# Combining Weak and Strong Gravitational Lensing

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with:
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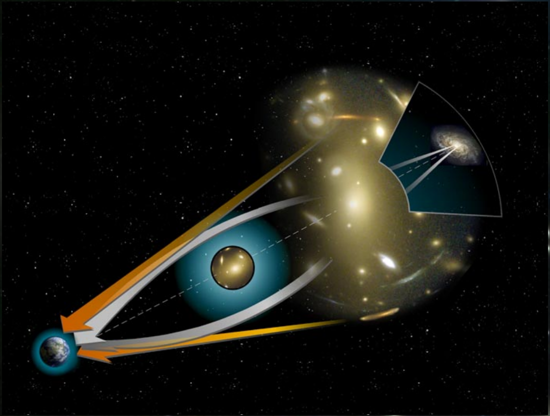
# Gravitational Lensing



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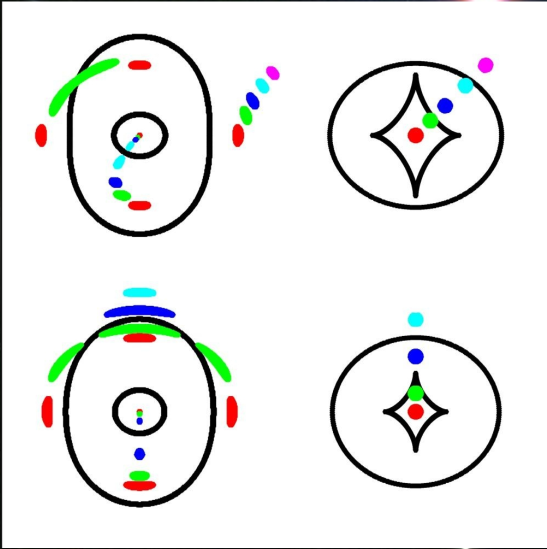

## The lensing potential

$$\alpha_1 = \psi_{,1} \qquad \alpha_2 = \psi_{,2}$$

$$\gamma_1 = \frac{1}{2}\left(\psi_{,11} - \psi_{,22}\right) \qquad \gamma_2 = \psi_{,12}$$

$$\kappa = \frac{1}{2}\left(\psi_{,11} + \psi_{,22}\right)$$

# Combining Weak and Strong Lensing (JM et al. 2009)

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$$\frac{\partial \chi^2(\psi)}{\partial \psi} \stackrel{!}{=} 0$$

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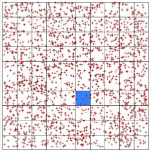

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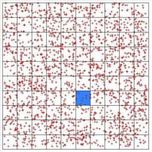

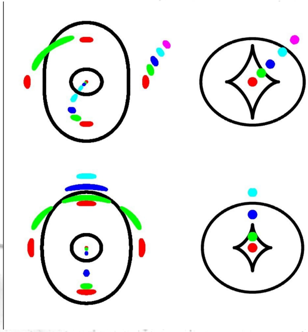

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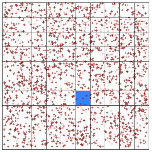

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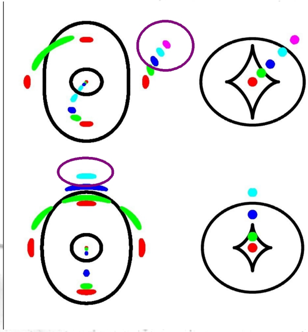

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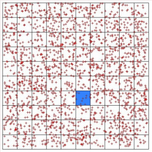
$$\left|(1-\kappa)^2 - (\gamma)^2\right|_\mathrm{crit} = 0$$



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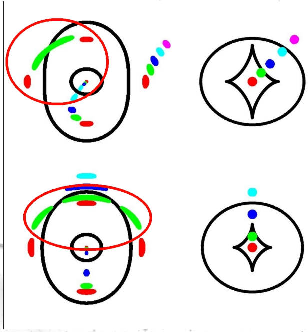
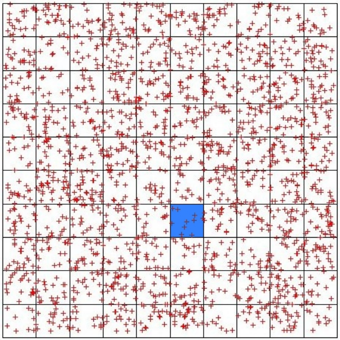
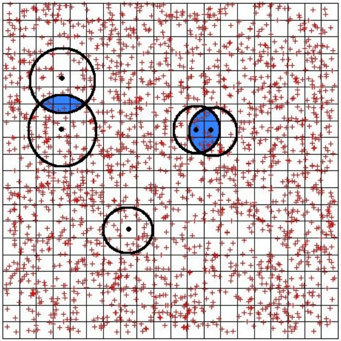


