Momentum equation

$$\frac{D\vec{u}}{Dt}+(0,0,g)+\frac{1}{\rho}\nabla p+2\Omega\times\vec{u}=\vec{F}_{\text{friction}}$$

$$(\ldots,\partial_i u_j u_i,\ldots.)$$ 2nd order differential eq.

Heating

$$\frac{DQ}{Dt}=c_p\frac{DT}{Dt}-\frac{1}{\rho}\frac{Dp}{Dt}$$

(Cartesian)

Mass continuity

$$\nabla_p\cdot\vec{u}_p=\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_p}{\partial p}=0$$

(Pressure)

Hydrostatic equilibrum

$$\frac{\partial p}{\partial z}=-\rho g$$

(Cartesian)

Equation of state

$$\rho=\rho(p,T)=\frac{p}{T}\frac{R}{M}$$

(Pressure)

Held & Suarez: Simple Earth

Surface friction $$\frac{\partial v}{\partial t} = \cdots - k_v(\sigma) v$$

Newtonian cooling $$\frac{\partial T}{\partial t} = \cdots - k_T(\phi, \sigma)\left[T - T_{eq}(\phi, p)\right]$$

$$T_{eq} = \max\left\{200\text{K}, \left[315\text{K} - (\Delta T)_y \sin^2\phi - \qquad \right]\left(\frac{p}{p_0}\right)^{\kappa}\right\}$$

Held & Suarez: Simple Earth

k_v= 1 day^{-1} over surface

Surface friction $$\frac{\partial v}{\partial t} = \cdots - k_v(\sigma)v$$

$$\sigma \propto p$$
$$u = v$$

Newtonian cooling $$\frac{\partial T}{\partial t} = \cdots - k_T(\phi,\sigma)\left[T - T_{eq}(\phi,p)\right]$$

k_T= 1-1/40 day^{-1}

Numerical stabilization

Adiabtic cooling with height

$$T_{eq} = \max\left\{200\text{K}, \left[315\text{K} - (\Delta T)_y \sin^2\phi - (\Delta\theta)_z \log\left(\frac{p}{p_0}\right)\cos^2\phi\right]\left(\frac{p}{p_0}\right)^{\kappa}\right\}$$

Lower stratosphere temperature

Equator temperature

Δ T Equator $\leftarrow\rightarrow$ Pole

$$\kappa = \frac{R_{specific}}{c_P} = \frac{2}{7}$$

Held & Suarez 1994

Can you see the forcing in the model?

- Do you notice what is apparently missing?

- What ingredients are missing in this recipe for real life Earth?

Held & Suarez: Simple Earth

$$\theta = \left(\frac{p}{p_0}\right)^{\kappa}$$

Adiabtic cooling with height

$$T_{eq} = \max\left\{200\mathrm{K}, \left[315\mathrm{K} - (\Delta T)_y \sin^2\phi - (\Delta\theta)_z \log\left(\frac{p}{p_0}\right)\cos^2\phi\right]\left(\frac{p}{p_0}\right)^{\kappa}\right\}$$

Lower stratosphere temperature

Equator temperature

Δ T Equator $\leftarrow\rightarrow$ Pole

$$\kappa = \frac{R_{specific}}{c_P} = \frac{2}{7}$$

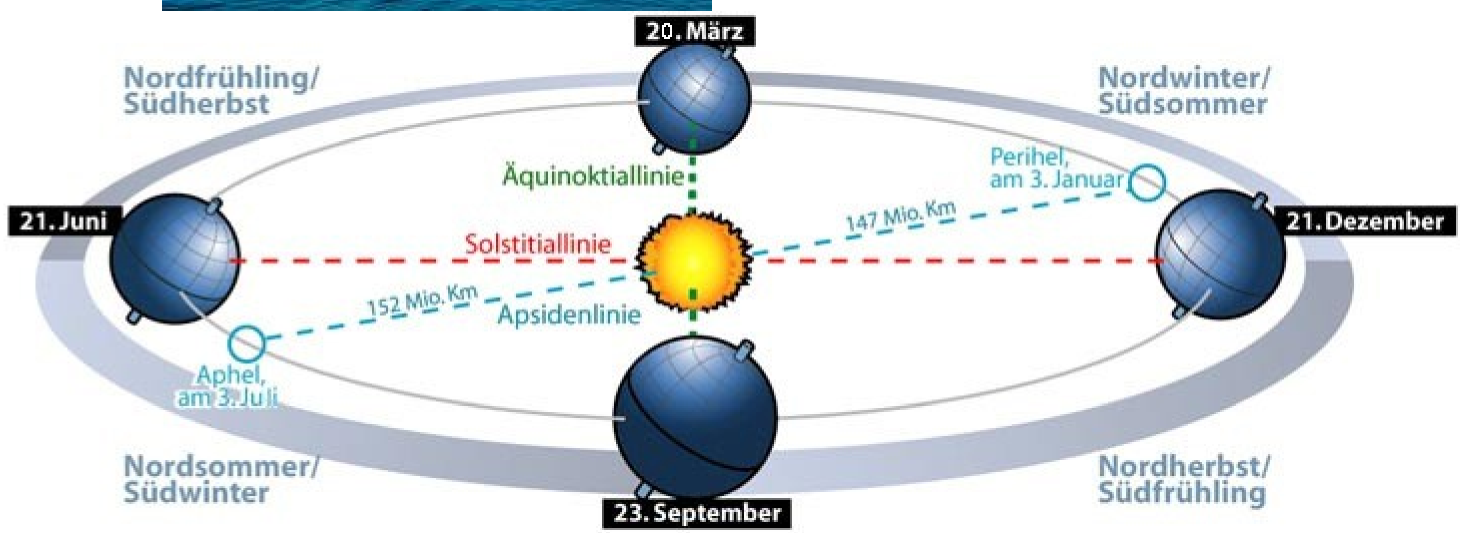
Held & Suarez 1994

Held & Suarez: Missing

- Topography

- Oceans

- Seasons
- Day/night



We don't care: b/c we look at climate

- Time average
  - Monthly
  - seasonally
  - yearly

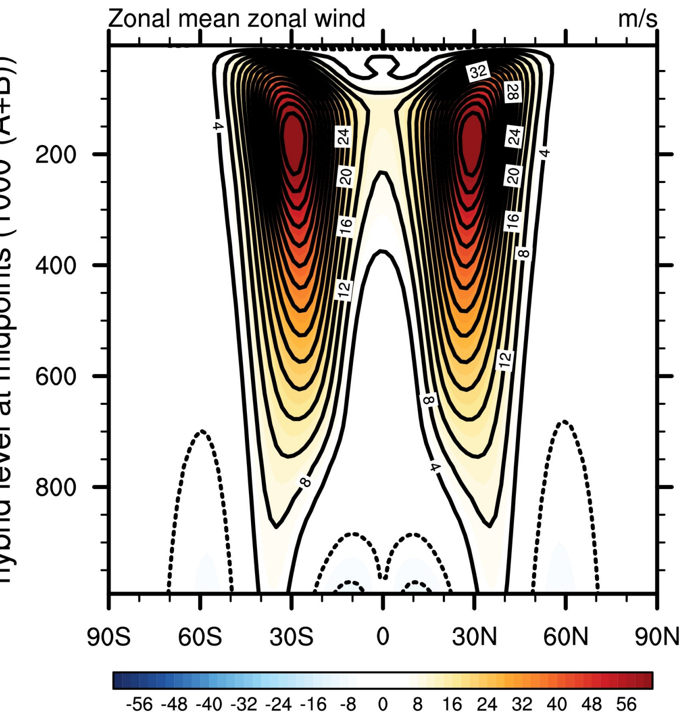
- Zonal average (from east to west)

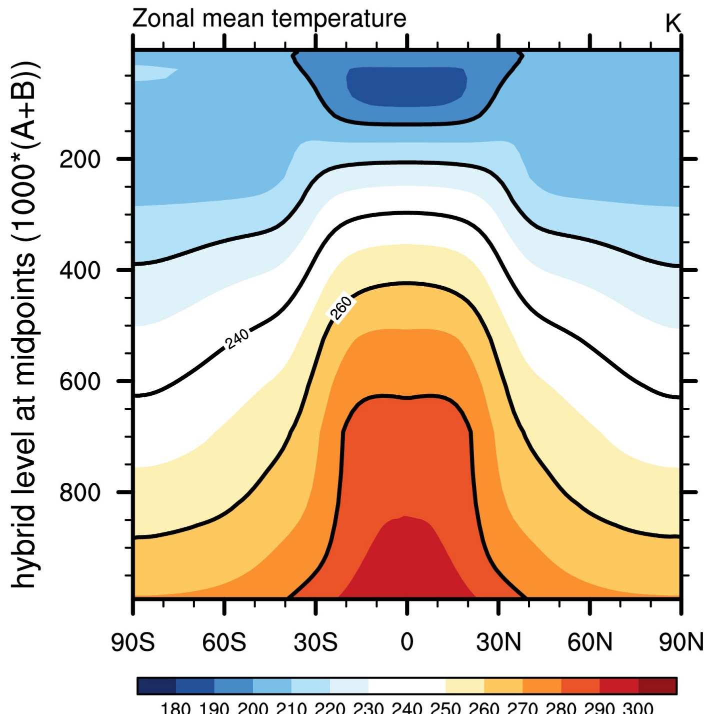
Plot zonal mean: Temp, wind(U), circulation

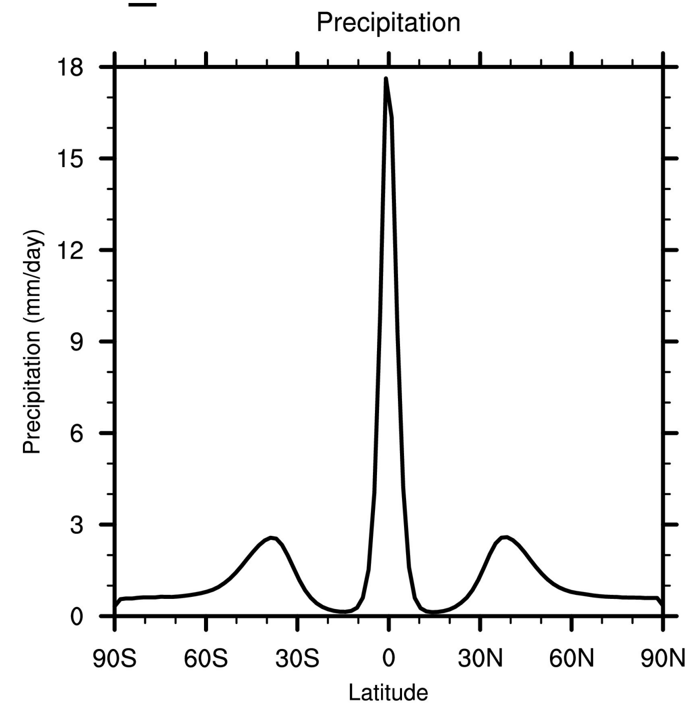
- https://gcmtools.readthedocs.io/en/latest/notebooks/demo.html

