Combining Weak and Strong Lensing in Galaxy Cluster Mass Reconstruction

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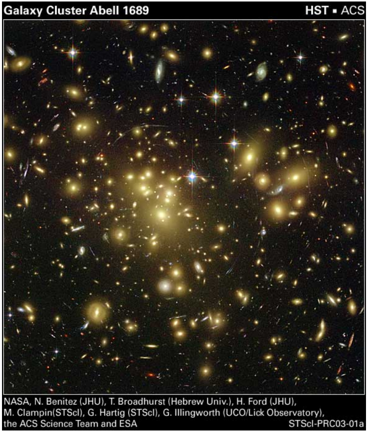
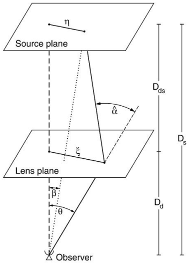
Galaxy Clusters

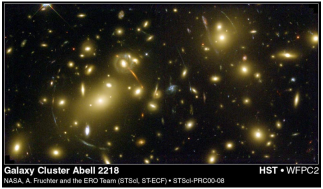
- Galaxy clusters (GC's) are the largest gravitationally bound objects in the universe. $(10^{13} - 10^{15} M_{\odot})$
- GC's are very important for cosmology
  - ΛCDM predicts bottom-up-scenario
  - linear regime
  - mass function of dark matter halos
- Different methods of mass determination
  - dynamical analysis
  - X-Ray emission
  - gravitational lensing



Gravitational Lensing by Galaxy Clusters

- Bending of light rays induced by gravitation
- Predicted by Einstein's General Theory of Relativity
- Lensing effects by galaxy clusters are distortions of the images of background galaxies. The cluster is acting as the lens.
- Two important effects are observed and well understood
    - strong lensing (spectacular)
    - weak lensing (statistical)





Lens Equation

$$\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \frac{4G}{c^2} \int d^2\xi' \Sigma(\boldsymbol{\xi}') \frac{\boldsymbol{\xi} - \boldsymbol{\xi}'}{|\boldsymbol{\xi} - \boldsymbol{\xi}'|^2}$$

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \frac{D_{ds}}{D_s} \hat{\boldsymbol{\alpha}}(D_d \boldsymbol{\theta}) \equiv \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

Lensing Quantities

$$\boldsymbol{\alpha} = \nabla\psi$$

$$\kappa = \frac{1}{2}(\psi_{,11} + \psi_{,22})$$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22})$$

$$\gamma_2 = \psi_{,12}$$

Lensing Potential

$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma(D_d \boldsymbol{\theta})}{\Sigma_{cr}} \quad \text{with}$$

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}}$$

$$\psi(\boldsymbol{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}_2} d^2\theta' \kappa(\boldsymbol{\theta}') \ln|\boldsymbol{\theta} - \boldsymbol{\theta}'|$$

Magnification and Critical Curve

$$\mu = \frac{1}{\det\mathcal{A}} = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

Recent Lensing Highlights

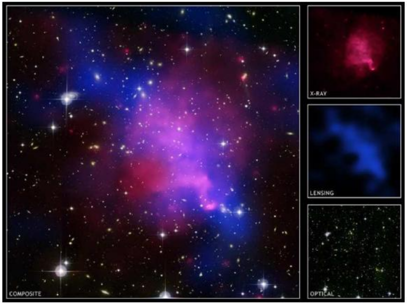


Figure: The "Cosmic Train Wreck" Abell 520. (Mahdavi et al, 2007)

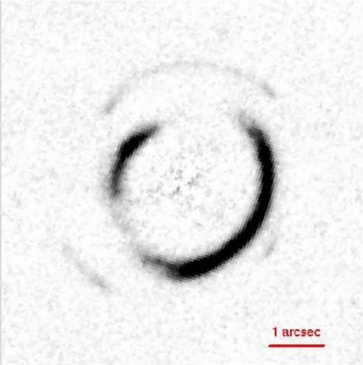
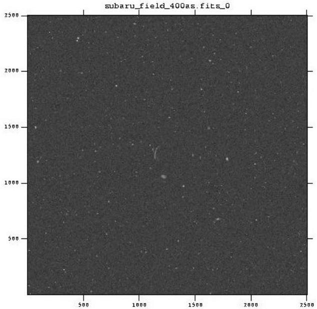