# A Matter of Scale: weak lensing

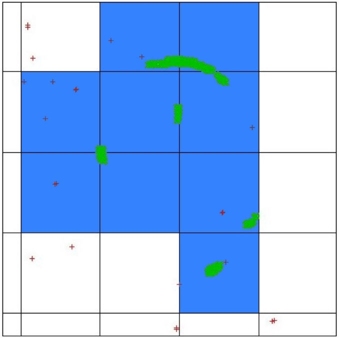
$$\chi^2_w(\psi) = \sum_{i,j} \left(\varepsilon - \frac{Z(z)\gamma(\psi)}{1 - Z(z)\kappa(\psi)}\right)_i \mathcal{C}^{-1}_{ij} \left(\varepsilon - \frac{Z(z)\gamma(\psi)}{1 - Z(z)\kappa(\psi)}\right)_j$$

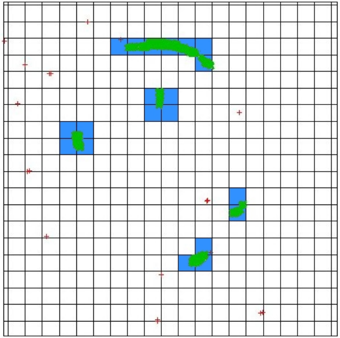




# A Matter of Scale: strong lensing

$$\chi_s^2(\psi) = \sum_i \frac{\left((1 - Z(z)\kappa(\psi))^2 - (Z(z)\gamma(\psi))^2\right)_i^2}{\sigma_i^2}$$





# Making it all work: Numerics

- $\alpha$, $\gamma$, $\kappa$, $\mathcal{F}$ and $\mathcal{G}$ can be expressed by derivatives of $\psi$ via finite differences.
- A specific finite difference can be written as a matrix multiplication

$$\kappa_i = \mathcal{K}_{ij}\psi_j.$$

- The minimisation of the $\chi^2$-function can be translated into a linear system of equations.
- Furthermore the code uses a 2-level iteration scheme.
- Runtime: 2 mins - 6 hrs.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| • 1 | -1 | $\frac{1}{2}$ | | $\frac{1}{2}$ | • $-\frac{1}{2}$ | $\frac{1}{2}$ | |
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| | | | | | | | |
| | | $\frac{1}{3}$ | $-\frac{1}{6}$ | $\frac{1}{3}$ | | | |
| | | $-\frac{1}{6}$ | • $-\frac{2}{3}$ | $-\frac{1}{6}$ | | | |
| | | $\frac{1}{3}$ | $-\frac{1}{6}$ | $\frac{1}{3}$ | | | |

## Implementation

- Parallel C++ code
- medium sized $\sim$ 12000 lines
- Uses GSL, LAPACK, ATLAS, MPI
- Possibly, GPU implementation. More on that soon.

# The Reconstruction Method

In our reconstruction method we try to combine the advantages of both lensing regimes into a joint method:

- Fully non-parametric, adaptive grid method (no initial model necessary).
- Reconstruction quantity is the lensing potential $\psi$.
- Maximum-likelihood method. We are searching for that lensing potential which is most likely to have caused the observations:

$$\chi^2(\psi) = \chi^2_w(\psi) + \chi^2_s(\psi)$$

- Input data are:
  1. Ellipticity catalogue
  2. Arc positions
  3. Flexion catalogue (come tomorrow if you are interested in that)
  4. Multiple image positions (Bradač et al. 2005-08)

