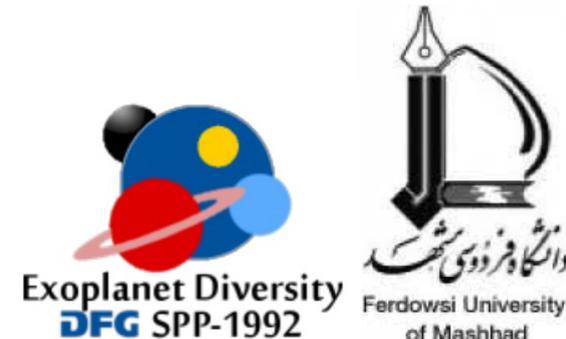


Hydrodynamic modelling of planet-disk interaction and planet migration

Hands-on Numerical Astrophysics School
for Exoplanetary Sciences
July 4-8, 2022
Hanau-Steinheim

5 July 2022, Sareh Ataiee



Outline

- What is a planet?
- Force, torque, and power
- Lindblad and co-rotation torques
- Gap opening
- Planetary migration
- Put your knowledge in practice using FARGO3D

What is a planet?

Our migration-wise definition:

A planet (in a disc)

- is a solid object (not a gas parcel)
- orbits around one (more) star(s)
- is moving mostly due to the gravitational forces

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$$\frac{d\mathbf{v}}{dt} = \mathbf{a}$$

$$\text{for a solid in a disc} \rightarrow \mathbf{a} = \mathbf{a}_{\text{star}} + \mathbf{a}_{\text{gas drag}} + \mathbf{a}_{\text{disc gravity}}$$

$$\text{if } \begin{cases} \mathbf{a}_{\text{gas drag}} \ll \mathbf{a}_{\text{disc gravity}} & \text{Planet} \\ \mathbf{a}_{\text{gas drag}} \gg \mathbf{a}_{\text{disc gravity}} & \text{Dust particle} \end{cases}$$

What is a planet?

Our migration-wise definition:

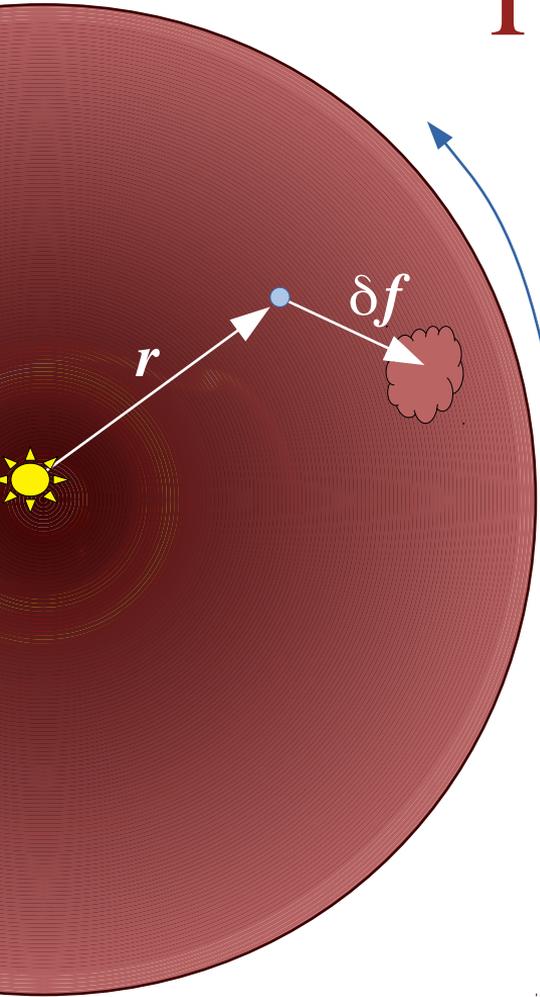
A planet (in a disc)

- is a solid object (not a gas parcel)
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Numerically:

- is a gravitational point source

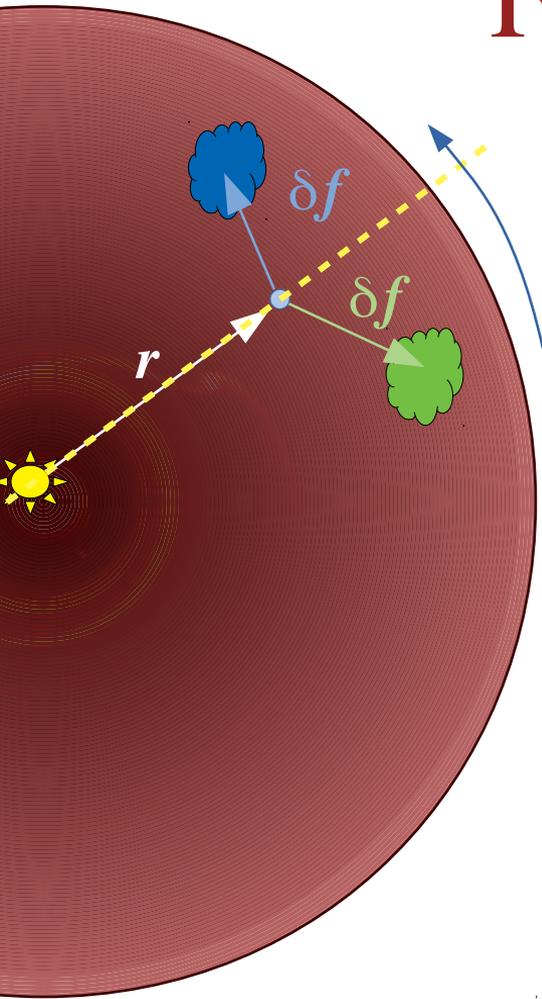
Force, torque, and power



$$\text{Torque: } \delta \Gamma = \mathbf{r} \times \delta \mathbf{f} = \delta \frac{dL_p}{dt} \quad L_p = m_p \sqrt{GM_* a} \sqrt{1-e^2} \mathbf{k}$$

$$\text{Power: } \delta P = \mathbf{v} \cdot \delta \mathbf{f} = \delta \frac{dE_p}{dt} \quad E_p = -\frac{Gm_p M_*}{2a}$$

Force, torque, and power



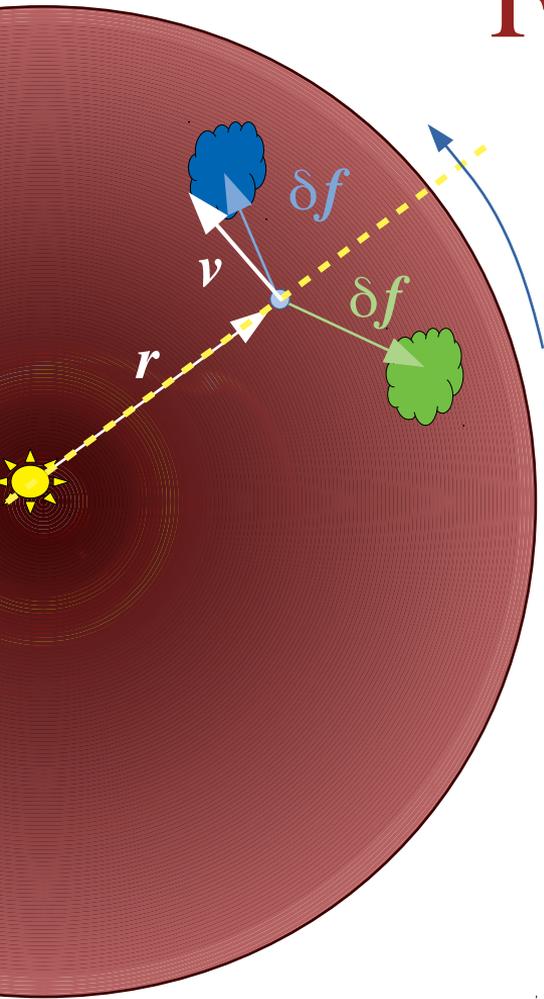
$$\text{Torque: } \delta \Gamma = \mathbf{r} \times \delta \mathbf{f} = \delta \frac{d \mathbf{L}_p}{dt} \quad \mathbf{L}_p = m_p \sqrt{G M_* a} \sqrt{1 - e^2} \mathbf{k}$$

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$$\delta \Gamma|_z = \mathbf{r} \times \delta \mathbf{f} < 0 \Rightarrow \delta \frac{d \mathbf{L}_p}{dt} |_z < 0 \quad \text{planet loses angular momentum}$$

$$\delta \Gamma|_z = \mathbf{r} \times \delta \mathbf{f} > 0 \Rightarrow \delta \frac{d \mathbf{L}_p}{dt} |_z > 0 \quad \text{planet gains angular momentum}$$

Force, torque, and power



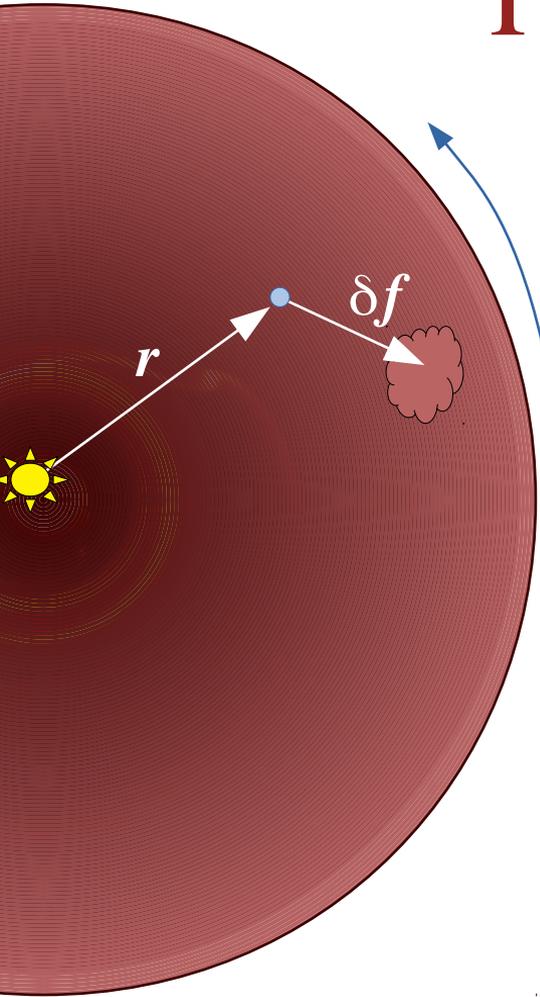
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$$\delta P = \mathbf{v} \cdot \delta \mathbf{f} < 0 \Rightarrow \delta \frac{d E_p}{dt} < 0 \quad \text{planet loses energy}$$

$$\delta P = \mathbf{v} \cdot \delta \mathbf{f} > 0 \Rightarrow \delta \frac{d E_p}{dt} > 0 \quad \text{planet gains energy}$$

Force, torque, and power



$$\text{Torque: } \delta \boldsymbol{\Gamma} = \mathbf{r} \times \delta \mathbf{f} = \delta \frac{d \mathbf{L}_p}{dt} \quad \mathbf{L}_p = m_p \sqrt{G M_* a} \sqrt{1 - e^2} \mathbf{k}$$

