Cuscuton Dark Energy

Julian Merten

Institut für Theoretische Astrophysik
Zentrum für Astronomie
Universität Heidelberg

December 4$^{\text{th}}$, 2008

Ref.:
R. Durrer and R. Maartens
Dark Energy and Modified Gravity
N. Afshordi, D. J. H. Chung, M. Doran and G. Geshnizjani
Cuscuton Cosmology: Dark Energy meets Modified Gravity

Overview

1. Introduction
2. Fundamental theories
3. Cuscuton and cosmology
4. Two examples
5. Summary

DARK ENERGY AND MODIFIED GRAVITY

Ruth Durrer[1] and Roy Maartens[2]
[1]*Départment de Physique Théorique, Université de Genève, 1211 Genève 4, Switzerland*
[2]*Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 2EG, UK*

Explanations of the late-time cosmic acceleration within the framework of general relativity are plagued by difficulties. General relativistic models are mostly based on a dark energy field with fine-tuned, unnatural properties. There is a great variety of models, but all share one feature in common – an inability to account for the gravitational properties of the vacuum energy, and a failure to solve the so-called coincidence problem. Two broad alternatives to dark energy have emerged as candidate models: these typically address only the coincidence problem and not the vacuum energy problem. The first is based on general relativity and attempts to describe the acceleration as an effect of inhomogeneity in the universe. If this alternative could be shown to work, then it would provide a dramatic resolution of the coincidence problem; however, a convincing demonstration of viability has not yet emerged. The second alternative is based on infra-red modifications to general relativity, leading to a weakening of gravity on the largest scales and thus to acceleration. Most examples investigated so far are scalar-tensor or brane-world models, and we focus on the simplest candidates of each type: $f(R)$ models and DGP models respectively. Both of these provide a new angle on the problem, but they also face serious difficulties. However, investigation of these models does lead to valuable insights into the properties of gravity and structure formation, and it also leads to new strategies for testing the validity of General Relativity itself on cosmological scales.

Cuscuton Cosmology: Dark Energy meets Modified Gravity

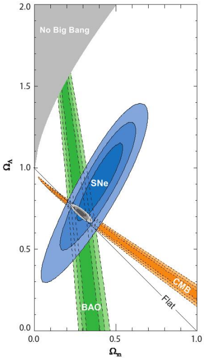
Niayesh Afshordi,[1,*] Daniel J.H. Chung,[2,†] Michael Doran,[3,‡] and Ghazal Geshnizjani[2,§]
[1]*Institute for Theory and Computation, Harvard-Smithsonian Center for Astrophysics, MS-51, 60 Garden Street, Cambridge, MA 02138, USA*
[2]*Department of Physics, University of Wisconsin, Madison, WI 53706, USA*
[3]*Institut fur Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany*
(Dated: October 17, 2008)

In a companion paper [1], we have introduced a model of scalar field dark energy, *Cuscuton*, which can be realized as the incompressible (or infinite speed of sound) limit of a k-essence fluid. In this paper, we study how *Cuscuton* modifies the constraint sector of Einstein gravity. In particular, we study *Cuscuton* cosmology and show that even though *Cuscuton* can have an arbitrary equation of state, or time dependence, and is thus inhomogeneous, its perturbations do not introduce any additional dynamical degree of freedom and only satisfy a constraint equation, amounting to an effective modification of gravity on large scales. Therefore, *Cuscuton* can be considered to be a minimal theory of evolving dark energy, or a minimal modification of a cosmological constant, as it has no internal dynamics. Moreover, this is the only modification of Einstein gravity to our knowledge, that does not introduce any additional degrees freedom (and is not conformally equivalent to the Einstein gravity). We then study two simple *Cuscuton* models, with quadratic and exponential potentials. The quadratic model has the exact same expansion history as ΛCDM, and yet contains an early dark energy component with constant energy fraction, which is constrained to $\Omega_Q \lesssim 2\%$, mainly from WMAP Cosmic Microwave Background (CMB) and SDSS Lyman-α forest observations. The exponential model has the same expansion history as the DGP self-accelerating braneworld model, but generates a much smaller integrated Sachs-Wolfe (ISW) effect, and is thus consistent with the CMB observations. Finally, we show that the evolution is local on super-horizon scales, implying that there is no gross violation of causality, despite *Cuscuton*'s infinite speed of sound.