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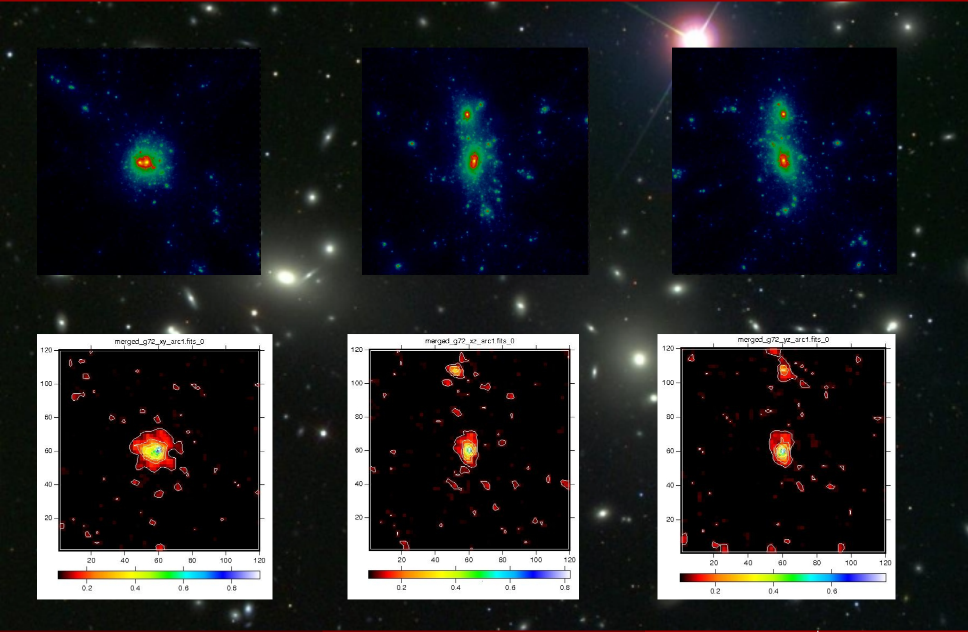

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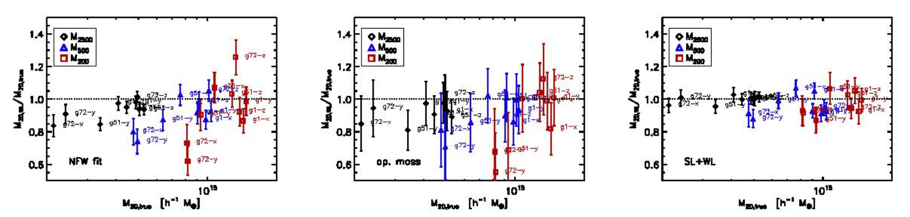
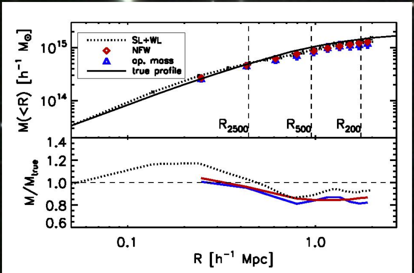
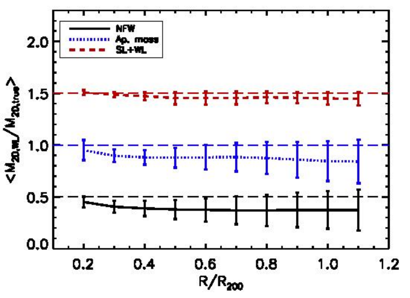
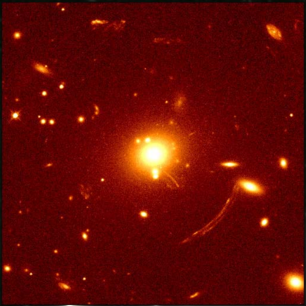
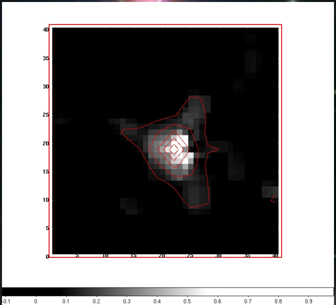


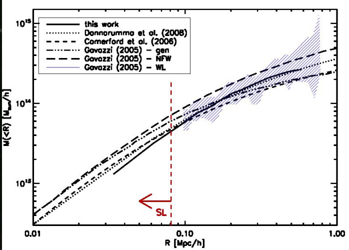



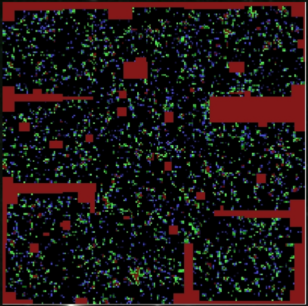
# Results: MS2137 (JM et al. 2009)