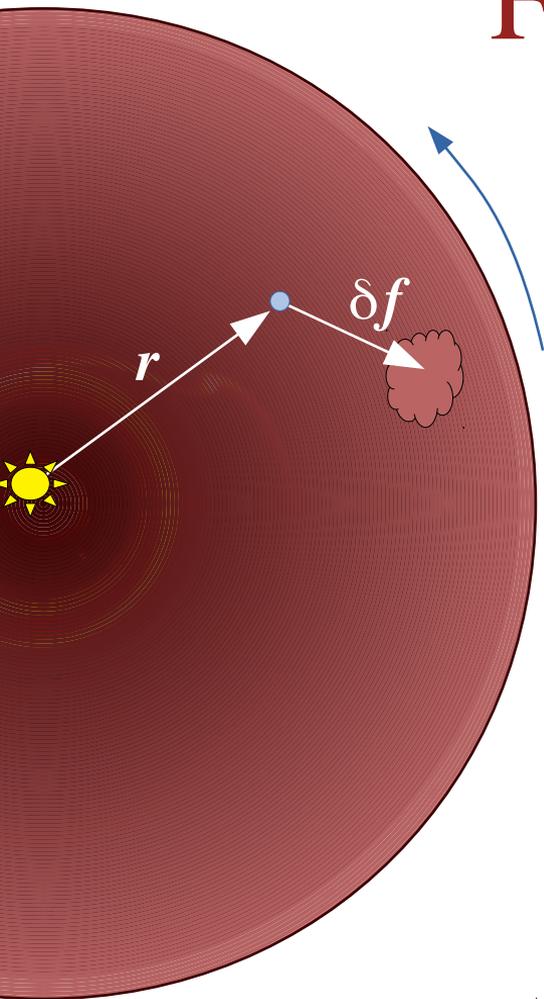
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$$\text{Torque: } \boldsymbol{\Gamma} = \int_{\text{disc}} \mathbf{r} \times \delta \mathbf{f} = \frac{d \mathbf{L}_p}{dt}$$

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Force, torque, and power



$$\text{Torque: } \boldsymbol{\Gamma} = \int_{\text{disc}} \mathbf{r} \times \delta \mathbf{f} = \frac{d \mathbf{L}_p}{dt} \quad \mathbf{L}_p = m_p \sqrt{GM_* a} \sqrt{1-e^2} \mathbf{k}$$

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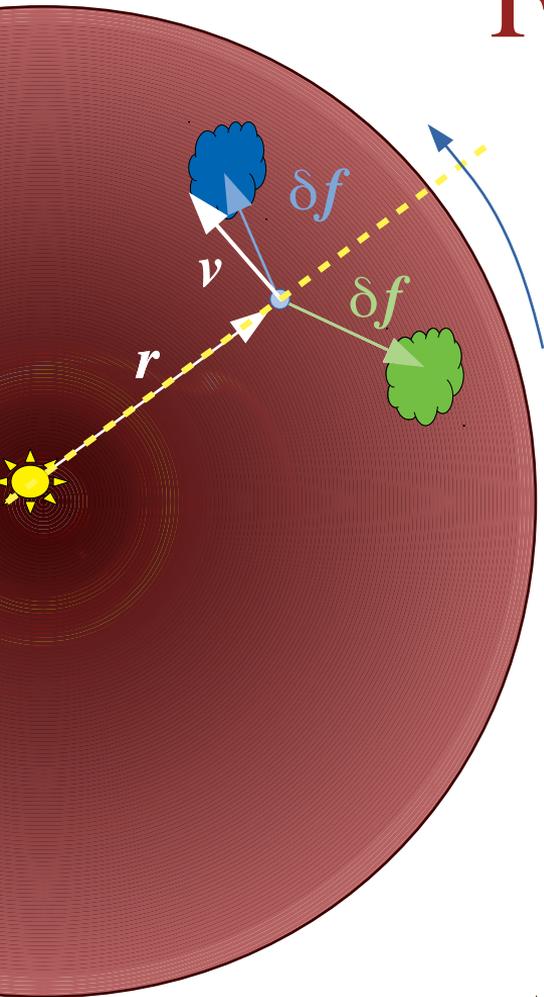
For a circular planet ($e = 0$):

$$\Gamma_z = \frac{dL_p}{dt} = m_p \frac{\sqrt{GM_*}}{2\sqrt{a}} \dot{a} = \frac{L_p}{2} \frac{\dot{a}}{a}$$

$$P = \frac{dE_p}{dt} = m_p \frac{GM_*}{2a^2} \dot{a} = -E_p \frac{\dot{a}}{a}$$

$$\Rightarrow \tau_a = \frac{a}{\dot{a}} = \frac{L_p}{2\Gamma} = \frac{|E_p|}{P}$$

Force, torque, and power



$$\text{Torque: } \mathbf{\Gamma} = \int_{\text{disc}} \mathbf{r} \times \delta \mathbf{f} = \frac{d\mathbf{L}_p}{dt} \quad \mathbf{L}_p = m_p \sqrt{GM_* a} \sqrt{1-e^2} \mathbf{k}$$

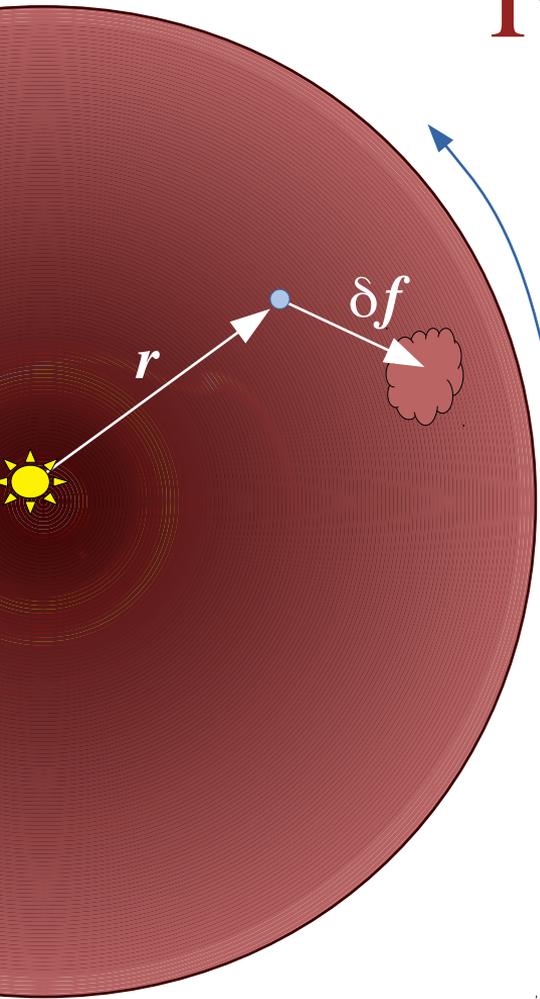
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For a circular planet ($e = 0$):

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Green part wins \rightarrow Torque and power on a planet are negative \rightarrow The planet's semi-major axis decreases

Force, torque, and power



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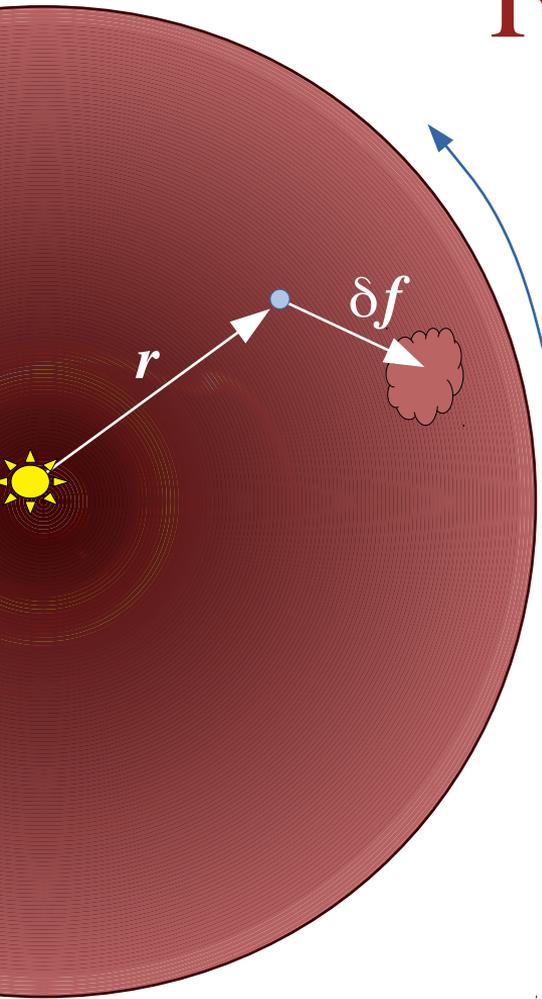
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Surface density perturbation \rightarrow exerts torque/power on the planet \rightarrow changes the planet's angular momentum/energy \rightarrow changes the planet's semi-major axis



Migration

Force, torque, and power



$$\text{Torque: } \boldsymbol{\Gamma} = \int_{\text{disc}} \mathbf{r} \times \delta \mathbf{f} = \frac{d \mathbf{L}_p}{dt} \quad \mathbf{L}_p = m_p \sqrt{G M_* a} \sqrt{1 - e^2} \mathbf{k}$$

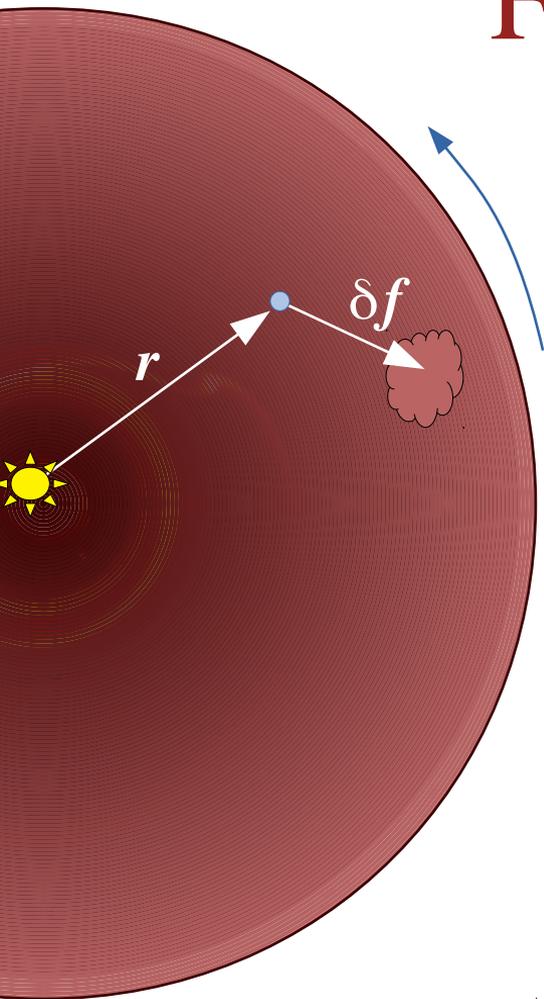
$$\text{Power: } P = \int_{\text{disc}} \mathbf{v} \cdot \delta \mathbf{f} = \frac{dE_p}{dt} \quad E_p = -\frac{G m_p M_*}{2a}$$

For an eccentric planet ($0 < e < 1$):

$$\Gamma_z = \frac{dL_p}{dt} = L_p \left(\frac{\dot{a}}{2a} - \frac{\dot{e}}{e} \frac{e^2}{1 - e^2} \right) \quad \tau_a = \frac{a}{\dot{a}} = \frac{|E_p|}{P}$$

$$P = \frac{dE_p}{dt} = m_p \frac{GM_*}{2a^2} = -E_p \frac{\dot{a}}{a} \quad \tau_e = \frac{e}{\dot{e}} = \frac{e^2}{1 - e^2} \left(\frac{P}{E_p} - \frac{\Gamma}{L_p} \right)^{-1}$$

Force, torque, and power



$$\text{Torque: } \Gamma = \int_{\text{disc}} \mathbf{r} \times \delta \mathbf{f} = \frac{d\mathbf{L}_p}{dt} \quad \mathbf{L}_p = m_p \sqrt{GM_* a} \sqrt{1-e^2} \mathbf{k}$$

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For an eccentric planet ($0 < e < 1$):

If a planet is **eccentric**,
torque does **NOT** give the
direction of migration.