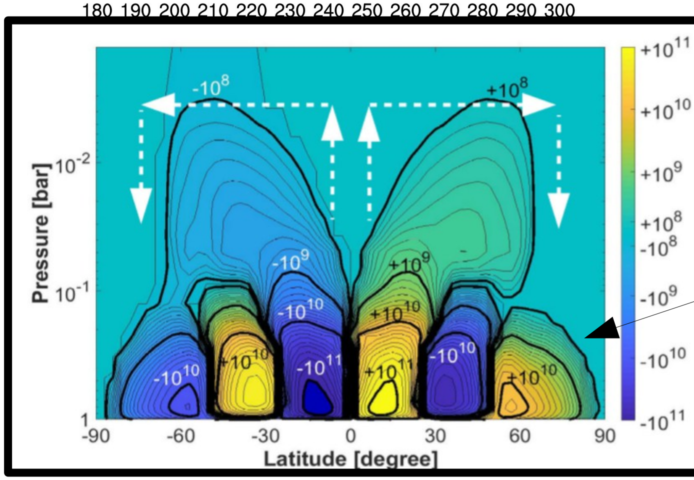
Held & Suarez: Simple Earth

https://www.cesm.ucar.edu/models/simpler-models/moist_hs/index.html





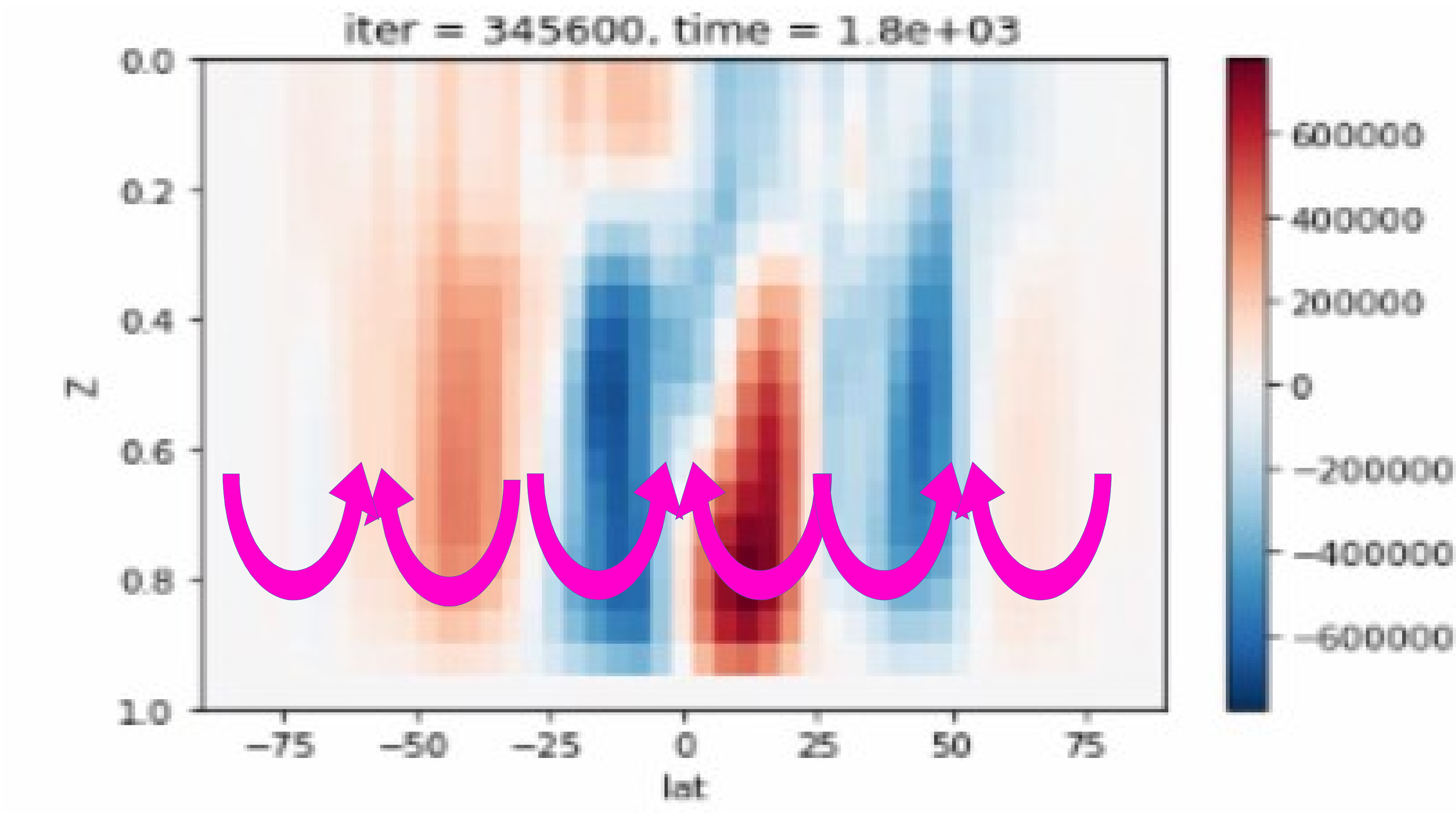




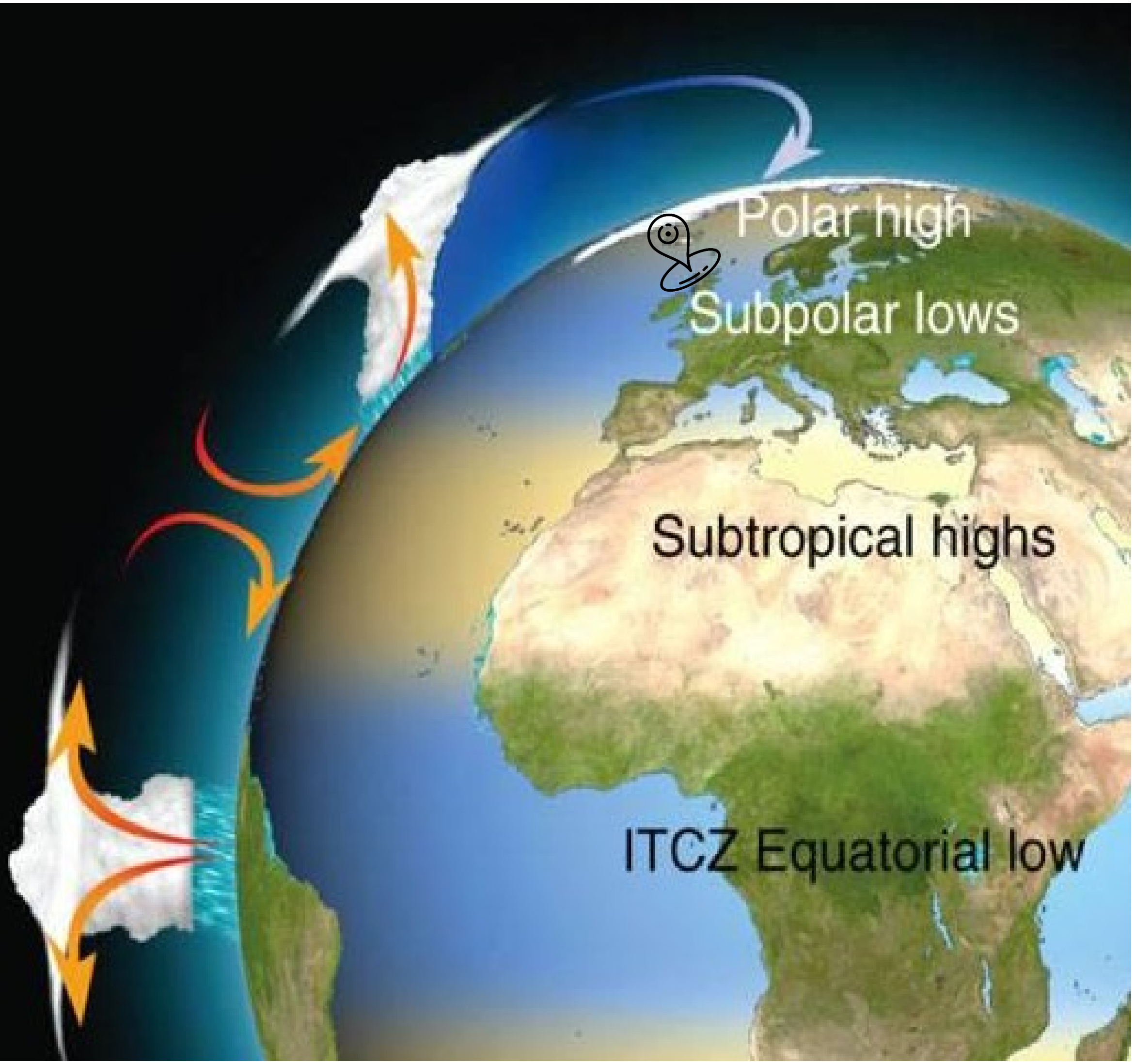
Circulation cells per hemisphere

Carone+2018

Circulation



Earth has three circulation cells per hemisphere



Wikipedia

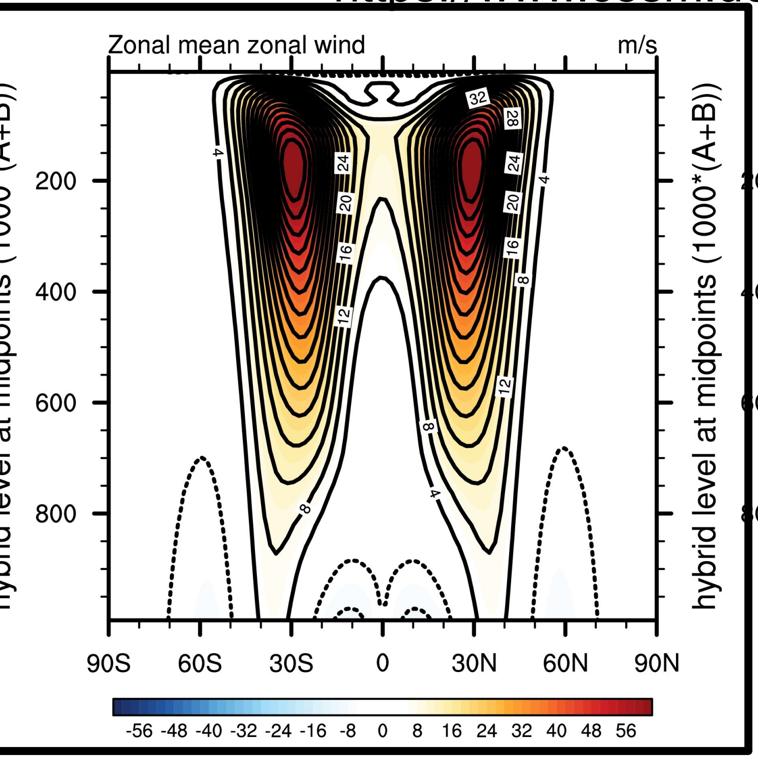
Polar cell