Figure: The first discovered double Einstein ring. (Gavazzi et al, 2008)

Maximum-Likelihood Reconstruction

- Reconstruction quantity is the lensing potential ψ
- $\chi^2(\psi) = \chi^2_w(\psi) + \chi^2_s(\psi)$
- χ^2-function is then minimised wrt to certain parameters
- Input data are:
  1. Ellipticity catalog
  2. Critical curve position
- The reconstruction field is divided into a grid.
  This allows a completely non-parametric reconstruction

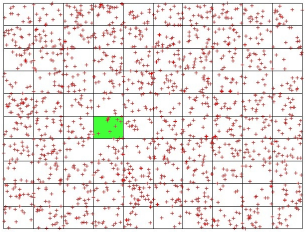


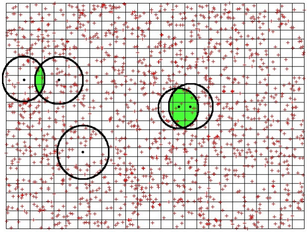
Collaboration

Marcello Cacciato (MPIA)
Massimo Meneghetti (INAF Bologna)
Matthias Bartelmann (ZAH/ITA)

Weak Lensing: Resolution Problems

- Weak lensing has to be treated statistically
  $\Rightarrow$ Averaging over ~ 10 background galaxies
- With state-of-the-art observations this would allow for a ($\sim$ 10x10) pixel reconstruction grid
- Furthermore galaxies are not distributed homogenously over the field
- Solution:
  Adaptive-averaging-process
  Problem:
  Grid points become correlated





Weak Lensing: χ^2-Function

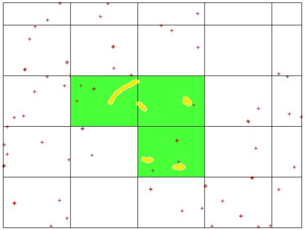
- Weak lensing observables are slight distortions of background galaxies
- Assuming a source to be point-like in its surface-brightness-distribution it would appear elliptical due to lensing
- This ellipticity can be measured but this is a difficult task
- The expectation value for the ellipticity of a source lensed by a deflector is the reduced shear:

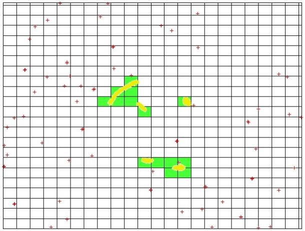
$$\langle\varepsilon\rangle = \begin{cases} \dfrac{Z(z)\gamma}{1 - Z(z)\kappa} & \text{for } |g| \leq 1 \\ \dfrac{1 - Z(z)\kappa}{Z(z)\gamma^*} & \text{for } |g| > 1 \end{cases} \quad \text{with} \quad g(\boldsymbol{\theta}) \equiv \frac{\gamma(\boldsymbol{\theta})}{1-\kappa(\boldsymbol{\theta})}$$

$$\chi^2_w(\psi) = \sum_{i,j} \left(\varepsilon - \frac{Z(z)\gamma(\psi)}{1-Z(z)\kappa(\psi)}\right)_i \mathcal{C}^{-1}_{ij} \left(\varepsilon - \frac{Z(z)\gamma(\psi)}{1-Z(z)\kappa(\psi)}\right)_j$$

Strong Lensing: Resolution Problems

- The exact position of the critical curve is not observable
- Position of arcs is a very good approximation for the location of the critical curve
- Arc positions are known with high accuracy
- Using weak lensing grid resolutions would result in information loss
- ⇒ Use of refined grid to resolve arc positions
- Also adaptive grids are implemented