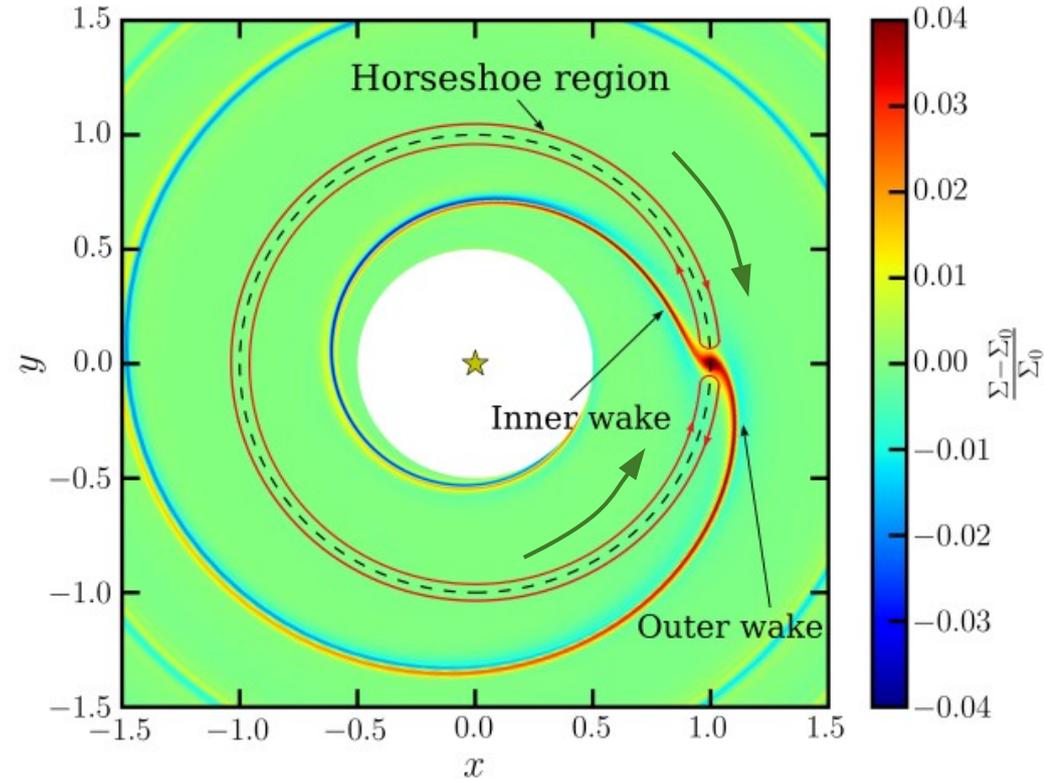
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$$\tau_e = \frac{e}{\dot{e}} = \frac{e^2}{1-e^2} \left(\frac{P}{E_p} - \frac{\Gamma}{L_p} \right)^{-1}$$

Lindblad and co-rotation torques

A planet divides the disc in two regions:

- far from its orbit (circulating)
- around its orbit (horseshoe)



Lindblad and co-rotation torques

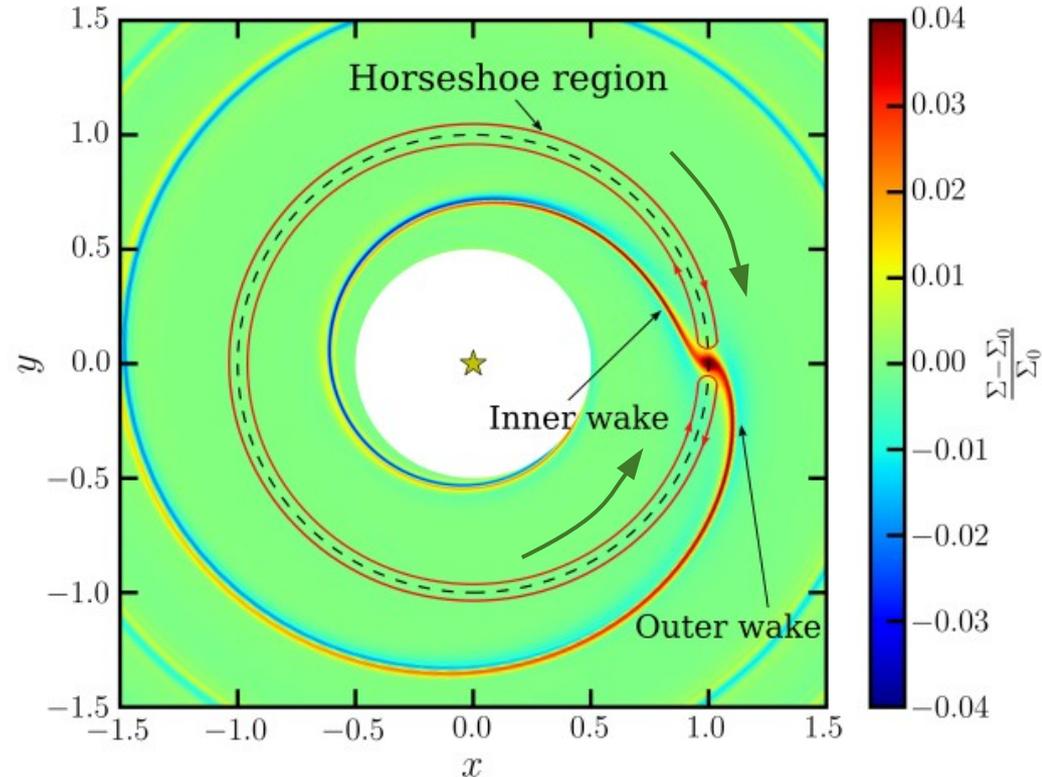
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Two main sources of surface density perturbation:

- Wakes resulting from the **Lindblad resonances**
- Horseshoe motion of gas in the **co-orbital region**



Lindblad and co-rotation torques

Goldreich & Tremaine 1979

$$\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_0 \cdot \nabla) \mathbf{v}_1 + (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_0 = -\nabla(\varphi_1 + \varphi_1^D + \eta_1),$$

$$\frac{\partial \sigma_1}{\partial t} + \nabla \cdot (\sigma_0 \mathbf{v}_1) + \nabla \cdot (\sigma_1 \mathbf{v}_0) = 0,$$

$$\eta_1 = c_0^2 (\sigma_1 / \sigma_0),$$

$$\nabla^2 \varphi_1^D = 4\pi G \sigma_1 \delta(z),$$

Lindblad and co-rotation torques

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$$\eta_1 = c_0^2 (\sigma_1 / \sigma_0),$$

$$\nabla^2 \varphi_1^D = 4\pi G \sigma_1 \delta(z),$$

$$X = X(r) \exp i(m\theta - \omega t)$$


$$u_1 = -\frac{i}{D} \left[(m\Omega - \omega) \frac{d}{dr} + \frac{2m\Omega}{r} \right] (\varphi_1 + \varphi_1^D + \eta_1),$$

$$v_1 = \frac{1}{D} \left[2B \frac{d}{dr} + \frac{m}{r} (m\Omega - \omega) \right] (\varphi_1 + \varphi_1^D + \eta_1),$$

Lindblad and co-rotation torques

Goldreich & Tremaine 1979

$$\left\{ \frac{d^2}{dr^2} + \left[\frac{d}{dr} \ln \left(\frac{\sigma r}{D} \right) \right] \frac{d}{dr} + \frac{2m\Omega}{r(m\Omega - \omega)} \left[\frac{d}{dr} \ln \left(\frac{\sigma\Omega}{D} \right) \right] - \frac{m^2}{r^2} \right\} (\varphi_1 + \varphi_1^D + \eta_1) = \frac{D\eta_1}{c^2}$$

$$D = \kappa^2 - (m\Omega - \omega)^2$$

Singularities:

- 1) $D = 0$ Lindblad resonance
- 2) $m\Omega - \omega = 0$ Co-rotation resonance