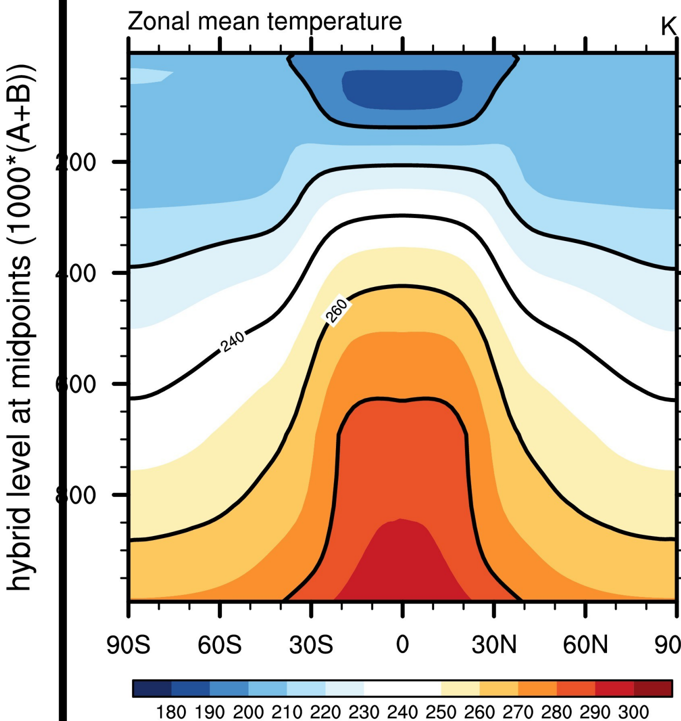
Ferrel cell

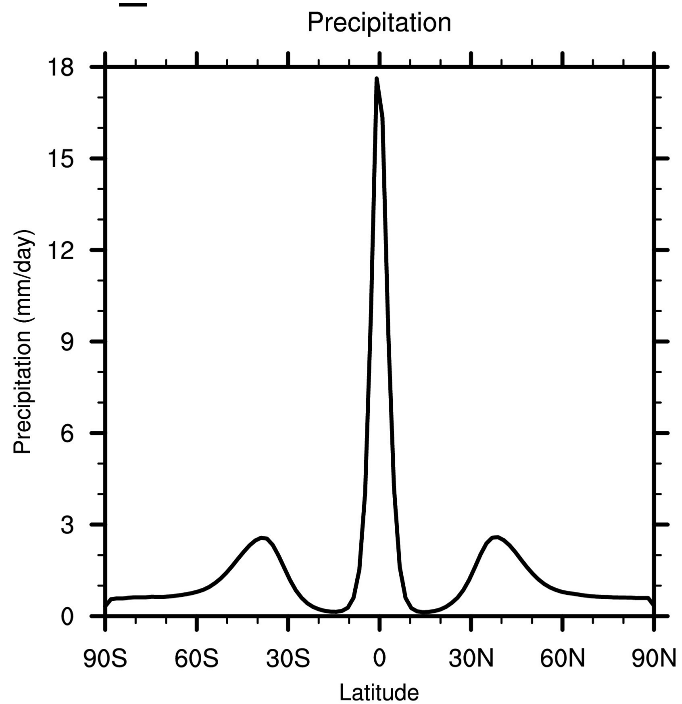
Hadley cell

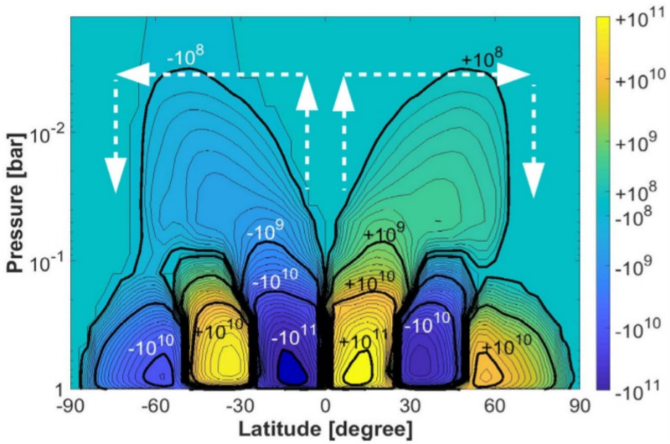
Held & Suarez: Simple Earth

https://www.cesm.ucar.edu/models/simpler-models/moist_hs/index.html

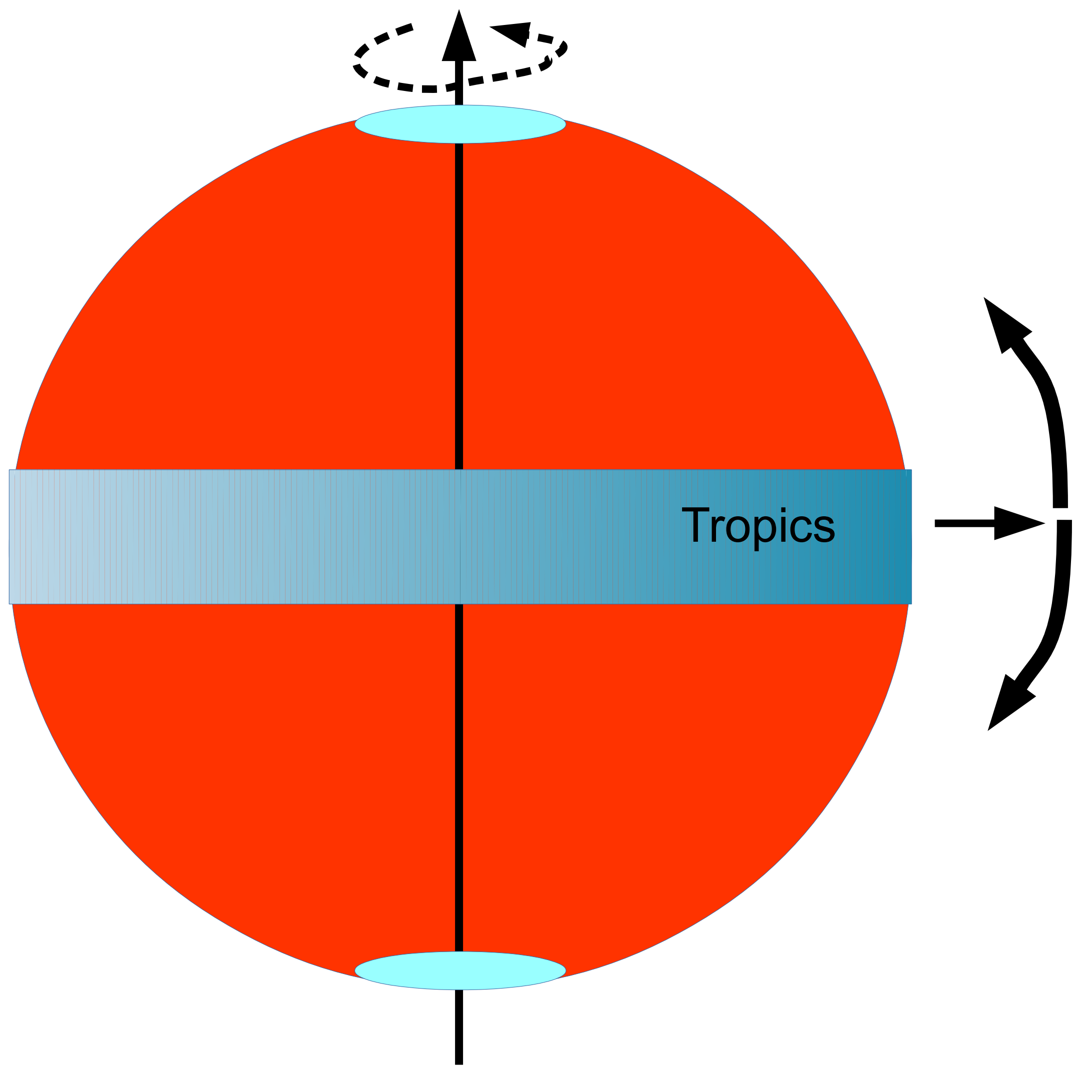




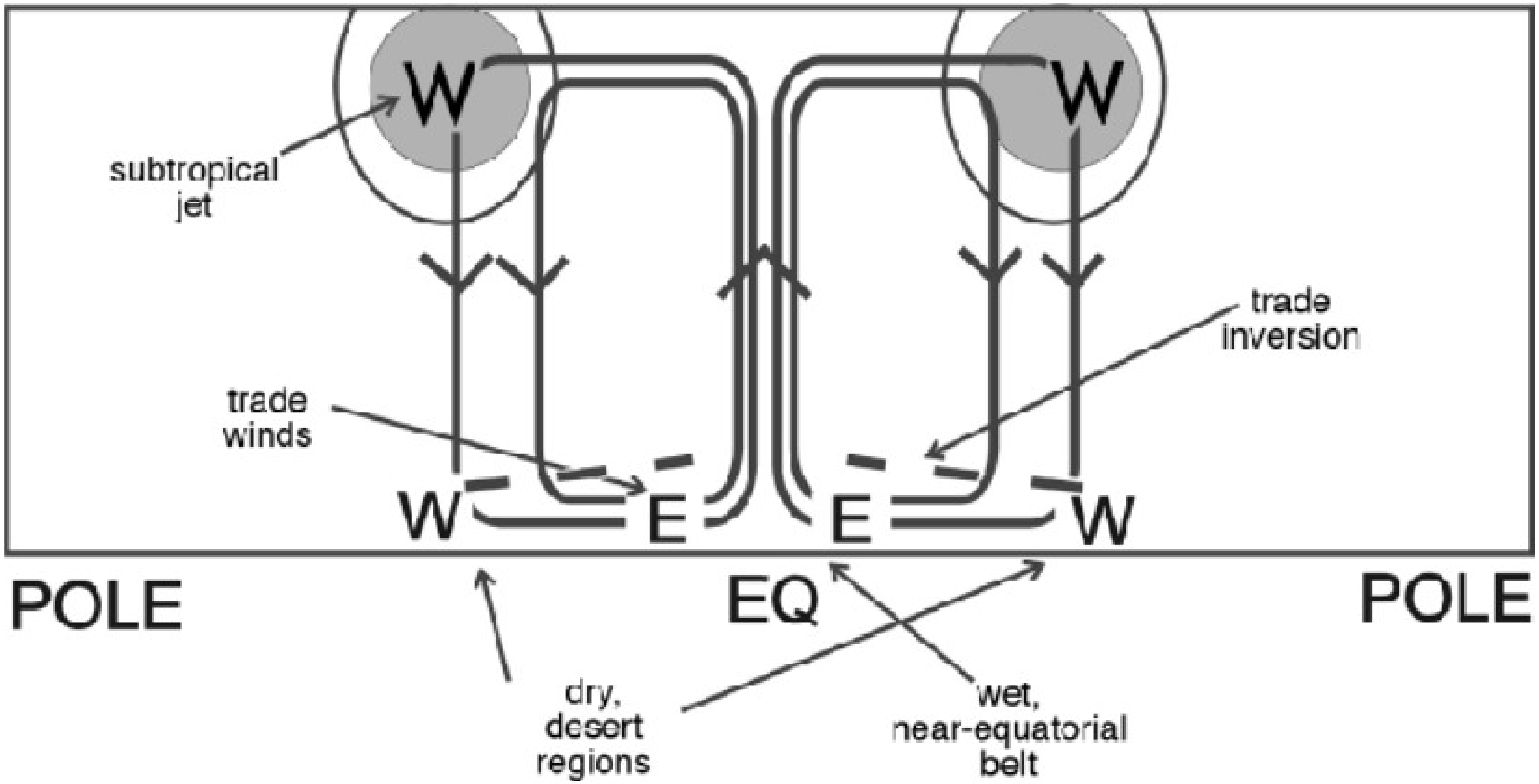




Pay attention to angular momentum

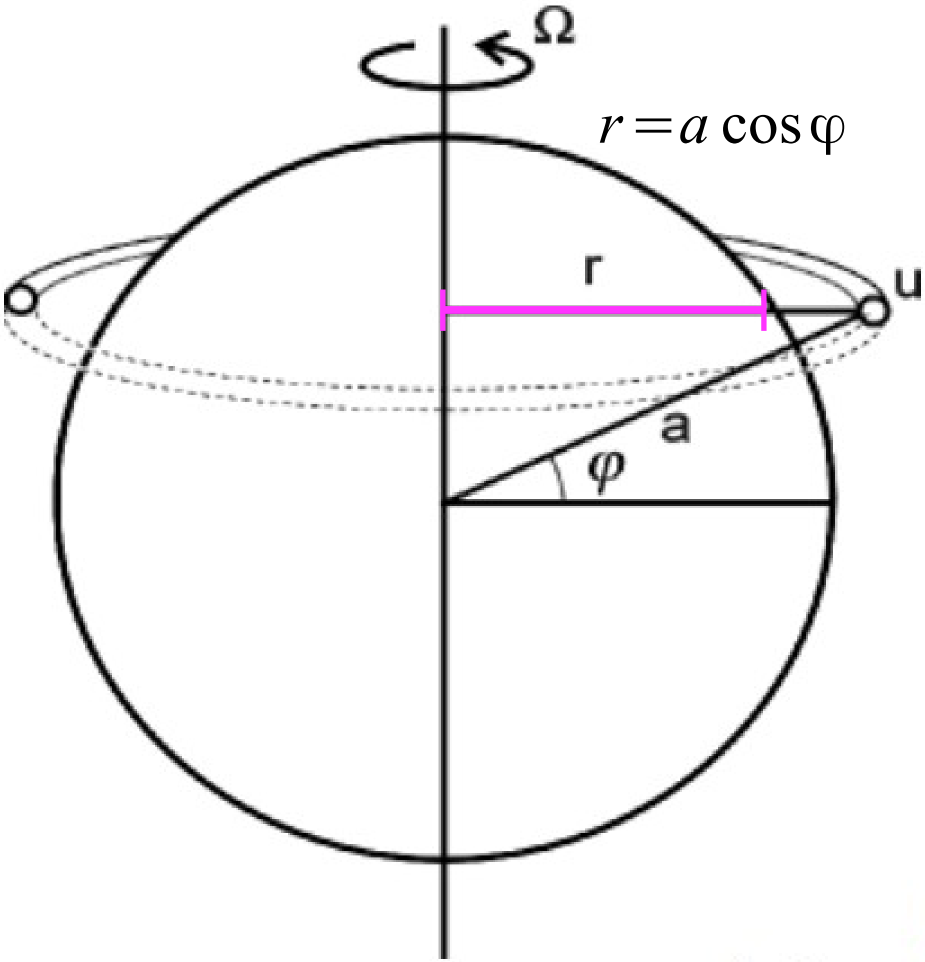