Strong Lensing: χ^2-Function

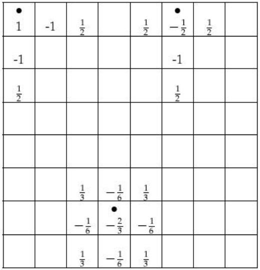
- At the position of the critical curve the determinant of the Jacobian vanishes
- This gives rise to the following strong lensing χ^2-function:

$$\chi_s^2(\psi) = \sum_i \frac{|\det A(\psi)|_i^2}{\sigma_{is}^2} = \sum_i \frac{|(1-Z(z)\kappa(\psi))^2 - |Z(z)\gamma(\psi)|^2|_i^2}{\sigma_{is}^2}$$

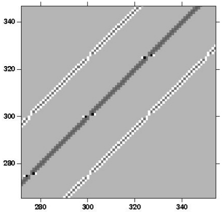
- The error estimation σ is mainly given by the pixelization of the grid
- Note that arc redshifts are crucial

Non-Parametric Approach: Finite Differences

- κ and γ are linear combinations of second derivatives of ψ
- Since a grid is used, they can be expressed through the potential via finite differences
- An elegant representation is a matrix multiplication:



$$\kappa_i = \mathcal{K}_{ij}\psi_j$$
$$\gamma_i^1 = \mathcal{G}_{ij}^1\psi_j$$
$$\gamma_i^2 = \mathcal{G}_{ij}^2\psi_j$$



- The total χ^2-function is now just a function of the potential on every grid position

Non-Parametric Approach: Solving Linear Systems

- To obtain the result which is the discrete representation of ψ on the reconstruction grid, we just have to minimise $\chi^2(\psi)$ wrt every potential value at grid position l

$$\frac{\partial \chi^2(\psi)}{\partial \psi_l} \stackrel{!}{=} 0$$

- This can be done by solving a linear system:

$$\mathcal{B}_{lk}\psi_k = \mathcal{V}_l,$$

- Building up the linear system and solving it is numerically not unproblematic but possible
- With the solution of the linear system the reconstruction is finished and from the potential a useful representation like a κ-map can be obtained easily

An Unrealistic Synthetic Test

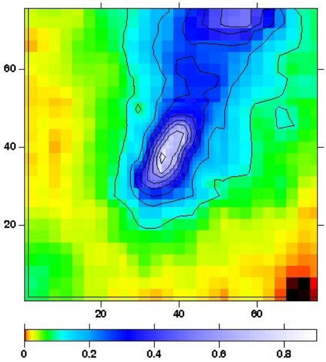


Figure: Reconstructed κ-map using the critical curve on a refined grid.

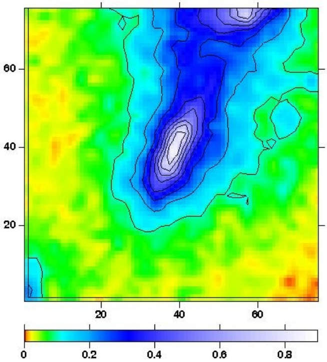
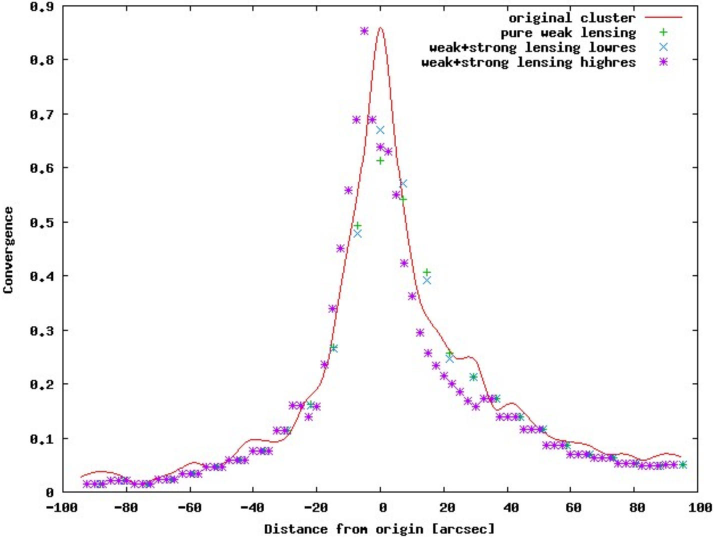


Figure: Original κ-map of the critical cluster rebinned to the reconstruction resolution.