Lindblad and co-rotation torques

Lindblad resonances $m(\Omega(r) - \Omega_p) = \pm \kappa \sqrt{(1 + m^2 h^2)}$

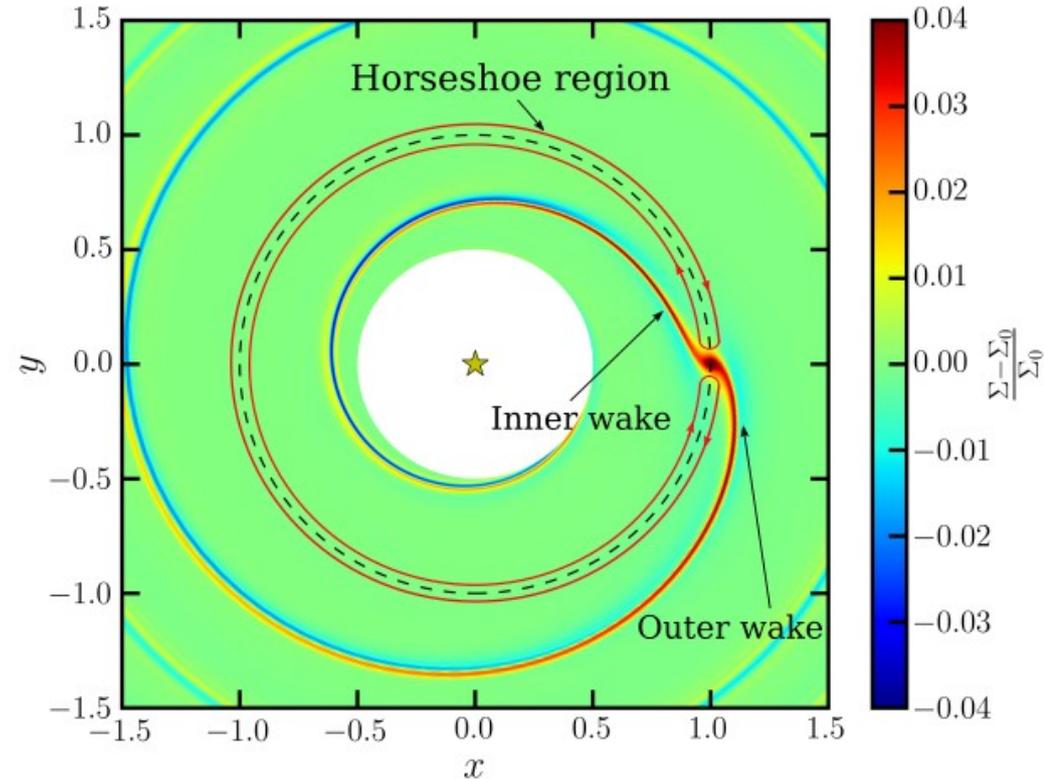
m : an integer

$\Omega(r)$: gas angular velocity

Ω_p : planet angular velocity

κ : epicyclic frequency

h : disc aspect ratio



Lindblad and co-rotation torques

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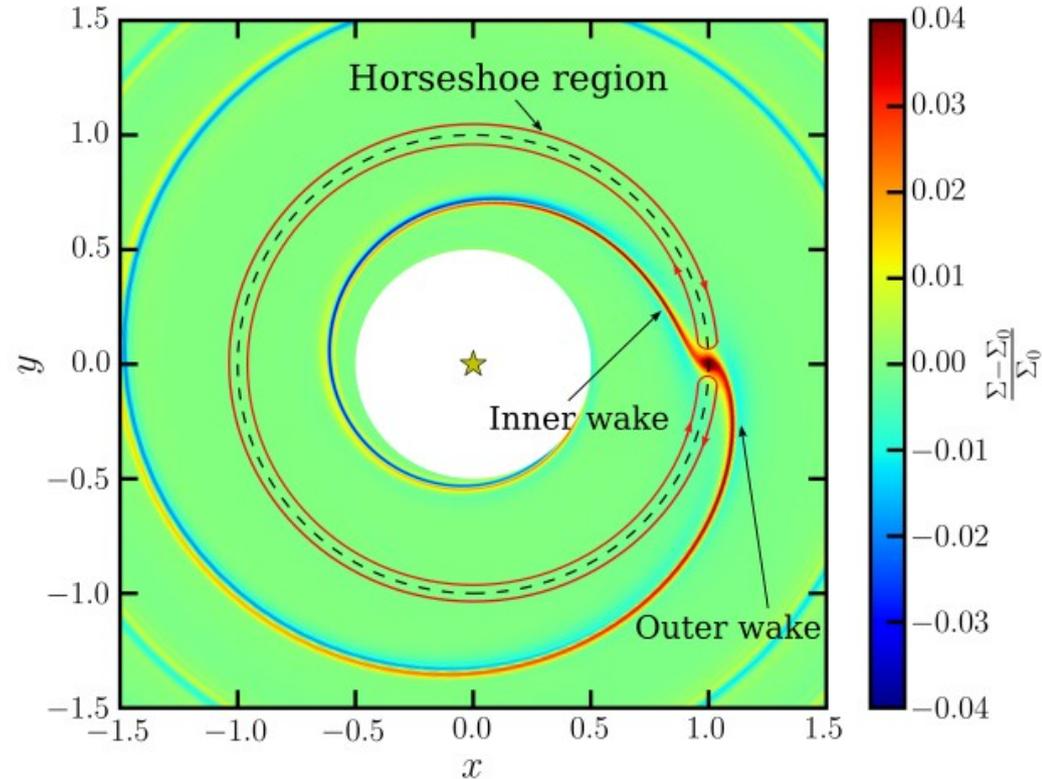
Sound waves launched at LRs

+

Differential rotation of the disc

↓

Spirals



Lindblad and co-rotation torques

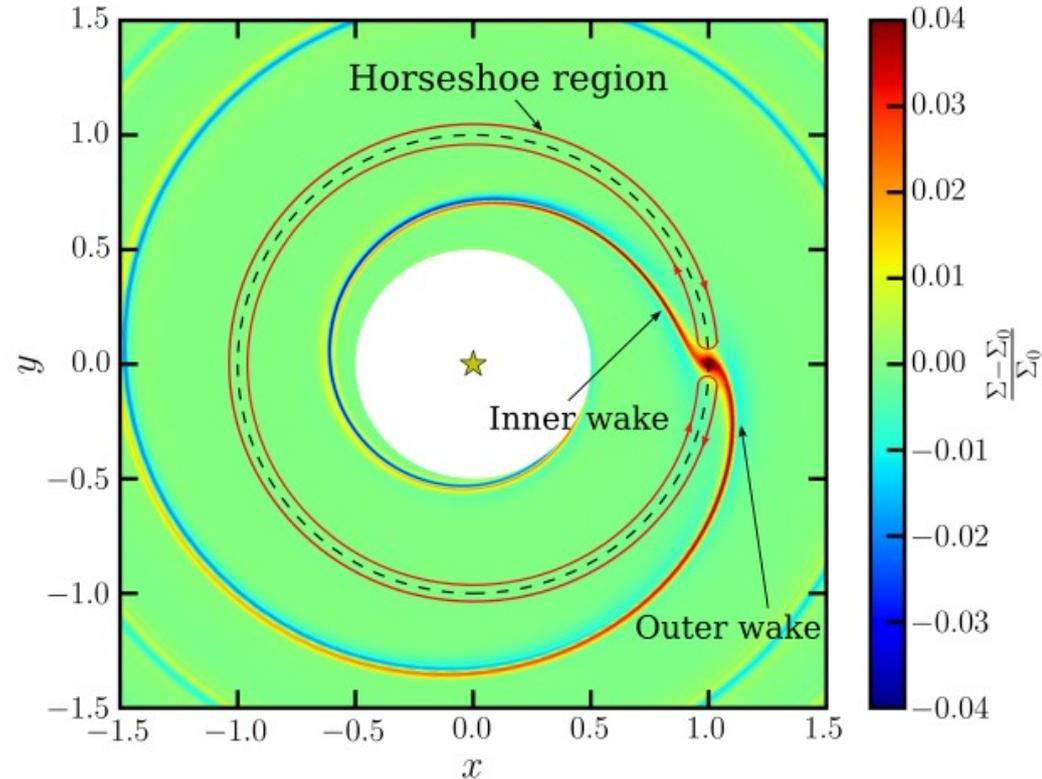
Lindblad resonances $m(\Omega(r) - \Omega_p) = \pm \kappa \sqrt{(1 + m^2 h^2)}$

Torque from the outer wake < 0

Torque from the inner wake > 0

→ Net torque determines the migration of the planet

Differential Lindblad Torque



Lindblad and co-rotation torques

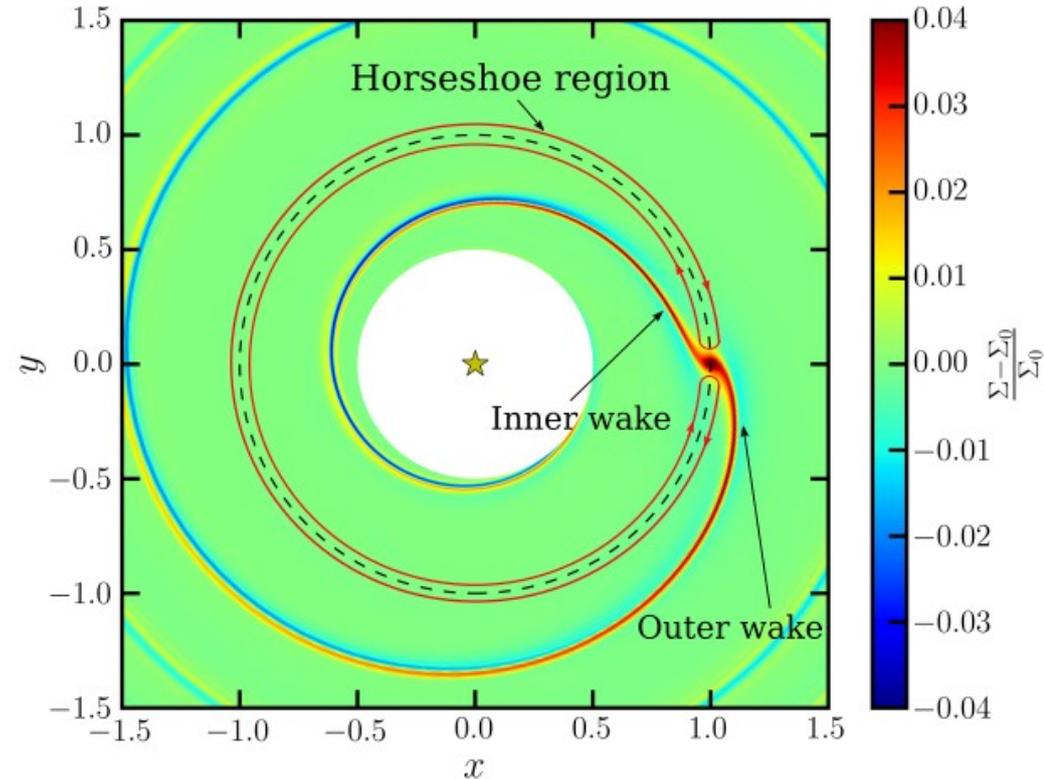
Lindblad resonances $m(\Omega(r) - \Omega_p) = \pm \kappa \sqrt{(1 + m^2 h^2)}$

$\Omega(r)$ depends on the disc pressure gradient



The **location** of Lindblad resonances depends on **pressure profile**:

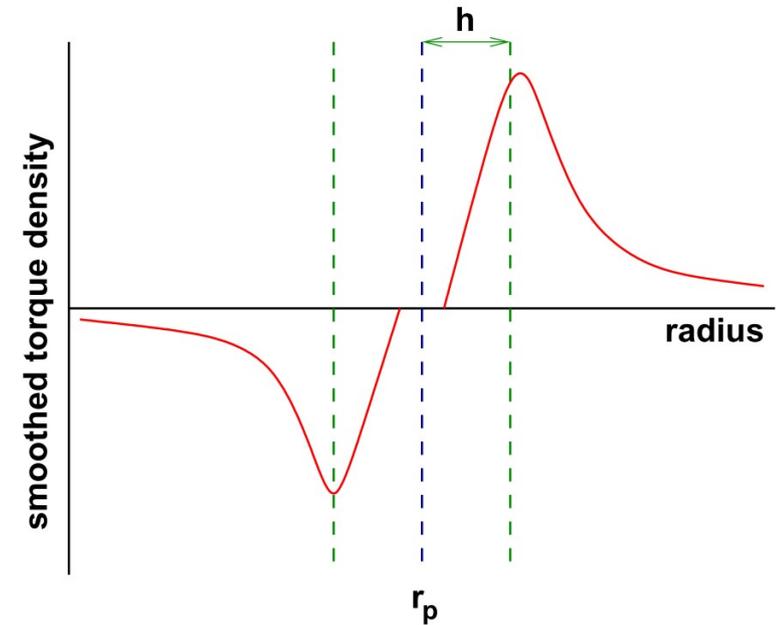
Density and **Temperature** gradient can change the Lindblad torque.