Hadley cell + angular momentum



Marshall & Plumb

Exercise: Check for yourself!



$$A_{ex} = \Omega a^2 \cos^2\varphi + ua\cos\varphi$$

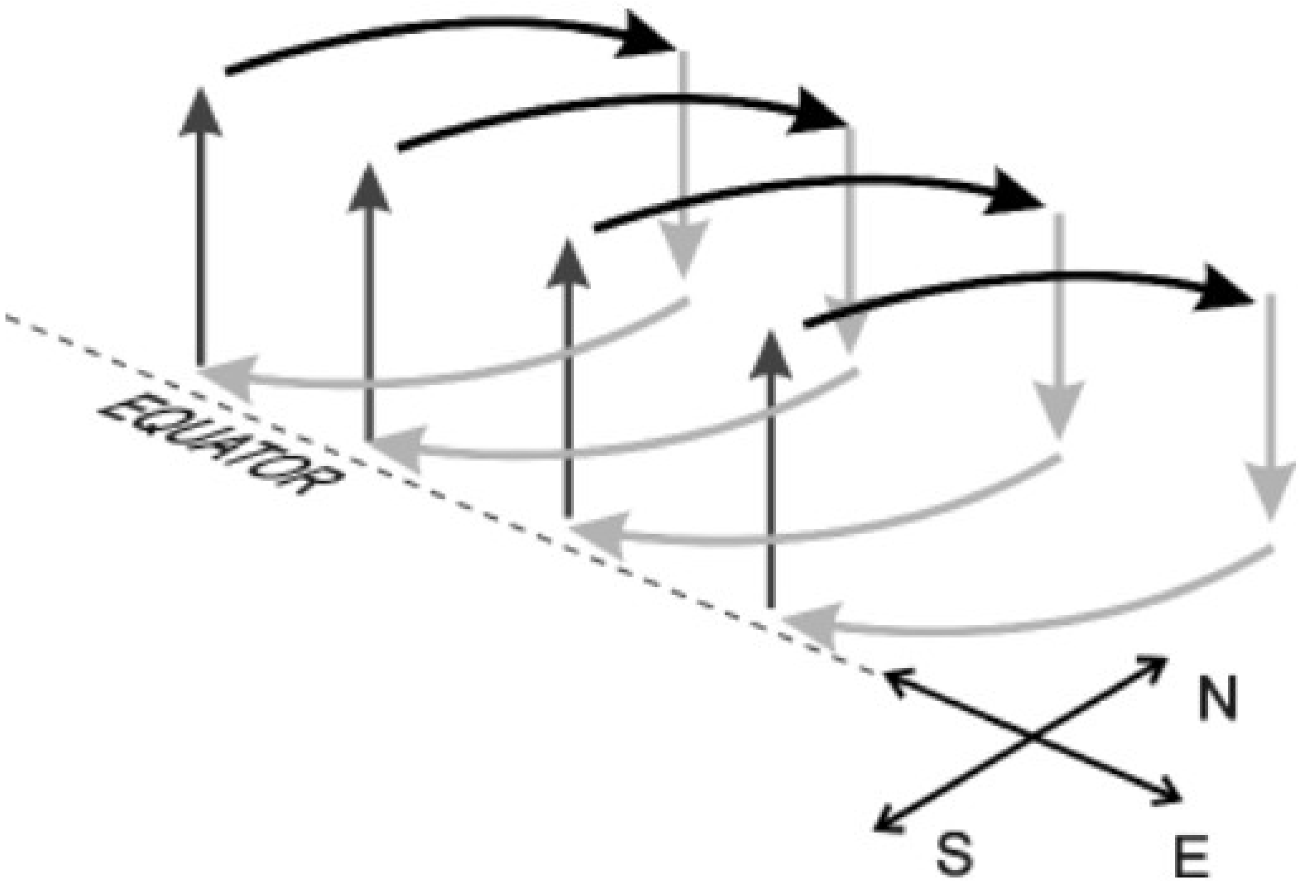
=

$$A_{eq} = \Omega r^2 + ur,$$

a^2 0

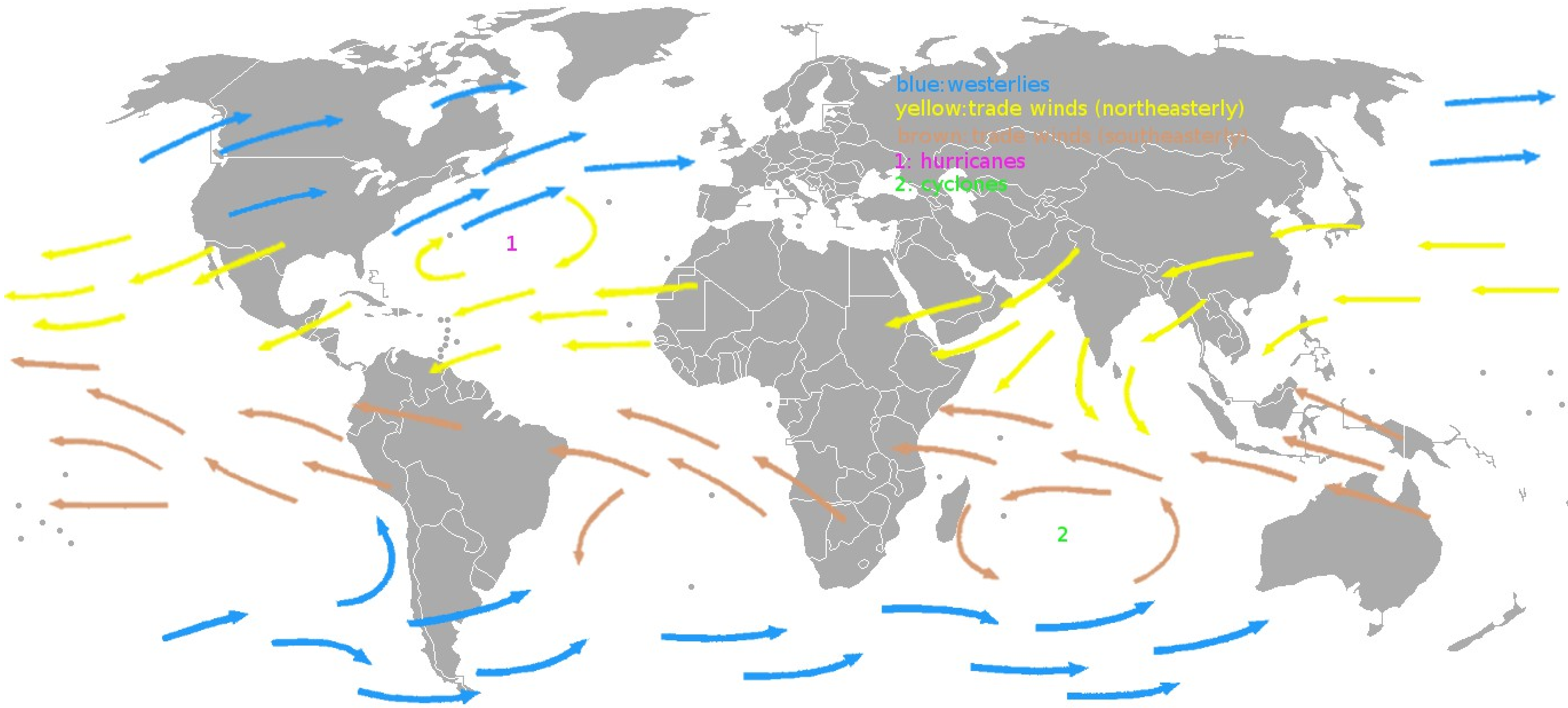
$$u(\varphi) = \frac{\Omega\left(a^2 - a^2\cos^2\varphi\right)}{a\cos\varphi} = \Omega a \frac{\sin^2\varphi}{\cos\varphi}.$$