The need for dark energy

Why Λ

- CMB power spectrum
- Flat universe is clearly favoured
  $\longrightarrow$ What is the rest?



Common realisations

- A cosmological constant
- A dynamical scalar field (quintessence, k-essence)
- Modified gravity (TeVeS, f(R), DGP, ...)
- Abandonment of FRW (Bubbles, voids, back-reaction...)

A dark overview

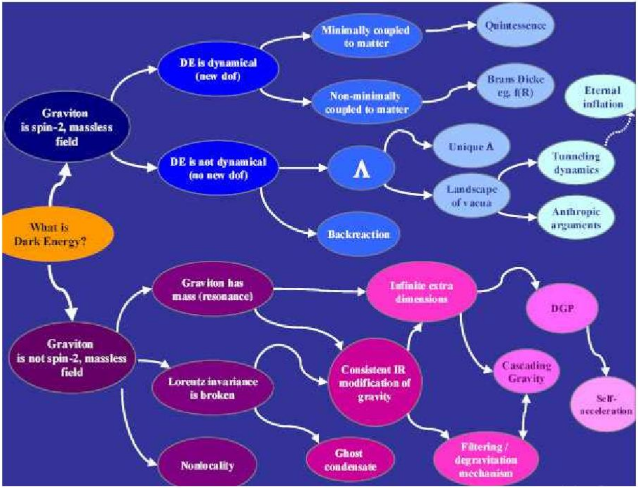


Figure: Stolen from R. Durrer, she stole it from de Rham & Tolley (2008)

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality

Some examples

Scalar fields (spin 0)

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)$$

QCD (spin 1/2 and spin 1)

$$\mathcal{L} = i\bar{\Psi}_{il}\not{D}_{ij}\Psi_{jl} - m_l\bar{\Psi}_l\Psi_l - \frac{1}{2}TR(F^{\mu\nu}F_{\mu\nu})$$

Electrodynamics (spin 1)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J^\mu A_\mu$$

QED (spin 1/2 and spin 1)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\Psi}\not{\partial}\Psi - m\bar{\Psi}\Psi + e\bar{\Psi}\gamma^\mu\Psi A_\mu$$

General relativity (spin 2)

$$\mathcal{L} = \frac{1}{16\pi G}R\sqrt{-g}$$

Different possibilities

After having constructed a "well-behaving" Lagrangian, one can construct:

Classical theories

- Variational principle delivers EOM's
- Very elegant formalism
- No particle creation
- Examples: Maxwell's theory, general relativity

Quantum theories

- Canonical quantisation or path-integral formalism delivers Feynman rules for the theory
- Mathematically difficult
- Allows particle creation (infinite degrees of freedom)
- Examples: The standard model, BCS theory

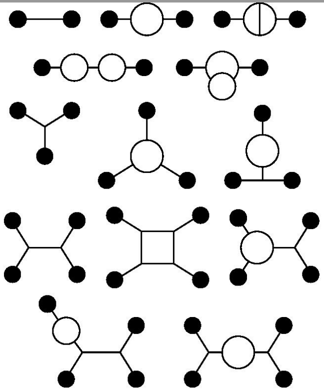
Full theory - straight line = quantum corrections

Different possibilities

After having constructed a "well-behaving" Lagrangian, one can construct:

Classical theories

Quantum theories



Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

$$\text{e.g. } F(\varphi, X) = \varphi^{-2} f(X) \quad \text{with an epoch during which} \quad c_s^2 > 1$$

Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

$$\text{e.g. } F(\varphi, X) = \varphi^{-2} f(X) \quad \text{with an epoch during which} \quad c_s^2 > 1$$

Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

$$\text{e.g. } F(\varphi, X) = \varphi^{-2} f(X) \quad \text{with an epoch during which} \quad c_s^2 > 1$$

Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

$$\text{e.g. } F(\varphi, X) = \varphi^{-2} f(X) \quad \text{with an epoch during which} \quad c_s^2 > 1$$

Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2X F_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

e.g. $F(\varphi, X) = \varphi^{-2} f(X)$ with an epoch during which $c_s^2 > 1$

- These theories are meant to solve the coincidence problem but allow for superluminal motion and therefor violate causality

Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

$$\text{e.g. } F(\varphi, X) = \varphi^{-2} f(X) \quad \text{with an epoch during which} \quad c_s^2 > 1$$

- These theories are meant to solve the coincidence problem but allow for superluminal motion and therefor violate causality

Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

$$L_\varphi = F(\varphi, X) - V(\varphi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

- Field fluctuation propagate with the sound speed :

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2X F_{,XX}}$$

- Quintessence, standard Lagrangian:

$$F(\varphi, X) = X \quad \text{with} \quad c_s^2 = 1$$

- k-essence with non-standard kinetic term:

e.g. $F(\varphi, X) = \varphi^{-2} f(X)$ with an epoch during which $c_s^2 > 1$

  - These theories are meant to solve the coincidence problem but allow for superluminal motion and therefor violate causality

Defining an action

- One starts with a general k-essence Lagrangian

$$S_\varphi = \int d^4x \sqrt{-g} \left[\frac{1}{2} F(X, \varphi) - V(\varphi) \right]$$

- In the homogeneous limit $F(X, \varphi(x)) \to F(\dot{\varphi}^2, \varphi(t))$ we want the field to lose its dynamics (kinetic term can be written as total derivative with $\dot{\varphi} \neq 0$)

$$S_\varphi^{\text{homog.}} = -\int d^4x \, \sqrt{-g} \, V(\varphi)$$

- When coupled to another field we have not added an additional dynamical degree of freedom, but obtain only a constraint equation

$$S_\chi = \int d^4x \left[\mathcal{L}_\chi(\chi, \varphi) - V(\varphi) \right]$$

$$\Rightarrow -\frac{\partial V}{\partial \varphi} + \frac{\mathcal{L}_\varphi(\chi, \varphi)}{\partial \varphi} = 0$$

Defining an action

- One starts with a general k-essence Lagrangian

$$S_\varphi = \int d^4x \sqrt{-g} \left[\frac{1}{2} F(X, \varphi) - V(\varphi) \right]$$

- In the homogeneous limit $F(X, \varphi(x)) \rightarrow F(\dot{\varphi}^2, \varphi(t))$ we want the field to lose its dynamics (kinetic term can be written as total derivative with $\dot{\varphi} \neq 0$)

$$S_\varphi^{\text{homog.}} = -\int d^4x \, \sqrt{-g} \, V(\varphi)$$

- When coupled to another field we have not added an additional dynamical degree of freedom, but obtain only a constraint equation

$$S_\chi = \int d^4x \left[\mathcal{L}_\chi(\chi, \varphi) - V(\varphi) \right]$$

$$\Rightarrow -\frac{\partial V}{\partial \varphi} + \frac{\mathcal{L}_\varphi(\chi, \varphi)}{\partial \varphi} = 0$$

Defining an action

- One starts with a general k-essence Lagrangian

$$S_\varphi = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(X,\varphi) - V(\varphi)\right]$$

- In the homogeneous limit $F(X,\varphi(x)) \rightarrow F(\dot{\varphi}^2,\varphi(t))$ we want the field to lose its dynamics (kinetic term can be written as total derivative with $\dot{\varphi} \neq 0$)

$$S_\varphi^{\text{homog.}} = -\int d^4x\ \sqrt{-g}\ V(\varphi)$$

- When coupled to another field we have not added an additional dynamical degree of freedom, but obtain only a constraint equation

$$S_\chi = \int d^4x\left[\mathcal{L}_\chi(\chi,\varphi) - V(\varphi)\right]$$

$$\Rightarrow -\frac{\partial V}{\partial\varphi} + \frac{\mathcal{L}_\varphi(\chi,\varphi)}{\partial\varphi} = 0$$

Defining an action

- One starts with a general k-essence Lagrangian

$$S_\varphi = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(X,\varphi) - V(\varphi)\right]$$

- In the homogeneous limit $F(X,\varphi(x)) \rightarrow F(\dot{\varphi}^2,\varphi(t))$ we want the field to lose its dynamics (kinetic term can be written as total derivative with $\dot{\varphi} \neq 0$)

$$S_\varphi^{\text{homog.}} = -\int d^4x\ \sqrt{-g}\ V(\varphi)$$

- When coupled to another field we have not added an additional dynamical degree of freedom, but obtain only a constraint equation

$$S_\chi = \int d^4x\,[\mathcal{L}_\chi(\chi,\varphi) - V(\varphi)]$$

$$\Rightarrow -\frac{\partial V}{\partial\varphi} + \frac{\mathcal{L}_\varphi(\chi,\varphi)}{\partial\varphi} = 0$$

Cusculton action

(Obviously)

$$S_\varphi = \int d^4x\sqrt{-g}\left[\mu^2\sqrt{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|} - V(\varphi)\right]$$

does the job.