Lindblad and co-rotation torques

Lindblad resonances $m(\Omega(r) - \Omega_p) = \pm \kappa \sqrt{(1 + m^2 h^2)}$

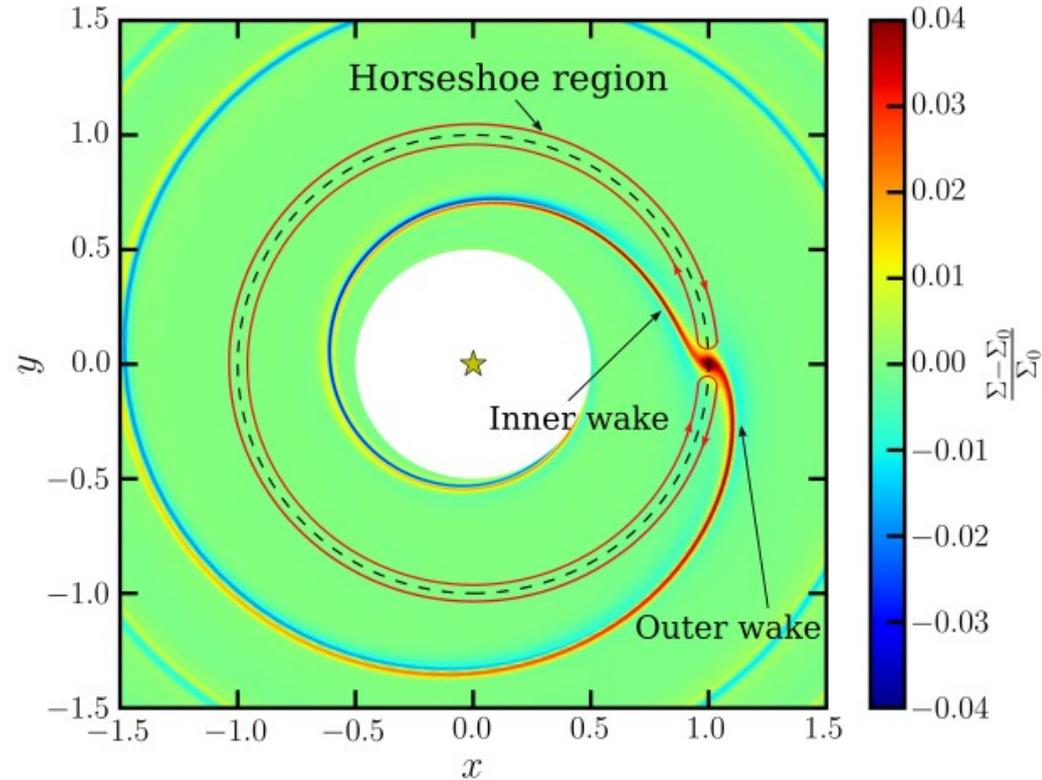
There is a **pile-up** of resonance location about $H \rightarrow$ the main contribution should come from this distance



Armitage 2017

Lindblad and **co-rotation** torques

Co-rotation resonance $\Omega(r) = \Omega_p$

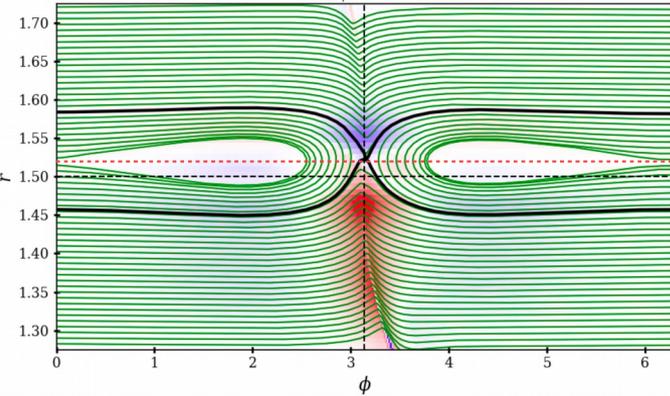


Lindblad and co-rotation torques

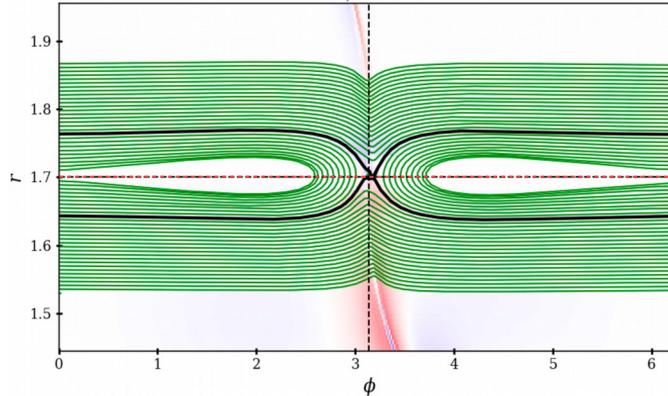
Co-rotation resonance $\Omega(r) = \Omega_p$

- r_{co} is not necessarily r_p

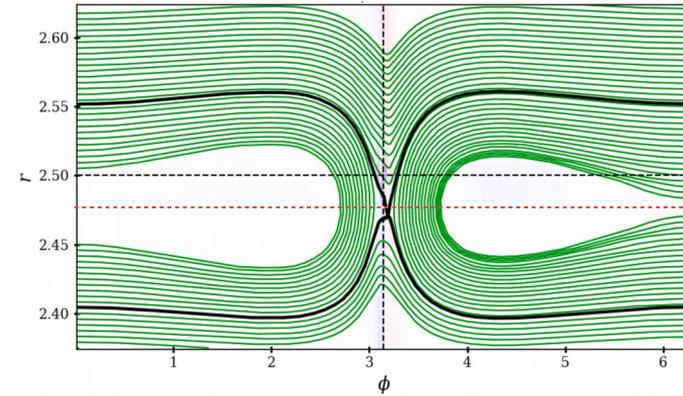
$\nabla P > 0$



$\nabla P = 0$



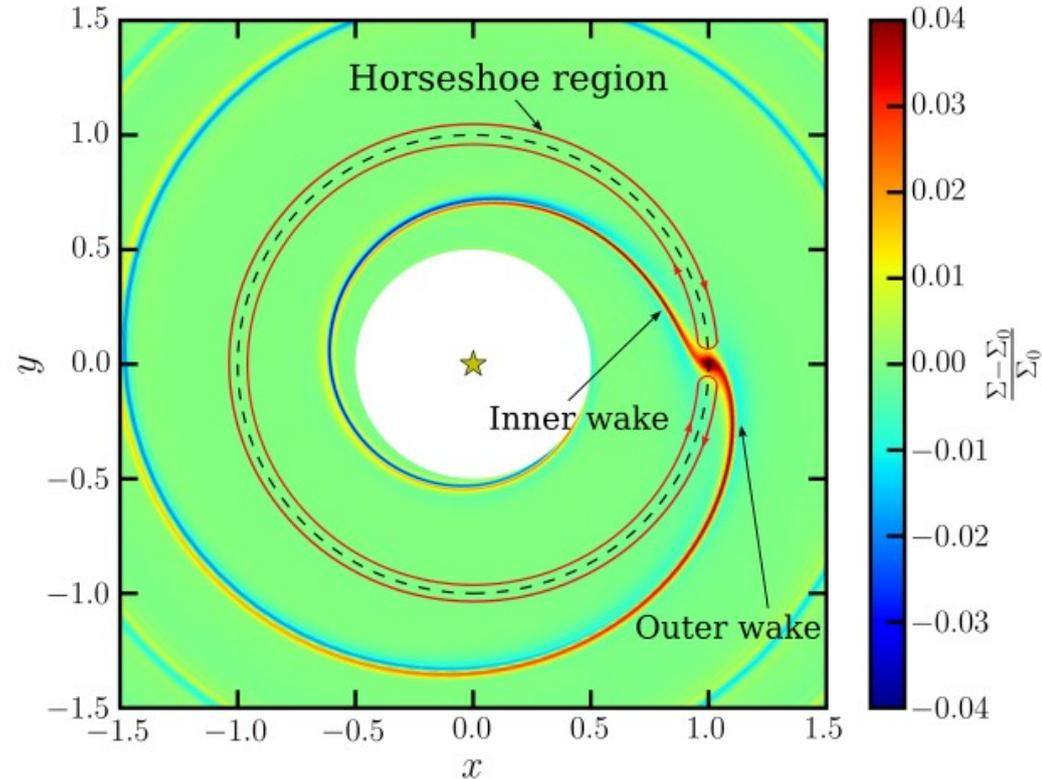
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Lindblad and co-rotation torques

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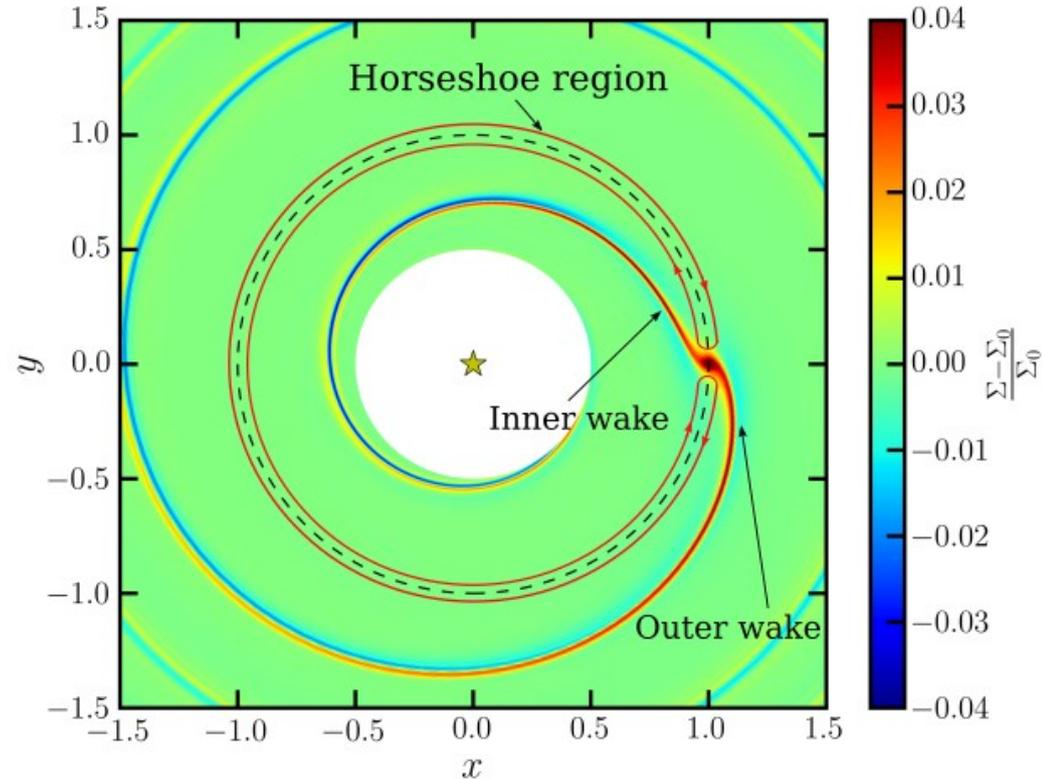
- r_{co} is not necessarily r_p
- Linear co-rotation torque is small



Lindblad and co-rotation torques

Co-rotation resonance $\Omega(r) = \Omega_p$

- r_{co} is not necessarily r_p
- Linear co-rotation torque is small
- Horseshoe drag is non-linear
(Paardekooper & Papaloizou 2009)