Hadley cell + angular momentum



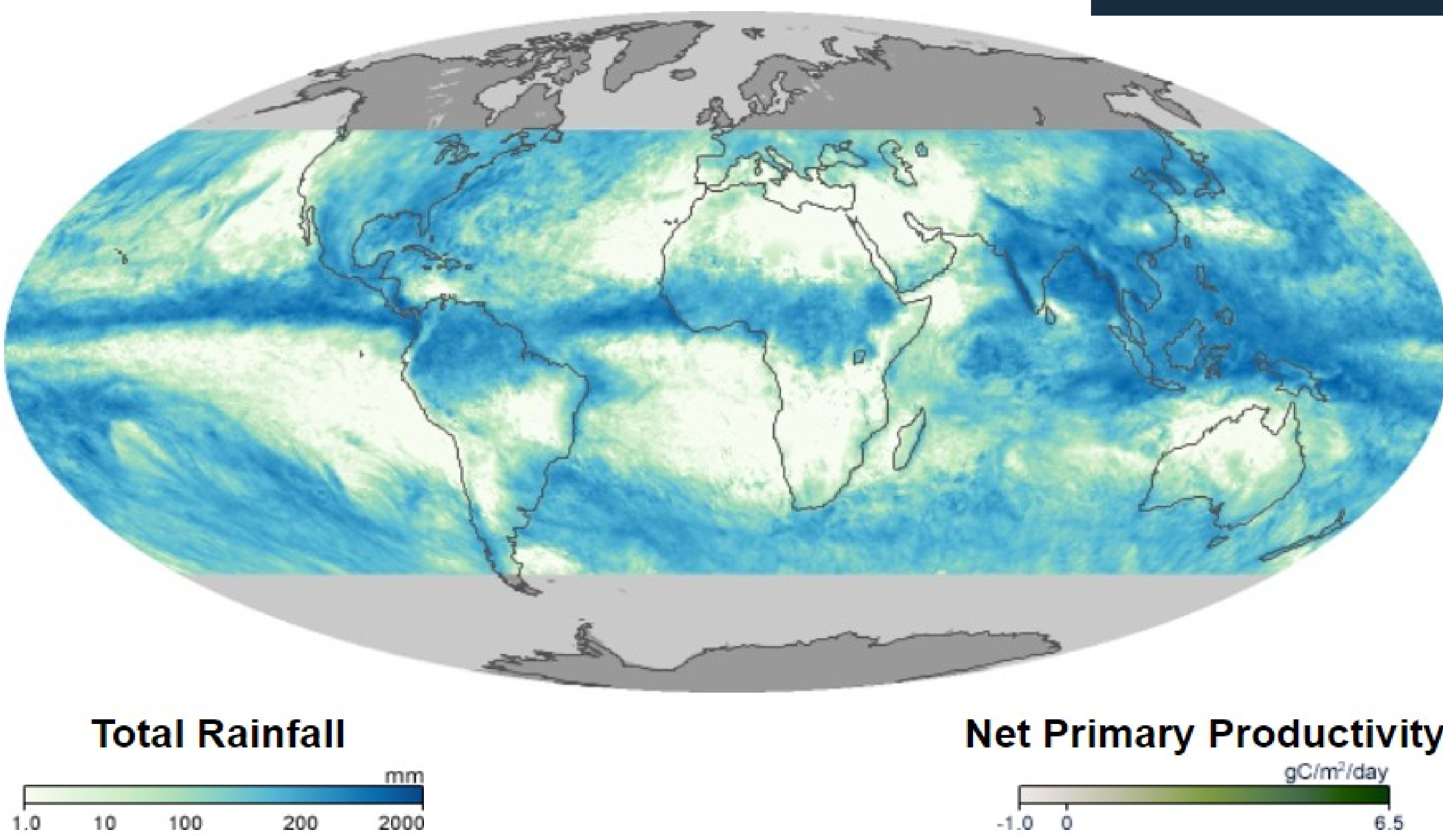
Marshall & Plumb

Plot horizontal slice over surface (Temp/Wind)



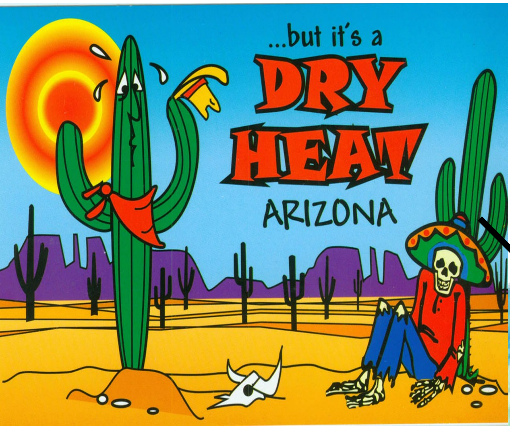
https://upload.wikimedia.org/wikipedia/commons/1/18/Map_prevailing_winds_on_earth.png

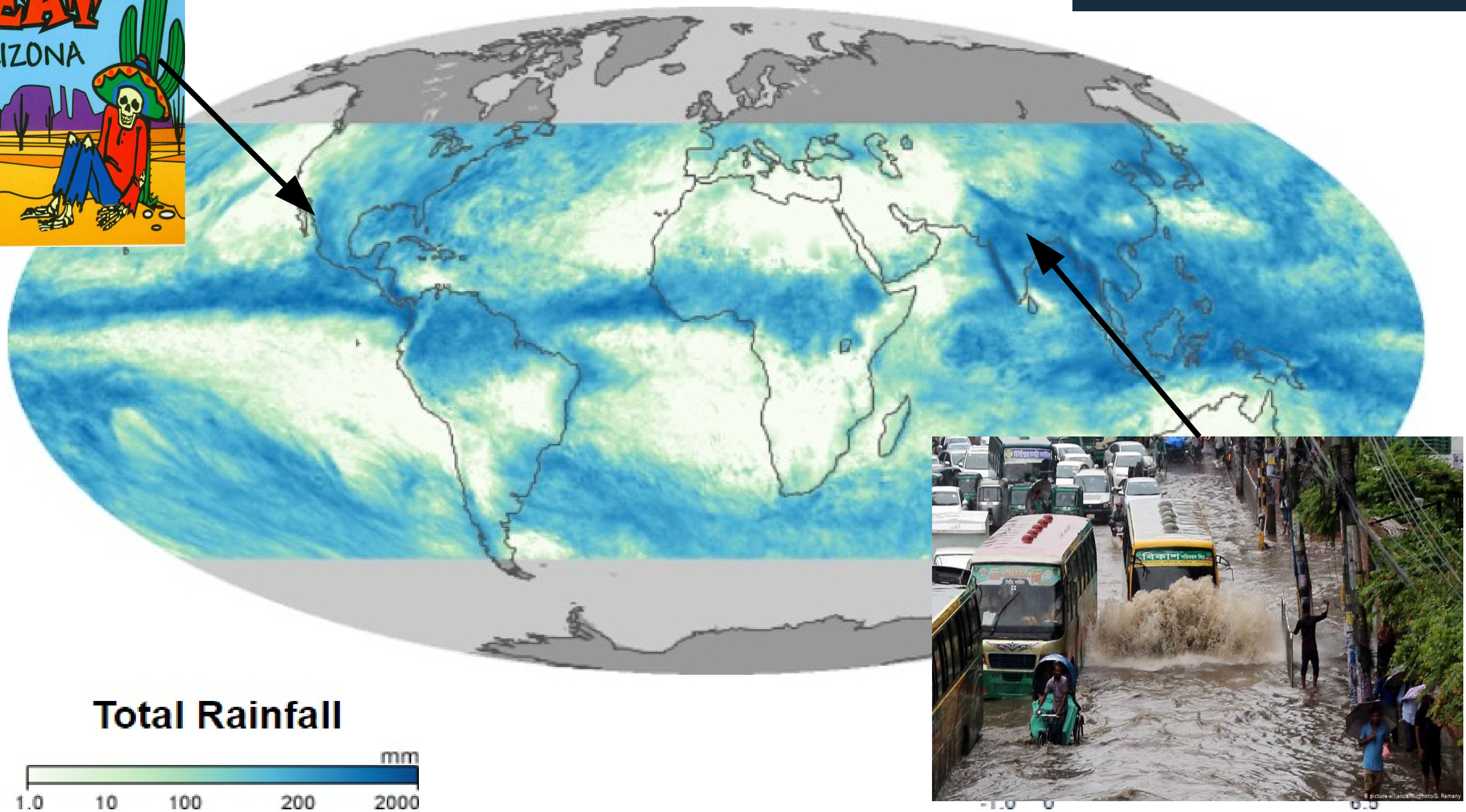
Fluid Dynamics on Earth: A world with conveyor belts



July 2013

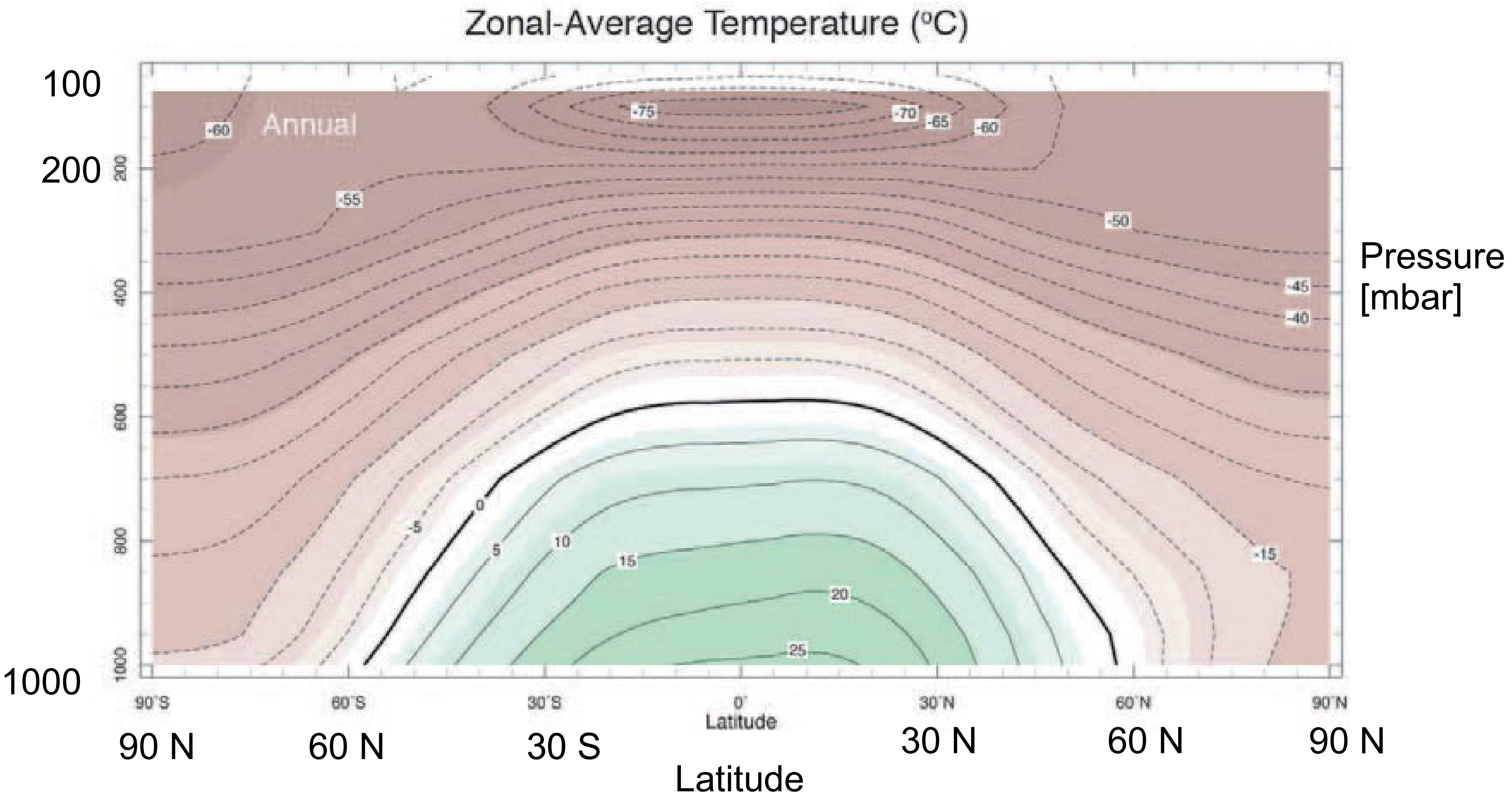
Fluid Dynamics on Earth: a matter of habitability