Properties (Afshordi, Chung and Geshnizjani 2006):

- $c_s = \infty$!
- Still, the Cuscuton field is causal.
  (phase space volume of linear perturbations vanishes in the homogeneous limit)
- Hypersurfaces of constant φ have constant mean curvature (spacetime soap bubbles)
- Cuscuton field is protected against quantum corrections at low energies.

Defining a name

- Cuscuta, dodder
- German: Seide, Teufelszwirn oder Kletterhur
- 100-170 species of parasitic plants
- wraps around host and connects to vascular system
- roots only exist in seed-phase but die when host is found

Why Cuscuton?

More "natural" than quintessence:
Minimal model for evolving dark energy

- No additional, dynamical degree of freedom
- Just adds a constraint-equation, but still affects other fields
  $\Rightarrow$ Might be observable

Could provide a viable low energy effective theory

- The cosmological constant for example is protected very bad against quantum corrections
  $\Rightarrow$ fine-tuning problem
- Also many quintessence models suffer under this problem

Cuscuton cosmology

1. Constraints
   - Homogeneous Universe
   - Inhomogeneous Universe
2. A quadratic potential ($\sim \Lambda$CDM)
3. An exponential potential ($\sim$ DGP)

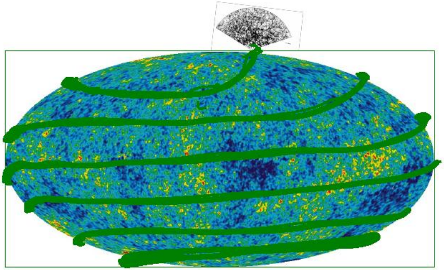


Figure: Stolen from N. Afshordi

Expansion history

We will use the reduced Planck mass $M_p = (8\pi G)^{-1/2}$

- Starting with a FRW metric and an homogeneous field configuration

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i \quad \text{and} \quad \varphi = \varphi(t) \; ,$$

the Cuscuton action becomes

$$S = \int a^3 dt \left[\mu^2 |\dot{\varphi}| - V(\varphi) \right] \; .$$

- Varying this with respect to φ gives the field equation

$$(3\mu^2 H) \; \mathrm{sgn}(\dot{\varphi}) + V'(\varphi) = 0$$

- Remember the ${}^0{}_0$Einstein equation in an homogeneous fluid

$$H^2 = \frac{\rho_{\mathrm{tot}}}{3M_p^2}$$

Expansion history

We will use the reduced Planck mass $M_p = (8\pi G)^{-1/2}$

- Starting with a FRW metric and an homogeneous field configuration

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i \quad \text{and} \quad \varphi = \varphi(t) \ ,$$

the Cuscuton action becomes

$$S = \int a^3 dt \left[\mu^2 |\dot{\varphi}| - V(\varphi)\right] \ .$$

- Varying this with respect to φ gives the field equation

$$(3\mu^2 H) \ \mathrm{sgn}(\dot{\varphi}) + V'(\varphi) = 0$$

- Remember the ${}^0{}_0$Einstein equation in an homogeneous fluid

$$H^2 = \frac{\rho_{\mathrm{tot}}}{3M_p^2}$$

Expansion history

We will use the reduced Planck mass $M_p = (8\pi G)^{-1/2}$

- Starting with a FRW metric and an homogeneous field configuration

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i \quad \text{and} \quad \varphi = \varphi(t) \ ,$$