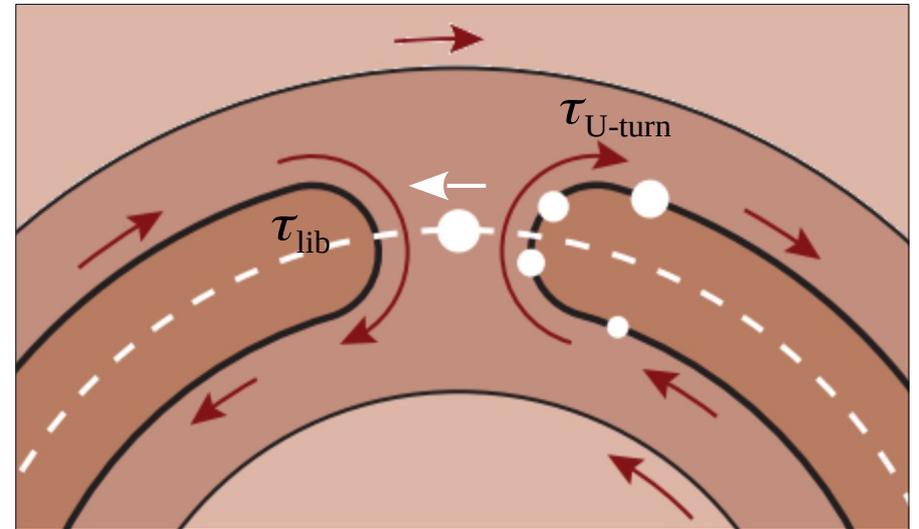
Lindblad and co-rotation torques

Co-rotation resonance $\Omega(r) = \Omega_p$

- r_{co} is not necessarily r_p
- Linear co-rotation torque is small
- Horseshoe drag is non-linear
- Time matter

$$\tau_{\text{U-turn}} \sim \frac{H}{\chi_s} \tau_{\text{dyn}}$$

$$\tau_{\text{lib}} \sim \frac{8 \pi a}{3 \Omega_p \chi_s}$$



Kley & Nelson 2012

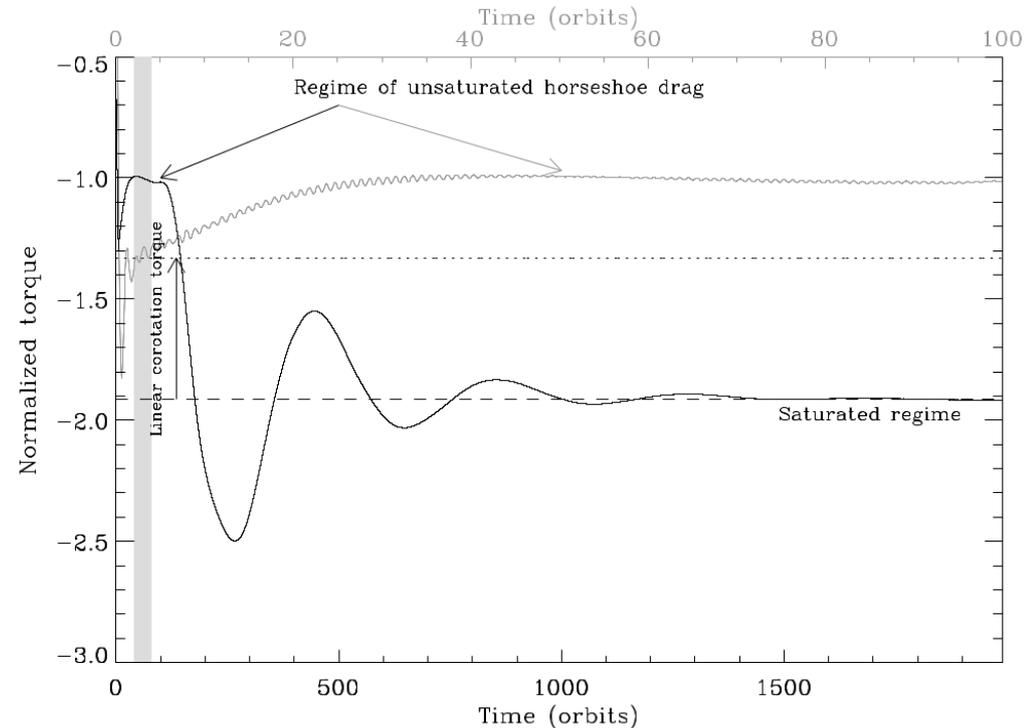
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Masset 2011

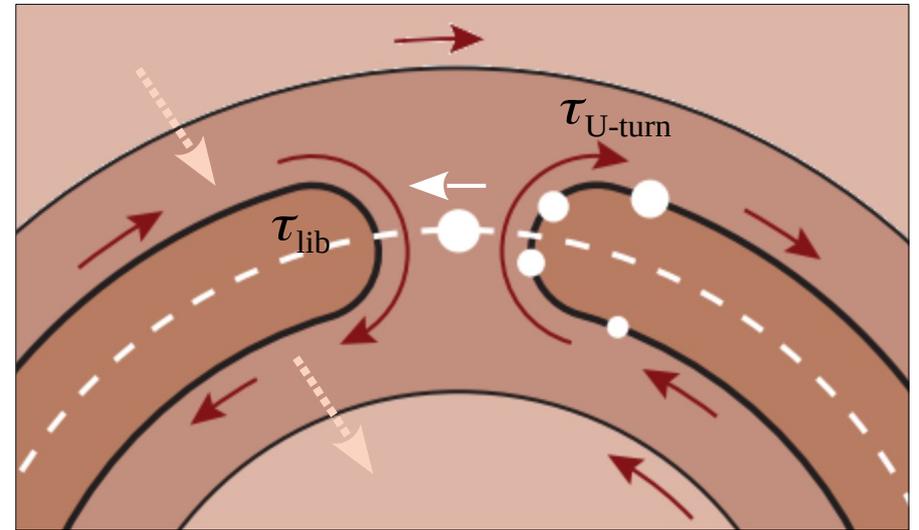
Lindblad and co-rotation torques

Co-rotation resonance $\Omega(r) = \Omega_p$

- r_{co} is not necessarily r_p
- Linear co-rotation torque is small
- Horseshoe drag is non-linear
- Time and diffusion matter

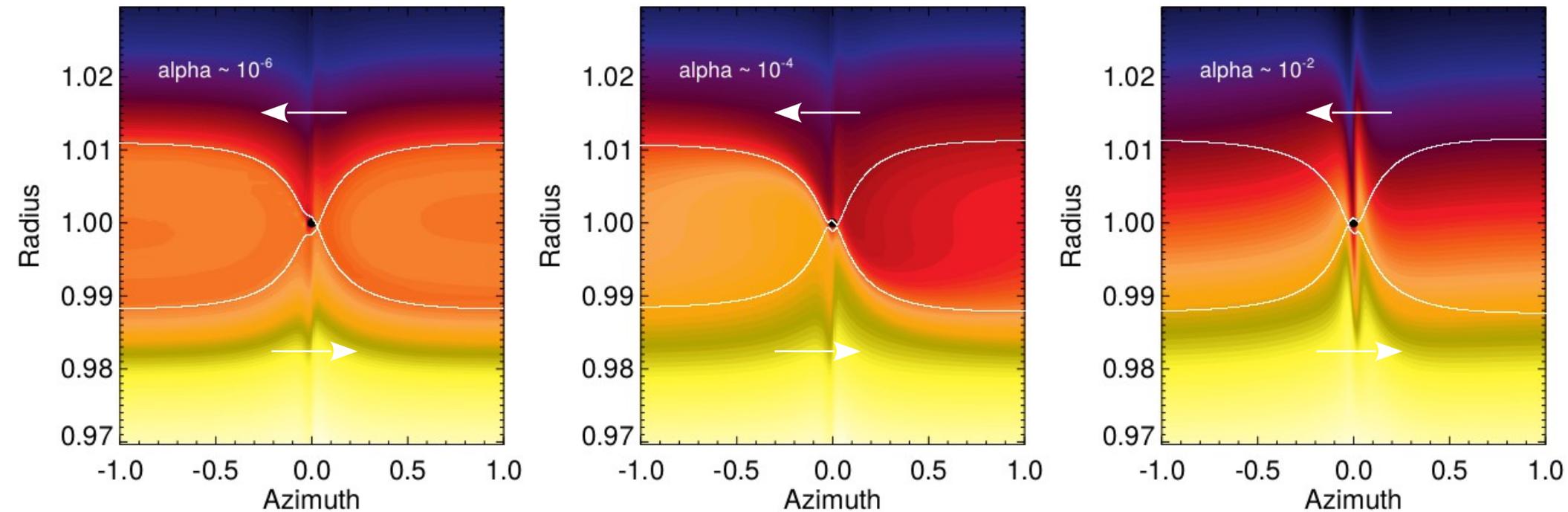
$$\tau_{\text{U-turn}} \sim \frac{H}{X_s} \tau_{\text{dyn}} \quad \tau_{\text{lib}} \sim \frac{8 \pi a}{3 \Omega_p X_s}$$

$$\tau_{\text{visc}} \sim \frac{X_s^2}{V_p}, \text{ or in general } \tau_{\text{diff}}$$



Kley & Nelson 2012

Lindblad and co-rotation torques

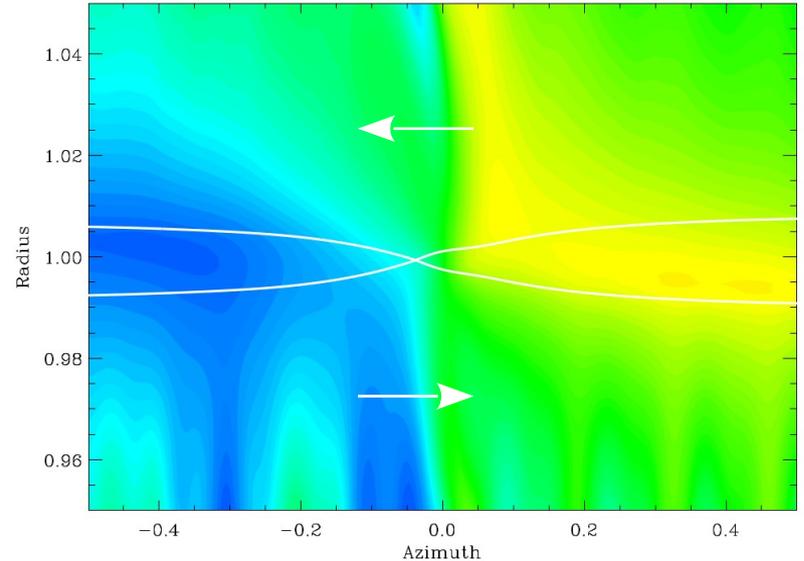


Baruteau & Masset 2013

Lindblad and co-rotation torques

Co-rotation resonance $\Omega(r) = \Omega_p$

- r_{co} is not necessarily r_p
- Linear co-rotation torque is small
- Horseshoe drag is non-linear
- Time and diffusion matter
- Vortensity and entropy gradients govern the horseshoe drag



