Flexion Revisited

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with:
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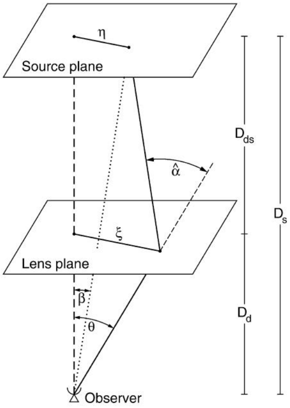
The linearized lens equation

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}}(D_d\boldsymbol{\theta}) \equiv \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) \quad (1)$$

$$I(\boldsymbol{\theta}) = I^{(s)}[\boldsymbol{\beta}(\boldsymbol{\theta})] \quad (2)$$

$$I(\boldsymbol{\theta}) = I^{(s)}[\boldsymbol{\beta_0} + \mathcal{A}(\boldsymbol{\beta_0}) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)] \quad (3)$$

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}} = \left(\delta_{ij} - \frac{\partial^2\psi(\boldsymbol{\theta})}{\partial\theta_i\partial\theta_j}\right) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_2 \end{pmatrix} \quad (4)$$



Second order lens equation

$$\beta_i = \frac{\partial \beta_i}{\partial \theta_j}\theta_j + \frac{1}{2}\frac{\partial^2 \beta_i}{\partial \theta_j \theta_k}\theta_j\theta_k + \mathcal{O}(\boldsymbol{\theta}^3) \tag{5}$$

$$D_{ijk} = \frac{\partial A_{ij}}{\partial \theta_k}, \tag{6}$$

$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix}, \quad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix} \tag{7}$$

$$I(\boldsymbol{\theta}) \simeq \left\{ 1 + \left[(A - I)_{ij}\theta_j + \frac{1}{2} D_{ijk}\theta_j\theta_k \right] \frac{\partial}{\partial \theta_i} \right\} I^{(s)}(\boldsymbol{\theta}) \tag{8}$$

Let's think about something more clever.

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \rightarrow v_1 + iv_2 \quad (9)$$

$$\partial \equiv \partial_1 + i\partial_2 \qquad \partial^\dagger \equiv \partial_1 - i\partial_2 \quad (10)$$

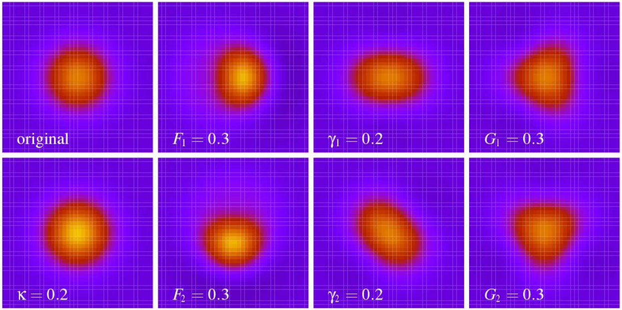
$$\partial\psi = \alpha \qquad s = 1 \quad (11)$$

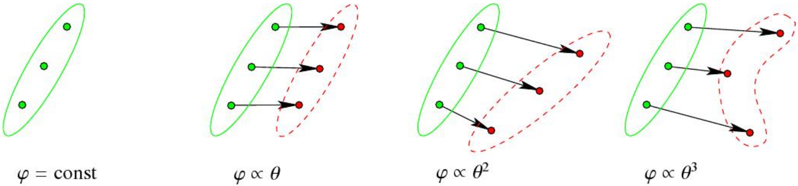
$$\partial^\dagger\partial\psi = 2\kappa \qquad s = 0 \quad (12)$$

$$\partial\partial\psi = 2\gamma \qquad s = 2 \quad (13)$$

$$\partial\partial^\dagger\partial\psi = 2\mathcal{F} \qquad s = 1 \quad (14)$$

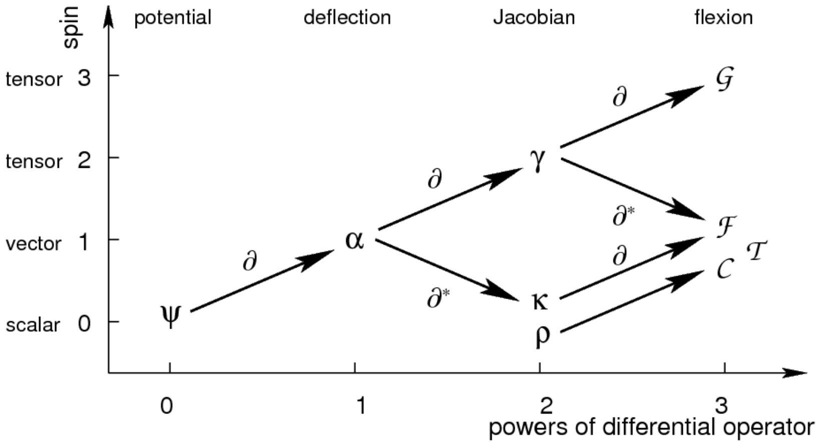
$$\partial^3\psi = 2\mathcal{G} \qquad s = 3 \quad (15)$$





(Figures stolen from P. Melchior and B.M. Schäfer)

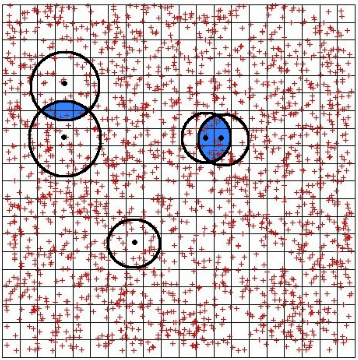
A beautiful formalism (Schäfer & Bacon in prep.)