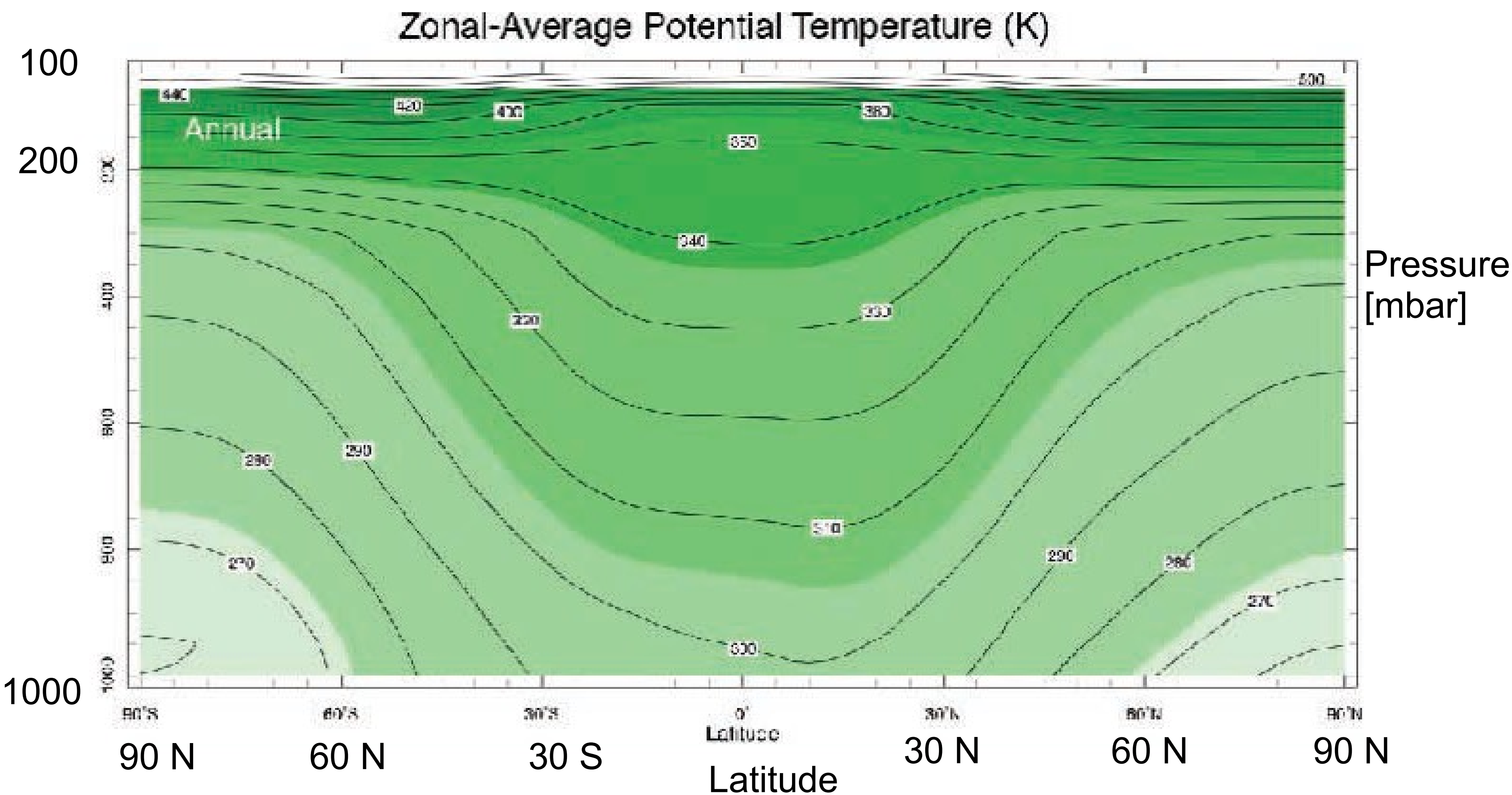
July 2013

A look at Earth



Plumb & Marshall: Introduction atmosphere, ocean & climate

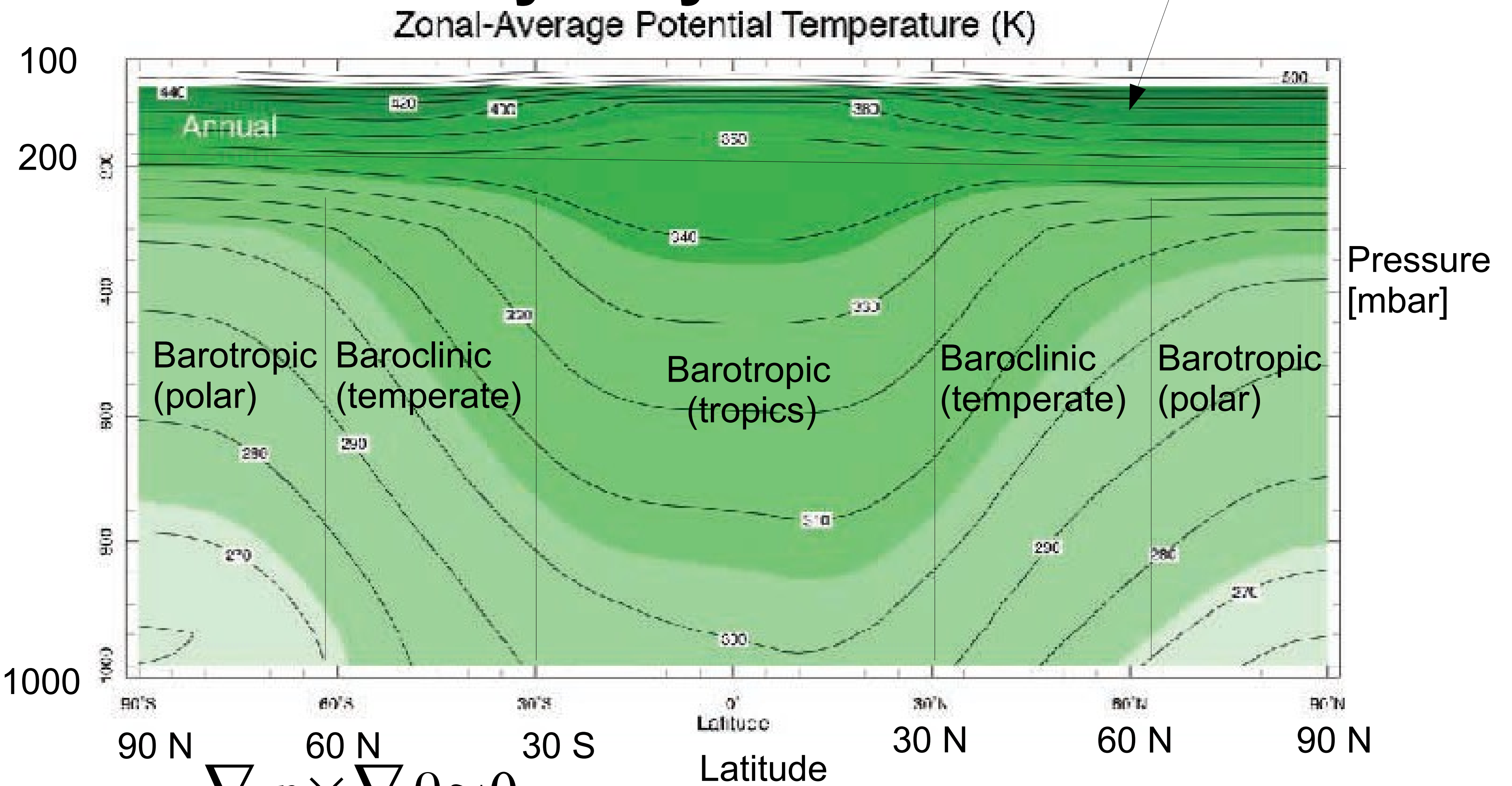
A look at Earth



Plumb & Marshall: Introduction atmosphere,
ocean & climate

A look at Earth by a dynamicist

$$\frac{\partial \theta}{\partial p} \gg 0$$



Barotropic: $\nabla p \times \nabla \theta \approx 0$

Baroclinic: $\nabla p \times \nabla \theta \,!= 0$

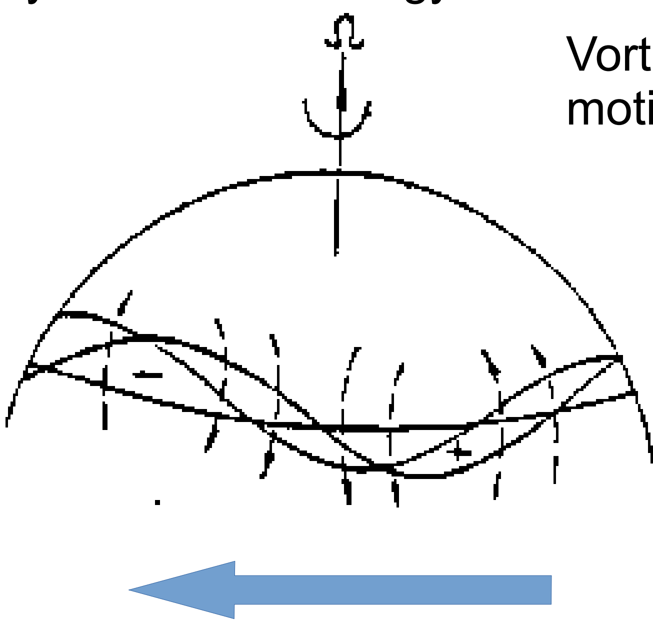
Plumb & Marshall: Introduction atmosphere, ocean & climate

Look at horizontal slices:

Do you notice something off the equator?

Rossby wave

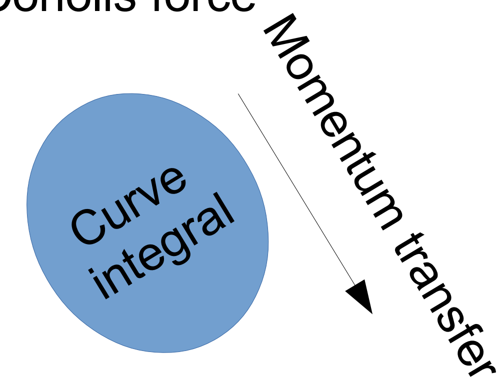
Holton, Dynamic meteorology



Vorticity conserving pertubation propagation motion due to latitudinal variation of Coriolis force

$$\eta = \zeta + f$$

$$\zeta = \vec{k} \cdot (\vec{\nabla} \times \vec{v})$$



Westward propagating wave (phase) induced by north- and southward displaced chain of air parcels, that start to rotate clockwise and anti-clockwise