  the Cuscuton action becomes

$$S = \int a^3 dt \left[\mu^2 |\dot{\varphi}| - V(\varphi)\right] \ .$$

- Varying this with respect to φ gives the field equation

$$(3\mu^2 H) \ \mathrm{sgn}(\dot{\varphi}) + V'(\varphi) = 0$$

- Remember the ${}^0{}_0$Einstein equation in an homogeneous fluid

$$H^2 = \frac{\rho_{\mathrm{tot}}}{3M_p^2}$$

Expansion history

We will use the reduced Planck mass $M_p = (8\pi G)^{-1/2}$

- Starting with a FRW metric and an homogeneous field configuration

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i \quad \text{and} \quad \varphi = \varphi(t) \ ,$$

  the Cuscuton action becomes

$$S = \int a^3 dt \left[\mu^2 |\dot{\varphi}| - V(\varphi)\right] \ .$$

- Varying this with respect to φ gives the field equation

$$(3\mu^2 H) \ \mathrm{sgn}(\dot{\varphi}) + V'(\varphi) = 0$$

- Remember the ${}^0{}_0$Einstein equation in an homogeneous fluid

$$H^2 = \frac{\rho_{\mathrm{tot}}}{3M_p^2}$$

What about ρ?

- Most general form of the energy momentum tensor for scalar fields a

$$T^{\mu\nu} \equiv -\frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \varphi_a(x))} \partial^\nu \varphi_a(x) + g^{\mu\nu} \mathcal{L}(x)$$

- Fortunately we look at an homogeneous universe and only at the ${}^0{}_0$component of a single field

$$\rho = \frac{1}{2} \left(\frac{d\varphi}{dt} \right)^2 + V(\varphi)\ .$$

- And remember how Cuscuton was constructed

$$\rho = V(\varphi)$$

- This gives us

$$\left(\frac{M_p^2}{3\mu^4} \right) V'^2(\varphi) - V(\varphi) = \rho_m\ .$$

What about ρ?

- Most general form of the energy momentum tensor for scalar fields a

$$T^{\mu\nu} \equiv -\frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \varphi_a(x))} \partial^\nu \varphi_a(x) + g^{\mu\nu} \mathcal{L}(x)$$

- Fortunately we look at an homogeneous universe and only at the ${}^0{}_0$component of a single field

$$\rho = \frac{1}{2} \left(\frac{d\varphi}{dt} \right)^2 + V(\varphi) \ .$$

- And remember how Cuscuton was constructed

$$\rho = V(\varphi)$$

- This gives us

$$\left(\frac{M_p^2}{3\mu^4} \right) V'^2(\varphi) - V(\varphi) = \rho_m \ .$$

What about ρ?

- Most general form of the energy momentum tensor for scalar fields a

$$T^{\mu\nu} \equiv -\frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \varphi_a(x))}\partial^\nu \varphi_a(x) + g^{\mu\nu}\mathcal{L}(x)$$

- Fortunately we look at an homogeneous universe and only at the ${}^0{}_0$component of a single field

$$\rho = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi)\ .$$

- And remember how Cuscuton was constructed

$$\rho = V(\varphi)$$

- This gives us

$$\left(\frac{M_p^2}{3\mu^4}\right) V'^2(\varphi) - V(\varphi) = \rho_m\ .$$

What about ρ?

- Most general form of the energy momentum tensor for scalar fields a

$$T^{\mu\nu} \equiv -\frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \varphi_a(x))} \partial^\nu \varphi_a(x) + g^{\mu\nu} \mathcal{L}(x)$$

- Fortunately we look at an homogeneous universe and only at the ${}^0{}_0$component of a single field

$$\rho = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi) \; .$$

- And remember how Cuscuton was constructed

$$\rho = V(\varphi)$$

- This gives us

$$\left(\frac{M_p^2}{3\mu^4}\right) V'^2(\varphi) - V(\varphi) = \rho_m \; .$$

What about ρ?

- Most general form of the energy momentum tensor for scalar fields a

$$T^{\mu\nu} \equiv -\frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \varphi_a(x))}\partial^\nu \varphi_a(x) + g^{\mu\nu}\mathcal{L}(x)$$