Baruteau & Masset 2013

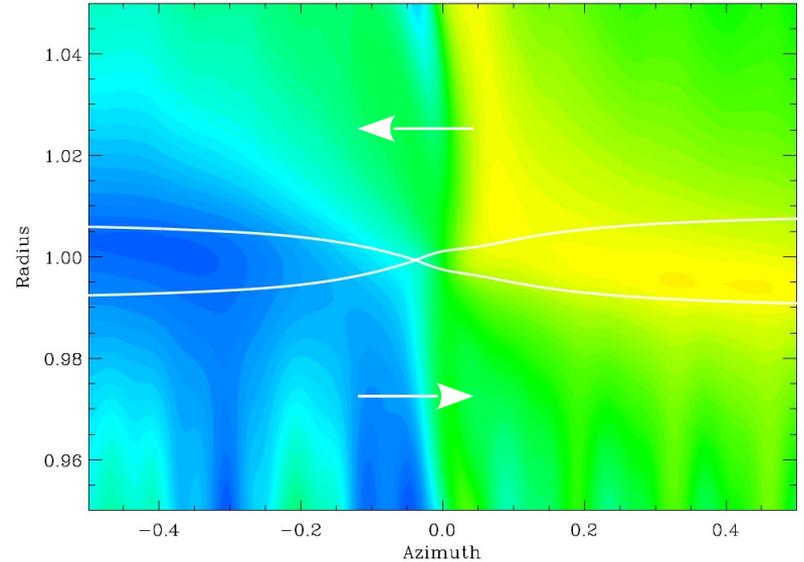
$$\frac{\delta \Sigma}{\Sigma} = \frac{\delta \Sigma}{\Sigma} \left(\frac{\delta l}{l}, \frac{\delta s}{s} \right)$$

l : vortensity, s : entropy

Lindblad and co-rotation torques

Co-rotation resonance $\Omega(r) = \Omega_p$

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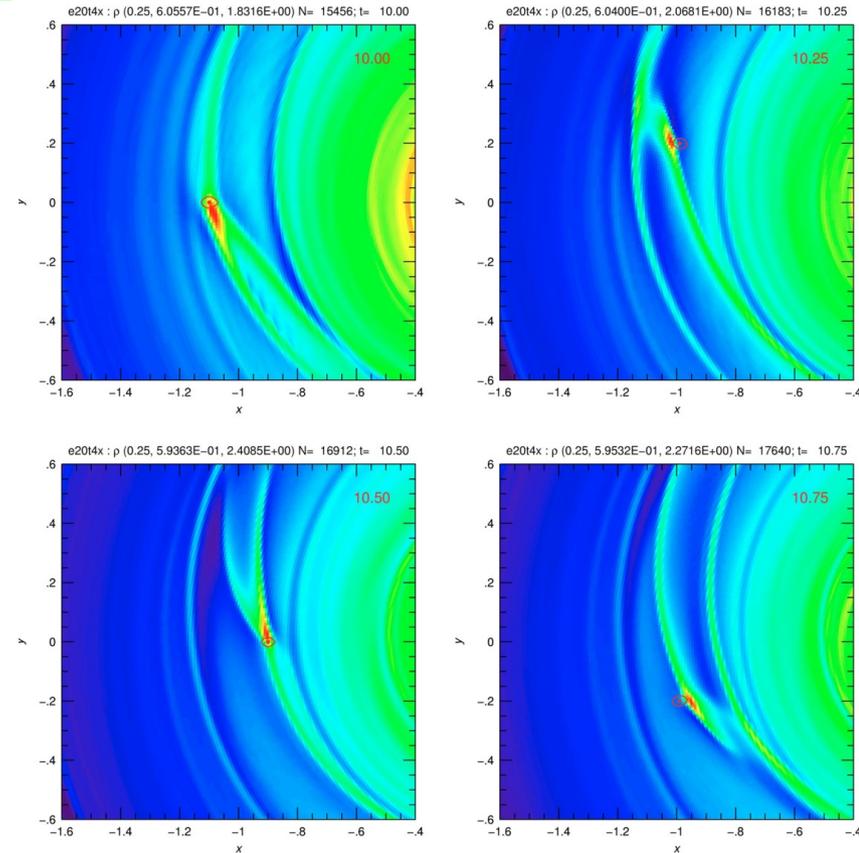
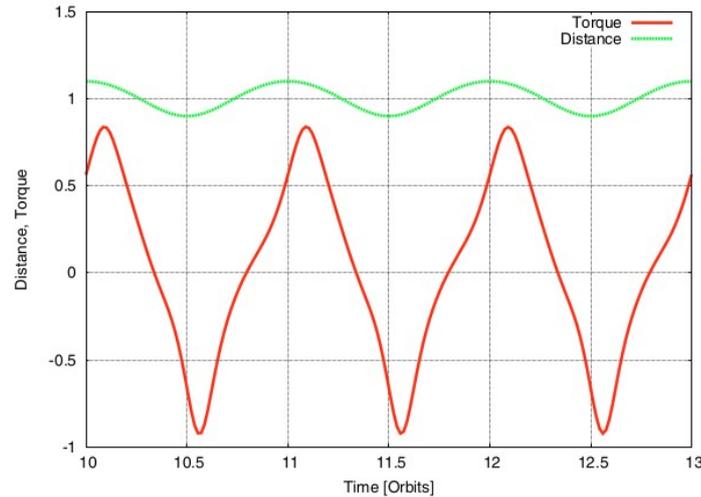
Baruteau & Masset 2013

For a detailed lecture check out:

http://clement.baruteau.free.fr/Bern2017/Baruteau_Bern2017.pdf

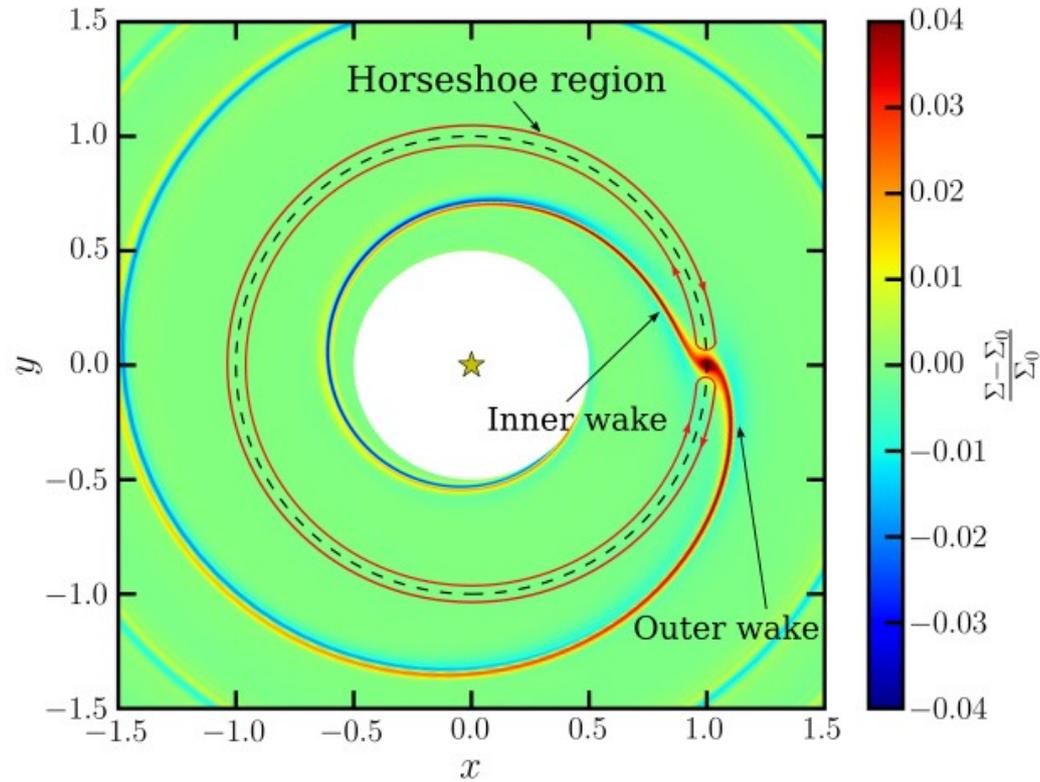
Lindblad and co-rotation torques

- $20 M_E$
- $e = 0.1$
- non-migrating

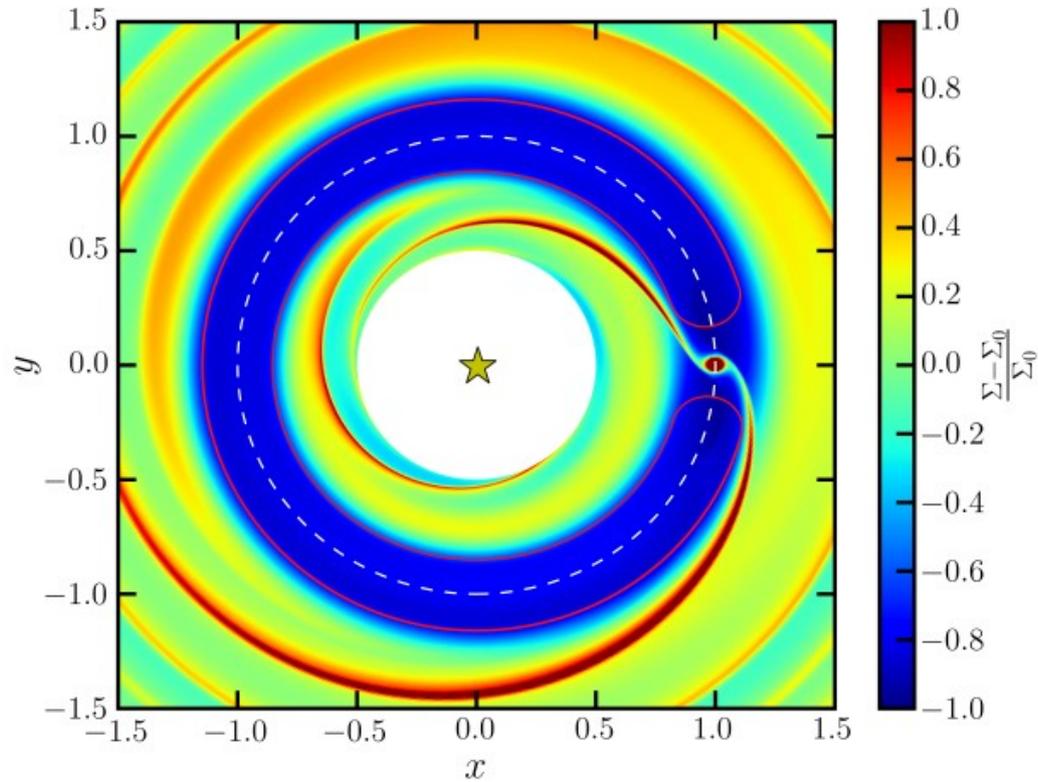


Cresswell+2007

Gap opening



Gap opening



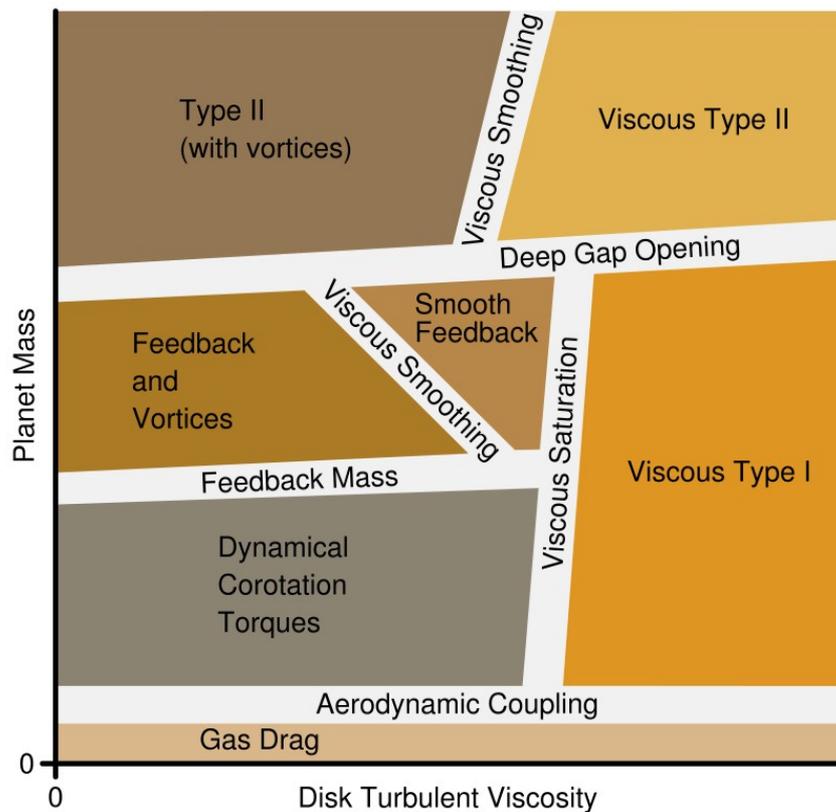
Planetary migration

Low-mass planet: type I migration

High-mass planet: type II migration

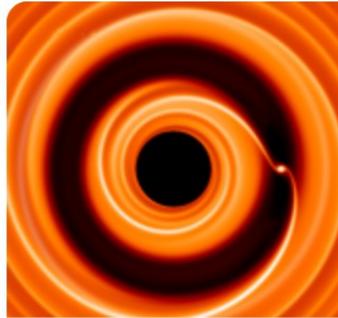
Medium-mass planet: (maybe) type III migration