Synthetic Test: κ-Profile



The Lensing Simulator: The Images

Developed by Meneghetti et al, 2007

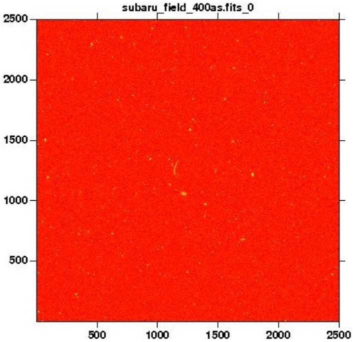


Figure: The full 2501x2501 pixel CCD image. The field size is 400"x400".

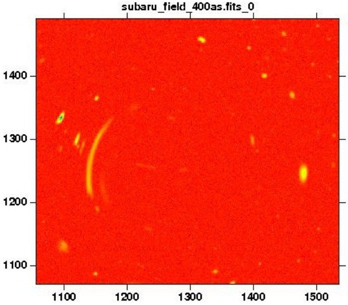


Figure: The arcs produced by the simulation. Field size is ∼50"x50".

The Lensing Simulator: High Resolution Reconstruction

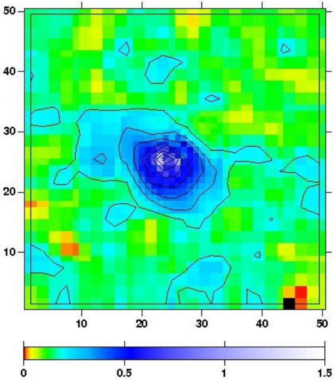


Figure: Reconstructed κ-map.

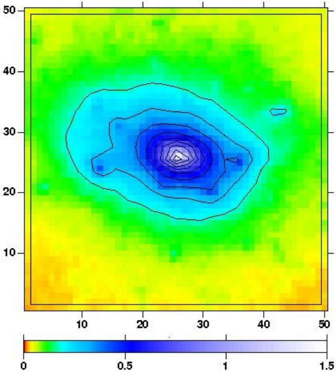


Figure: The original cluster at reconstruction resolution.

MS 2137: High Resolution Reconstruction

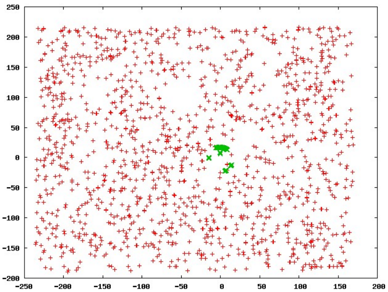


Figure: Galaxy distribution and arc positions in the MS 2137 field

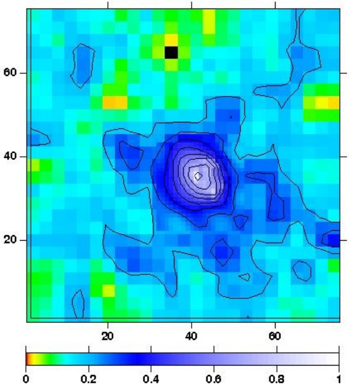


Figure: Reconstructed κ-map of MS 2137

Conclusions

- We developed and implemented a reconstruction method which successfully combines weak and strong lensing
- Simulations show that an optimal result is just obtained by the usage of both effects (if possible)
- Unfortunately accurate lensing reconstructions are still limited by image analysis (a lot of good work in progress)
- Application to MS 2137 shows that our method delivers reasonable results with real cluster data
- Several improvements are possible:
  - Addition of multiple image systems (Bradač et al, 2005)
  - Use of flexion, lensing to second order (P. Melchior, ZAH/ITA)
  - More elaborate usage of arcs as critical curve tracers
  - Numerical improvements (application to big datasets)