- Fortunately we look at an homogeneous universe and only at the ${}^0{}_0$component of a single field

$$\rho = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi)\ .$$

- And remember how Cuscuton was constructed

$$\rho = V(\varphi)$$

- This gives us

$$\left(\frac{M_p^2}{3\mu^4}\right) V'^2(\varphi) - V(\varphi) = \rho_m\ .$$

Constraints on the potential

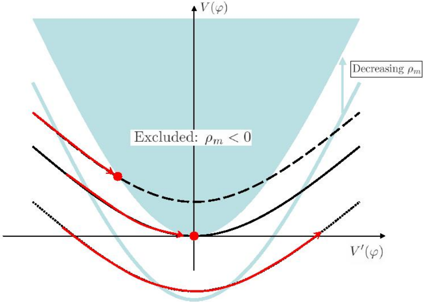
Now we can put constraints through ρ_m:

- Weak energy condition: $\rho_m > 0$

$$V(\varphi) < \left(\frac{M_p^2}{3\mu^4}\right) V'^2(\varphi)$$

- Null energy condition: $\rho_m + p_m > 0$

$$V''(\varphi) = \frac{1}{2}\frac{dV'^2(\varphi)}{dV(\varphi)} > \frac{3\mu^4}{2M_p^2}$$



Scalar metric perturbations

We now look at a linearly perturbed FRW metric in the presence of dust

- The field equation becomes:

$$3\dot{\varphi}(\dot{\Phi} + H\Phi) + a^{-2}\nabla^2\delta\varphi - \mu^{-2}|\dot{\varphi}|V''(\varphi)\delta\varphi = 0$$

giving for the field perturbations in Fourier space:

$$\delta\varphi = \frac{3\dot{\varphi}(\dot{\Phi} + H\Phi)}{\frac{k^2}{a^2} - 3\dot{H}}$$

- For the metric perturbations you find the Poisson equation:

$$\left(\frac{k^2}{a^2}\right)\Phi + \left[3H + \frac{9H(2\dot{H} + 3H^2\Omega_m)}{2\left(\frac{k^2}{a^2} - 3\dot{H}\right)}\right]\left(\dot{\Phi} + H\Phi\right) + (2M_p^2)^{-1}\delta\rho_m = 0$$

- $\delta\varphi$ drops out and Cuscuton acts as a modification of gravity

Another thing on modified gravity

Using the homogeneous field equations, we find:

$$H^2 = \frac{1}{3M_p^2}\left\{\rho_m + V\left[V'^{-1}(3\mu^2 H)\right]\right\},$$

where V'^{-1} is the inverse function of V'.

This again shows, that Cuscuton can be interpreted as a modification of gravity.

Look at a quadratic potential:

$$V = \frac{1}{2}m^2\varphi^2.$$

Inserting this, we obtain again a Friedmann equation with a modified Planck mass:

$$M_p^2 \rightarrow M_p^2 - \frac{3\mu^4}{2m^2}$$

In contrast to other models, we have not introduced an additional degree of freedom.

CMB anisotropies

In addition to the change to the matter power spectrum, we have an induced CMB anisotropy due to the Fourier mode $\Phi_{\mathbf{k}}$:

$$\begin{aligned}\Theta_{l,\mathbf{k}} = &\int_0^{\eta_0} d\eta\ g(\eta)(\Theta_0 + \Phi_{\mathbf{k}})\ j_l[k(\eta_0 - \eta)] + \\ &\frac{1}{ik}\int_0^{\eta_0} d\eta\ v_b(\mathbf{k})\ g(\eta)\frac{\partial}{\partial\eta}\ j_l[k(\eta_0 - \eta)] + \\ &2\int_0^{\eta_0} d\eta\ \frac{\partial\Phi_{\mathbf{k}}}{\partial\eta}e^{-\tau(\eta)}\ j_l[k(\eta_0 - \eta)]\end{aligned}$$

The ISW contribution to the CMB anisotropies can be computed perturbatively:

$$\frac{\partial\Phi_{(1),\mathbf{k}}}{\partial\eta} = a_i\left(\frac{t(\eta)}{t_i}\right)^{-2}\int_{t_i}^{t(\eta)} dt'\left(\frac{t'}{t_i}\right)^{8/3} S_{\mathbf{k}}(t') \qquad S_{\mathbf{k}}(t) \equiv -\left\{3V + \frac{3t\dot{V}}{4} + \frac{3}{\frac{k^2}{a^2} + \frac{2}{t^2}}\left[\frac{\dot{V}}{t} + \frac{\ddot{V}}{2} + \left(\frac{\dot{V}}{2t}\right)\frac{\frac{4k^2}{3a^2} + \frac{4}{t^2}}{\frac{k^2}{a^2} + \frac{2}{t^2}}\right]\right\}\frac{\Phi_{(0),\mathbf{k}}}{3M_p^2}$$