Planetary migration



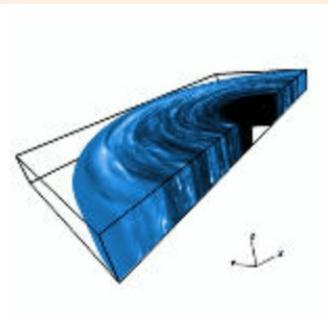
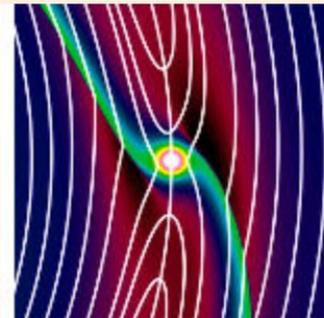
McNally+2019

Put your knowledge in practice using FARGO3D



FARGO3D

A versatile multifluid HD/MHD code that runs on clusters of CPUs or GPUs, with special emphasis on protoplanetary disks.



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FARGO: Fast **A**dvection in **R**otating **G**aseous **O**bjects (Masset 2000)

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FARGO3D: the **3D** successor of **FARGO** (Benítez- Llambay 2016)

Put your knowledge in practice using FARGO3D

FARGO: Fast **A**dvection in **R**otating **G**aseous **O**bjects (Masset 2000)

FARGO3D: the **3D** successor of **FARGO** (Benítez- Llambay 2016)

- Finite difference explicit Eulerian fixed grid code
- Cartesian, cylindrical or spherical geometry
- Multifluid capability (gas and different dust sizes)
- HD and MHD
- 5th order Runge-Kutta N-body solver

Put your knowledge in practice using FARGO3D

We assume you have already read document and

- installed FARGO3D with the mentioned correction
- have fargo3dplot and managed to import it
- had a look at the structure of the code

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```
arch  fargo3d  license.txt  outputs  README.md  setups  std  utils
bin  in  Makefile  planets  scripts  src  test_suite
```

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arch  fargo3d  license.txt  outputs  README.md  setups  std  utils
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Our problem → Setup files → Parallel or Serial → Make and Run → Read the outputs → Analyse

Put your knowledge in practice using FARGO3D

Our problem → **Setup files** → Parallel or Serial → Make and Run → Read the outputs → Analyse

Units:

- MKS: all quantities must be given in MKS
- CGS: all quantities must be given in CGS
- **Scale-free (default): $G=1, M_*=1, R_0=1 \Rightarrow T=2\pi$**

Put your knowledge in practice using FARGO3D

Our problem → **Setup files** → Parallel or Serial → Make and Run → Read the outputs → Analyse

There are four main setup files:

- setups/<setup_name>/<setup_name>.opt
- setups/<setup_name>/<setup_name>.par
- planets/<planet_name>.cfg
- setups/<setup_name>/<setup_name>.bound

Put your knowledge in practice using FARGO3D

<setup_name>.opt

```
11 FLUIDS := 0
12 NFLUIDS = 1
13 FARGO_OPT += -DNFLUIDS=${NFLUIDS}
14
15 #Monitoring options
16 MONITOR_SCALAR = MASS | MOM_X | TORQ
17 MONITOR_Y_RAW = TORQ
18
19 #Damping zones in the active mesh
20
21 FARGO_OPT += -DSTOCKHOLM
22
23 FARGO_OPT += -DX
24 FARGO_OPT += -DY
25
26 #Equation of State
27 FARGO_OPT += -DISOTHERMAL
28
29 #Coordinate System.
30 FARGO_OPT += -DCYLINDRICAL
31
32 #Legacy files for outputs
33 FARGO_OPT += -DLEGACY
34
35 FARGO_OPT += -DPOTENTIAL
36
37 FARGO_OPT += -DALPHAVISCOSITY
38
```

de Val-Borro+2006

Put your knowledge in practice using FARGO3D

<setup_name>.par

```
1  Setup          test1
2
3  ### Disk parameters
4
5  AspectRatio    0.05
6  Sigma0         6.3661977237e-4
7  Alpha          1.0e-4
8  SigmaSlope     1.5
9  FlaringIndex   0.5
10
11 # Radial range for damping (in period-ratios). Values smaller than one
12 # prevent damping.
13
14 DampingZone 1.15
15
16 # Characteristic time for damping, in units of the inverse local
17 # orbital frequency. Higher values means lower damping
18
19 TauDamp 0.3
20
21 ### Planet parameters
22
23 PlanetConfig    planets/jupiter.cfg
24 ThicknessSmoothing 0.6
25 Eccentricity   0.0
26 ExcludeHill    no
27 IndirectTerm   Yes
```

Thickness over Radius in the disc

$$h = \text{AspectRatio} \left(\frac{r}{R_0=1} \right)^{\text{FlaringIndex}}$$

Slope for the aspect-ratio

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<setup_name>.par

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1 Setup          test1
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7 Alpha          1.0e-4
8 SigmaSlope     1.5            $\Sigma = \text{Sigma0} \left( \frac{r}{R_0=1} \right)^{-\text{SigmaSlope}}$ 
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$$\Sigma = \text{Sigma0} \left(\frac{r}{R_0=1} \right)^{-\text{SigmaSlope}}$$

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7  Alpha          1.0e-4            $v = \alpha c_s H$  viscosity
8  SigmaSlope     1.5             Slope for the surface density
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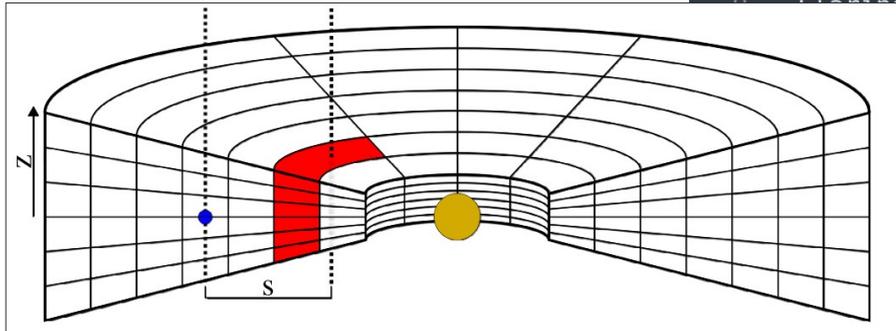
$$y_{\text{inf}} = Y_{\text{min}} \text{DampingZone}^{2/3}$$

$$y_{\text{sup}} = Y_{\text{max}} \text{DampingZone}^{-2/3}$$

$$T_{\text{damp in/out}} = \text{TauDamp} T_{\text{in/out}}$$

Put your knowledge in practice using FARGO3D

<setup_name>.par



Müller1+2012

```
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nt damping.

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characteristic time for damping, in units of the inverse local
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19 TauDamp 0.3
```

```
21   ### Planet parameters
```

```
22
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```

$$\phi_p = \frac{-GM_*}{\sqrt{r^2 + \epsilon^2}}, \quad \epsilon = \text{ThicknessSmoothing } H$$

Put your knowledge in practice using FARGO3D

<setup_name>.par

```
29  ### Mesh parameters
30
31  Nx          256      Azimuthal number of zones
32  Ny          128      Radial number of zones
33  Xmin        -3.14159265358979323844
34  Xmax        3.14159265358979323844
35  Ymin        0.4      Inner boundary radius
36  Ymax        2.5      Outer boundary radius
37  OmegaFrame  1.0005   Angular velocity for the frame of reference (If Frame is F)
38  Frame       C        Method for moving the frame of reference
39
40  ### Output control parameters
41
42  DT          0.314159265359   Physical time between fine-grain outputs
43  Ninterm     20               Number of DTs between scalar fields outputs
44  Ntot        1000            Total number of DTs
45
46  OutputDir   @outputs/test1
47
48  ### Plotting parameters
49
50  PlotLog     yes
51  Spacing     |Log
52
```

x is ϕ
y is r

log only in r

Put your knowledge in practice using FARGO3D

<setup_name>.par

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46  OutputDir    @outputs/test1
47
48  ### Plotting parameters
49
50  PlotLog      yes
51  Spacing      Log
52
```

dt: time-step
DT: between two dots
Ninterm × DT: one output
Ntot/Ninterm: final output

Put your knowledge in practice using FARGO3D

<planet_name>.cfg

```
1 #####
2 # Planetary system initial configuration
3 #####
4
5 # Planet Name    Distance    Mass    Accretion    Feels Disk    Feels Others
6 Jupiter         1.0         0.001   0.0          NO            NO
7
```

Put your knowledge in practice using FARGO3D

<setup_name>.bound

```
1 #Boundaries configuration file for fargo.bound
2 #-----
3
4 Density:
5     Ymin: KEPLERIAN2DDENS
6     Ymax: KEPLERIAN2DDENS
7
8 Vx:
9     Ymin: KEPLERIAN2DVAZIM
10    Ymax: KEPLERIAN2DVAZIM
11
12 Vy:
13    Ymin: ANTISYMMETRIC
14    Ymax: ANTISYMMETRIC
15
```

Put your knowledge in practice using FARGO3D

Our problem → Setup files → Parallel or Serial → Make and Run → **Read the outputs** → Analyse

The outputs contains several types:

- **Field files:** binary, gas<field_name>%n.dat
- **Planet files:** ASCII
 - planet%n.dat (every field output)
 - bigplanet%n.dat, orbit%n.dat, tqwk%n.dat (every DT)
- **Grid files:** ASCII, domain_x.dat, domain_y.dat, domain_z.dat
- **Monitoring files:** ASCII and binary
- more...

Put your knowledge in practice using FARGO3D

What is the plan?

- Make a setup
- Lindblad torque for a static planet
- Migration of a low-mass planet
- Check out the torque on an eccentric planet
- Surface density perturbation by a massive planet
- See the saturation of the co-rotation torque

Note 1: If your computer is not fast enough, you can download the results from the given link in the exercise sheet.

Note 2: Discuss the questions in each section with people around you!

Have fun!

Good reviews for further reading:

Kley & Nelson 2012 

Baruteau & Masset 2013 

Baruteau+2014 

Baruteau+2016 

Paardekooper+2022 