Waves shift momentum around or Rossby waves numbers

- Amplitude of Rossby wave

Buoyancy frequency equiv. Vertical thermal stability

Scale height

$$N^2 = \frac{g}{\theta_E}\frac{d\theta_E}{dz},$$

Tropical

$$\lambda_R = \sqrt{\frac{N H}{2\beta}}$$

$$\beta = \frac{df}{dy} = \frac{2\Omega}{R_P}$$

Meridional change in Coriolis force at equator

Holton+2010
Kataria+2016
Carone+2020

Extra tropical

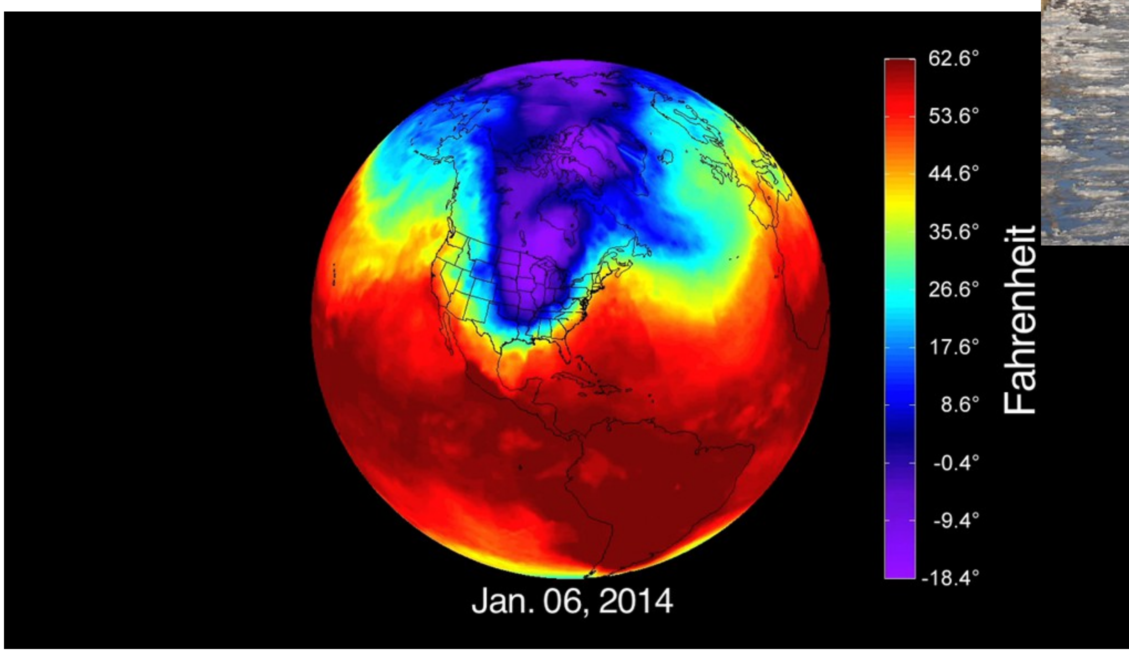
$$L_R = \frac{N H}{f}$$

Coriolis force at mid-latitude

Rossby wave(s) – Why care?

Philadelphia, January 7, 2014.
-16ºC

Zonal wave number = 4-6



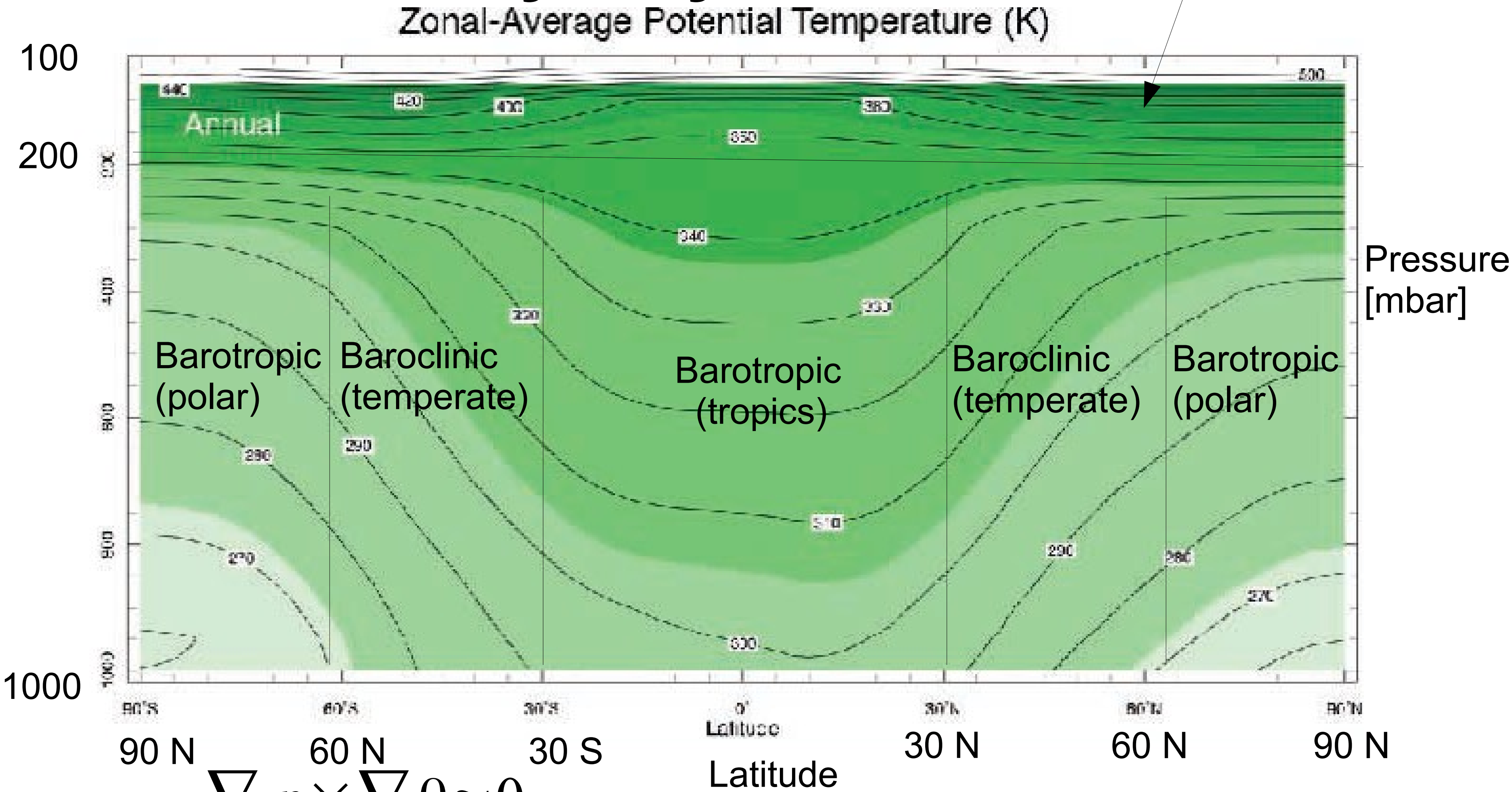
Wikimedia, User: Shuvaev

NASA's Goddard Space Flight
CenterVideo and images courtesy of
NASA/JPL
AIRS (Atmospheric InfraRed Sounder)

http://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=11451

A look at Earth by a dynamicist

$$\frac{\partial \theta}{\partial p} \gg 0$$



Barotropic: $\nabla p \times \nabla \theta \approx 0$

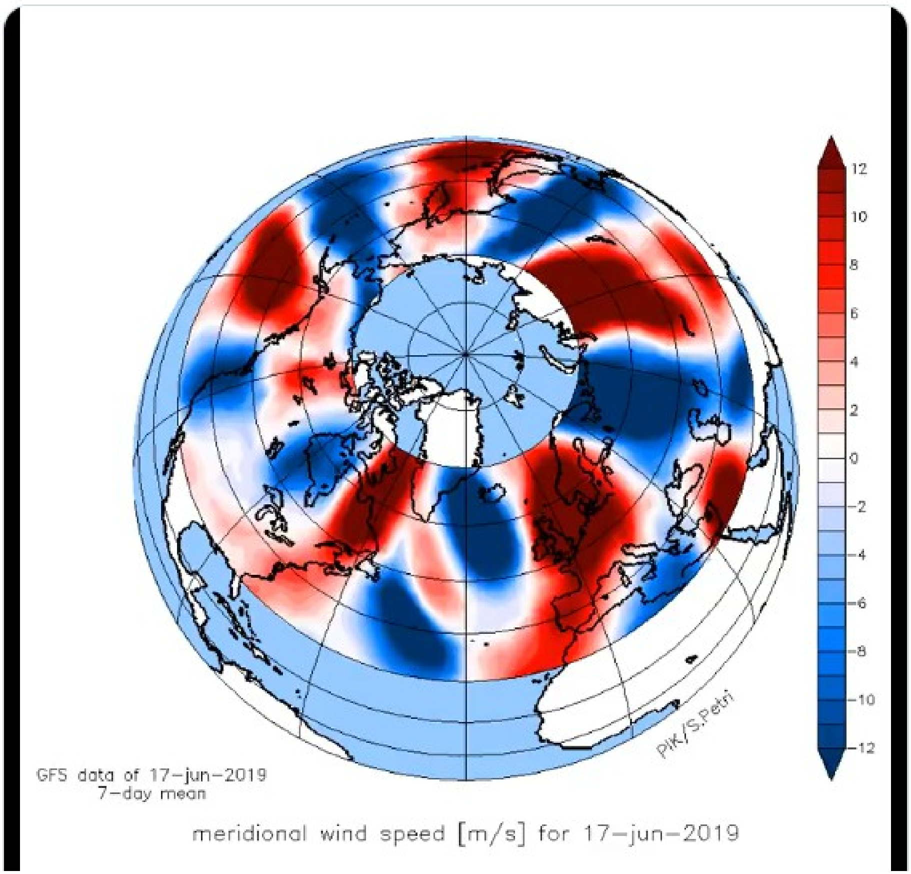
Baroclinic: $\nabla p \times \nabla \theta \,!= 0$

Plumb & Marshall: Introduction atmosphere, ocean & climate

Prof. Stefan Rahmstorf @rahmstorf

Strong Rossby wave activity in the northern mid-lats since early June! Red=northward flow, blue southward flow. Watch the red blob linger over Europe, bringing in warm air. 7-day averages centered on the stated day, using forecast at the end. #heatwave

Tweet übersetzen

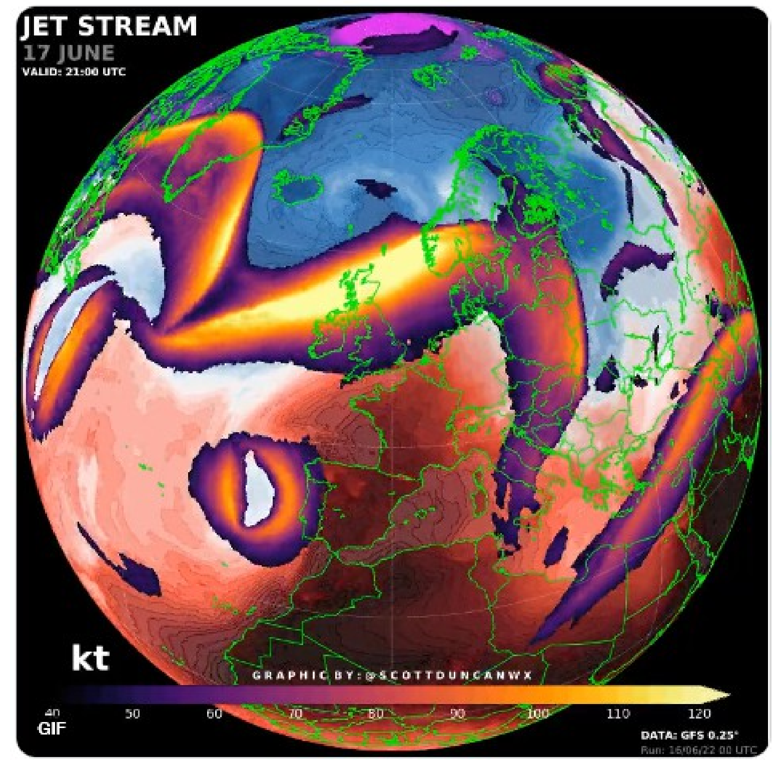


Scott Duncan @ScottDuncanWX

Heat has been building in south-west Europe and north-west Africa for a while. The cut-off low pressure spinning near Portugal acts like an engine to lift heat north.