A quadratic potential I

$$V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2$$

- V_0 is nothing else but a cosmological constant, but for the quadratic term we have:

$$\Omega_Q = \frac{\frac{1}{2}m^2\varphi^2}{\rho_{tot}} = \frac{3\mu^4}{2M_p^2 m^2} = \text{const.}$$

- This kind of tracking behaviour is identical to quintessence (EDE).
- Since the number of rel. neutrinos during radiation domination also does not change H(z) there is a degeneracy.
  $\Rightarrow \Omega_Q \lesssim 10\%$
- Large scale structure and CMB are putting much tighter constraints:
  $\Rightarrow \Omega_Q < 1.6\%$ (best constraint on EDE to the authors' knowledge)

A quadratic potential II

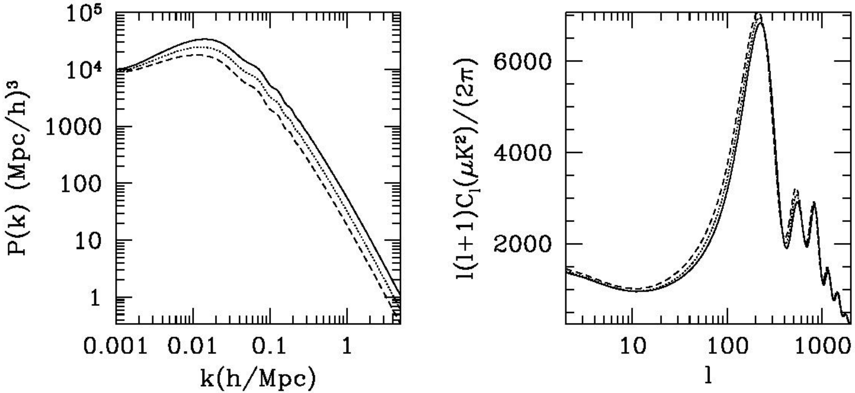


Figure: Ω_Q= 0, 0.05, 0.1; solid, dotted, dashed

An exponential potential I

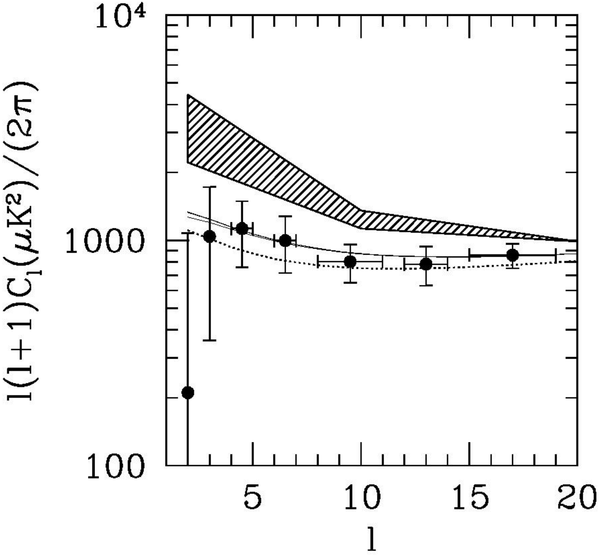
$$V(\varphi) = V_0 \; exp \left[- \left(\frac{\mu^2 r_c}{M_p^2} \right) \varphi \right]$$

- Substituting into the homogeneous equations yields:

$$H = \frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\rho_m}{3M_P^2}}$$

  which is exactly the DGP expansion history.
- As a result it is not possible to distinguish exponential Cuscuton from DGP with geometrical tests.
- Distinction is possible through e.g. ISW

An exponential potential II



Summary

1. Successful theories seem to and should have a common foundation and justification.
2. Cuscuton DE is a minimal extension to a cosmological constant.
3. The effect of the Cuscuton field can be tested with cosmological experiments.
4. Cuscuton blurs the line between dark energy and modified gravity models. (argument stolen from N. Afshordi)
5. A quadratic potential mimics the tracking behaviour of quintessence models.
6. An exponential potential gives the expansion history of DGP, but a distinction is possible.

HGSFP