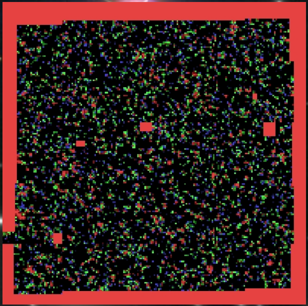


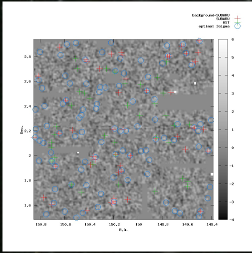


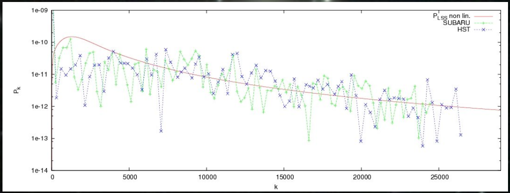




# Results: COSMOS (preliminary, with Matteo Maturi)







# Results: SUBARU Cluster sample (JM et al. in prep.)

with Masamune Oguri, Keiichi Umetsu and Tom Broadhurst

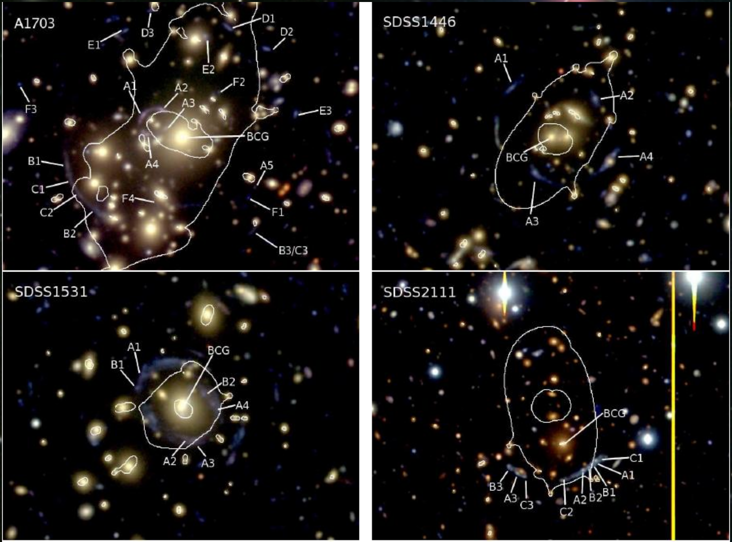


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TABLE 1

THE SUBARU DISTORTION MEASUREMENTS COMBINED WITH THE EINSTEIN-RADIUS CONSTRAINT

| Cluster | $z$ | Filters | Einstein Radius (arcsec) | $\langle D_{ds}/D_s \rangle$ | $\frac{d \log N(<m)}{dm}$ | $M_{vir}$ ($10^{15}\ M_\odot\ h_{70}^{-1}$) | $c_{vir}$ | $\chi^2$/dof |
|---|---|---|---|---|---|---|---|---|
| A1689 .............. | 0.183 | $V_J i'$ | 52 ($z_s$ = 3.05) | 0.704 | 0.150 | $1.59^{+0.24}_{-0.22}$ | $15.69^{+3.96}_{-2.88}$ | 4.94/9 |
| A1703 .............. | 0.258 | $g' r' i'$ | 33 ($z_s$ = 2.8) | 0.722 | 0.062 | $1.30^{+0.24}_{-0.20}$ | $9.92^{+2.39}_{-1.63}$ | 2.69/5 |
| A370 ................ | 0.375 | $B R_C z'$ | 43 ($z_s$ = 1.5) | 0.606 | 0.088 | $2.93^{+0.36}_{-0.32}$ | $7.75^{+1.12}_{-0.92}$ | 5.54/8 |
| RX J1347−11 ...... | 0.451 | $V_J R_C z'$ | 35 ($z_s$ = 1.8) | 0.553 | 0.066 | $1.47^{+0.26}_{-0.23}$ | $10.42^{+3.25}_{-2.13}$ | 6.25/7 |

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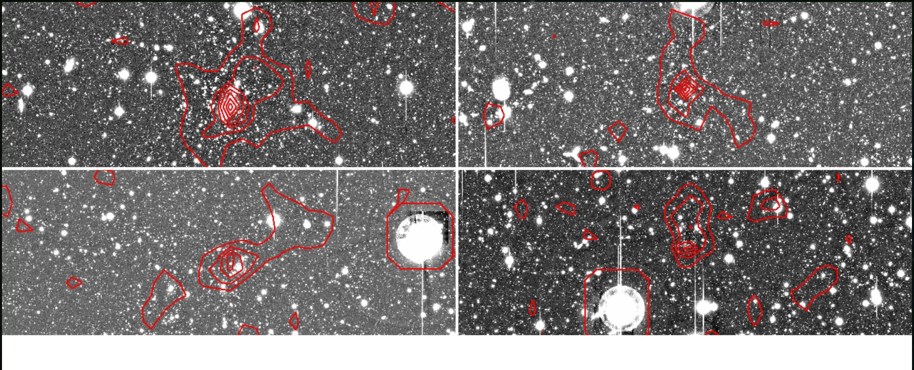

# Thank You