The strong jet racing across the Atlantic is also important for intensifying high pressure on the continent.

Tweet übersetzen

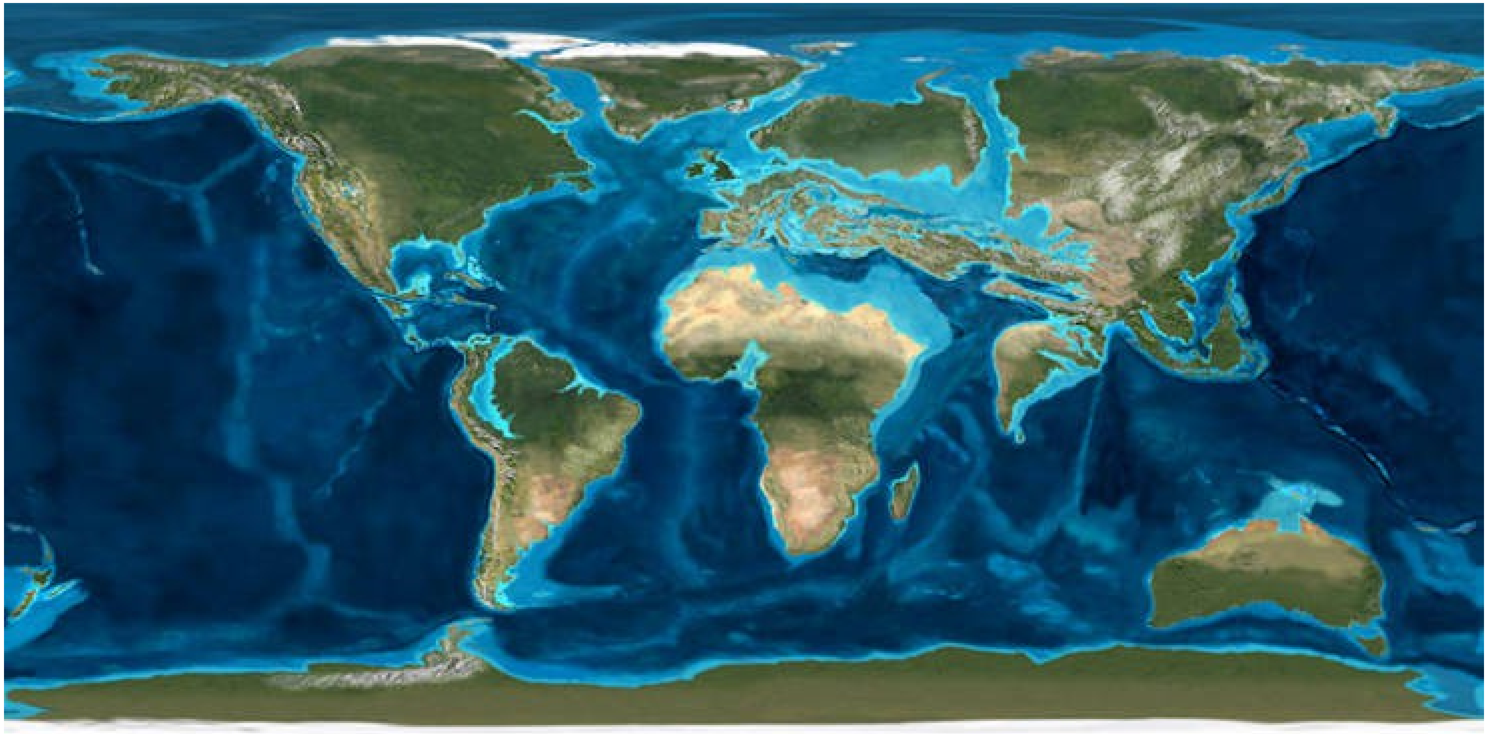


5:56 nachm. · 18. Juni 2022 · Twitter Web App

https://twitter.com/i/status/1143195454848544769

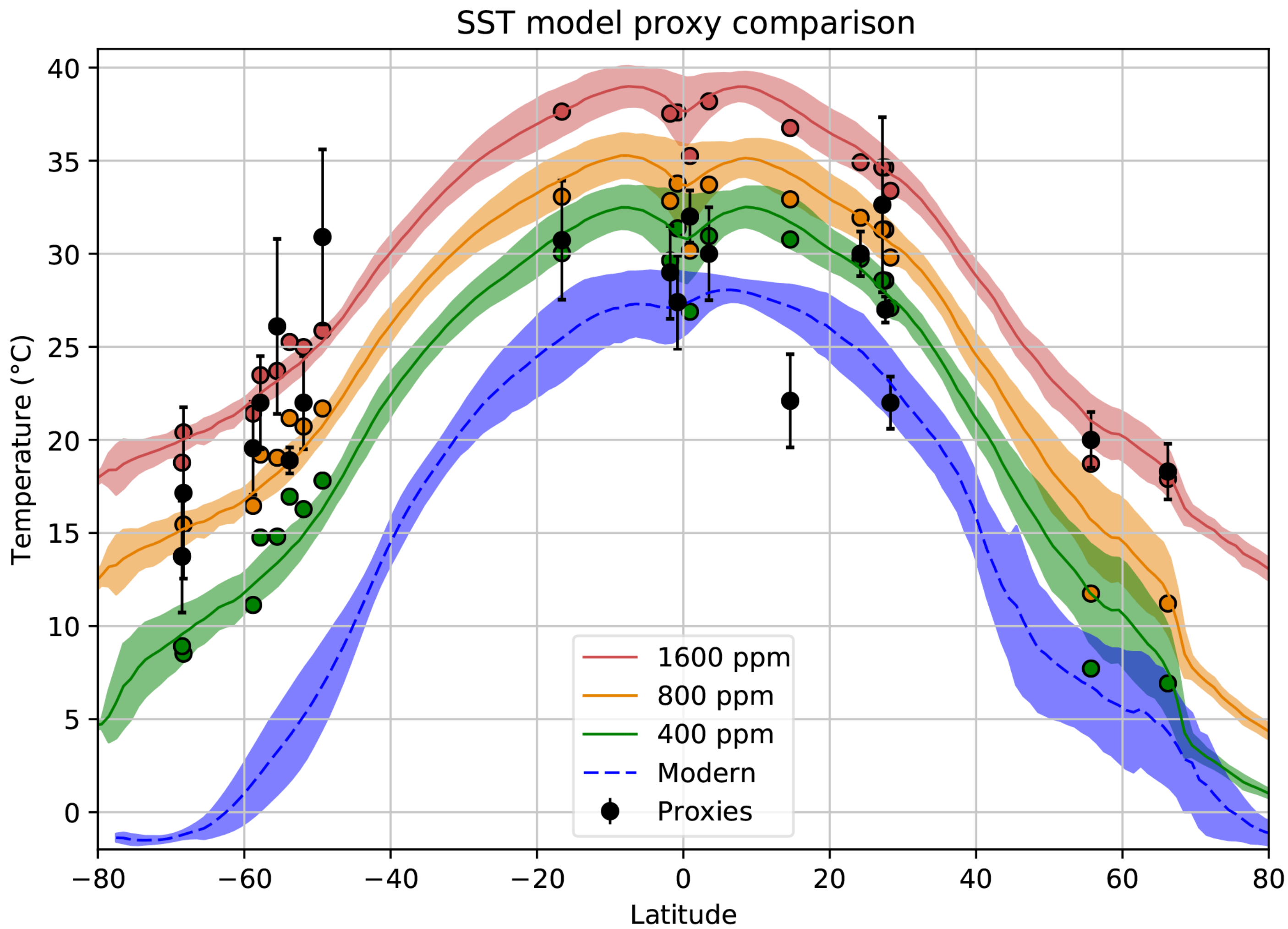
https://twitter.com/i/status/1538188867039379456

If you have some time: Eocene Earth 50 Myrs ago



https://cp.copernicus.org/articles/14/789/2018/cp-14-789-2018.html

If you have some time: Eocene Earth 50 Myrs ago

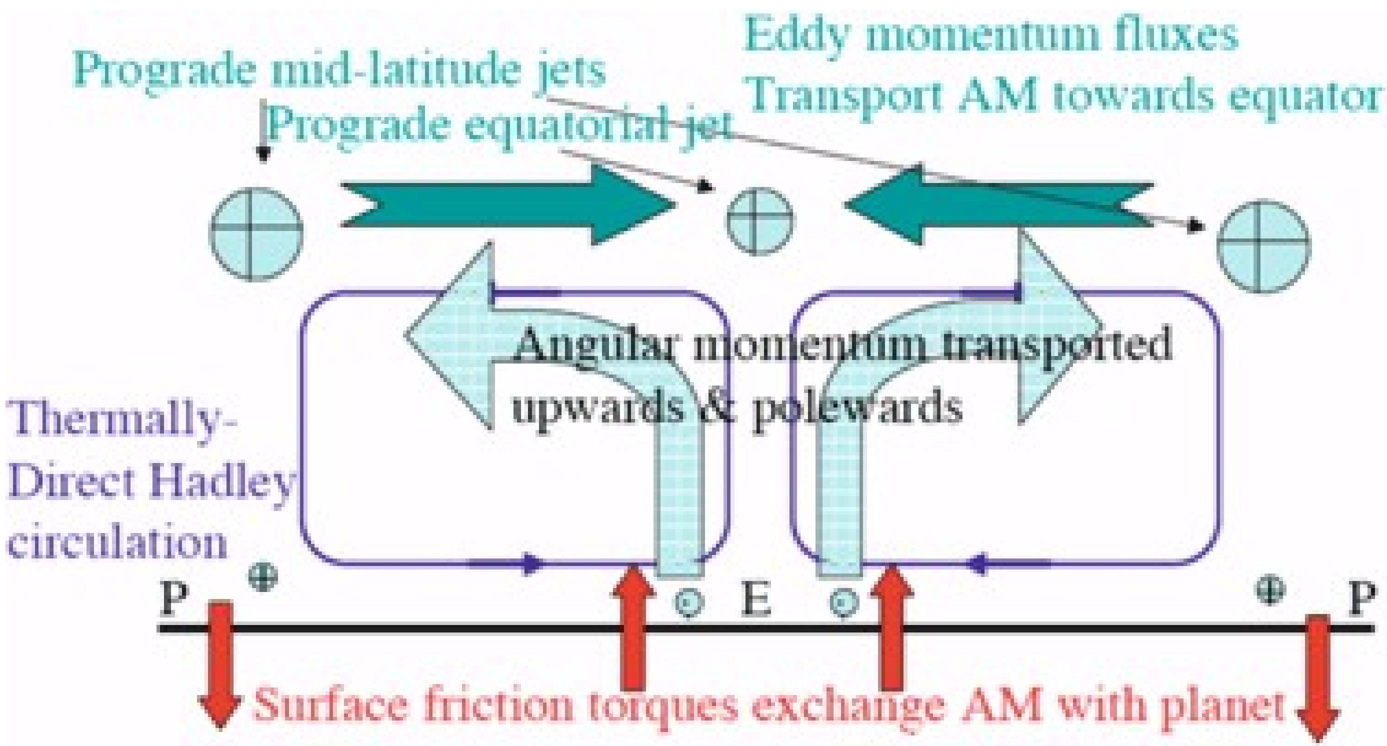


Model this in the H& S framework!

If you have some time: Change rotation (→ Circulation)

- 10 days

- 243 days



Gierasch-Rossow-Williams Mechanism

https://link.springer.com/article/10.1007/s11214-017-0389-x