(Figure stolen from B.M. Schäfer)

A combined reconstruction method in a nutshell

- Reconstruction quantity is the lensing potential ψ.



- Maximum-likelihood, grid-based approach.

$$\chi^2(\psi) = \chi^2_{\mathrm{w}}(\psi) + \chi^2_{\mathrm{s}}(\psi) + \chi^2_{\mathrm{f}}(\psi)$$

$$\frac{\partial \chi^2(\psi_{\mathrm{k}})}{\partial \psi_{\mathrm{l}}} \stackrel{!}{=} 0$$

- $\mathcal{F} = \frac{1}{2}\left[[\psi_{,111} + \psi_{,122}] + i[\psi_{,112} + \psi_{,222}]\right]$

  $\mathcal{G} = \frac{1}{2}\left[[\psi_{,111} - 3\psi_{,122}] + i[3\psi_{,112} - \psi_{,222}]\right]$

- $\chi^2_{fF}(\psi) = \sum_{i,j} \left(\langle \mathcal{F}_{\mathrm{obs}} \rangle - \mathcal{F}\right)_i \mathcal{C}^{-1}_{ij} \left(\langle \mathcal{F}_{\mathrm{obs}} \rangle - \mathcal{F}\right)_j$

  $\chi^2_{fG}(\psi) = \sum_{i,j} \left(\langle \mathcal{G}_{\mathrm{obs}} \rangle - \mathcal{G}\right)_i \mathcal{C}^{-1}_{ij} \left(\langle \mathcal{G}_{\mathrm{obs}} \rangle - \mathcal{G}\right)_j$

-1	2	-3/2	1/2		-1/4	1/4		-1/4	1/4 -
1	-1					1/2		-1/2	
-1/2	1/2					-1/4		1/4	
									-
-1/2	1/2								
1/2	1/2	-3/2	1/2			-1/4		1/4	
-1/2	1/2				-1/4	1		-1	1/4
+			+			-1/4		1/4	-

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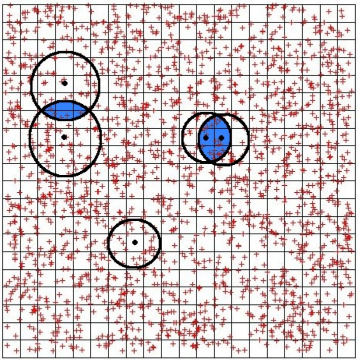

-1	2	-3/2	1/2		-1/4	1/4		-1/4	1/4 -
1	-1					1/2		-1/2	
-1/2	1/2					-1/4		1/4	
									-
-1/2	1/2								
1/2	1/2	-3/2	1/2			-1/4		1/4	
-1/2	1/2				-1/4	1		-1	1/4
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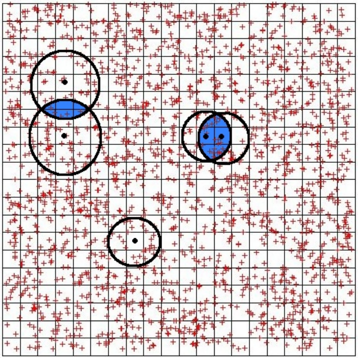

-1	2	-3/2	1/2		-1/4	1/4		-1/4	1/4 -
1	-1					1/2		-1/2	
-1/2	1/2					-1/4		1/4	
									-
-1/2	1/2								
1/2	1/2	-3/2	1/2			-1/4		1/4	
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+			+			-1/4		1/4	-

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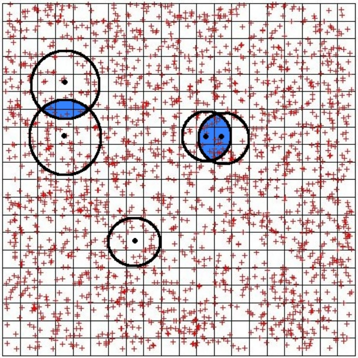

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1	-1					1/2		-1/2	
-1/2	1/2					-1/4		1/4	
									-
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1/2	1/2	-3/2	1/2			-1/4		1/4	
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+			+			-1/4		1/4	-

- $\mathcal{F} = \frac{1}{2}\left[[\psi_{,111} + \psi_{,122}] + i[\psi_{,112} + \psi_{,222}]\right]$

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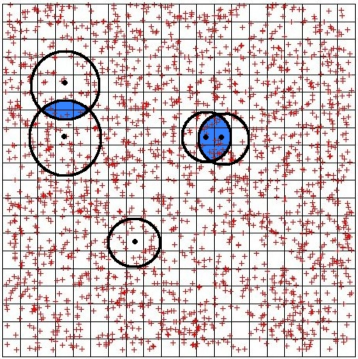

1	-1	1/2				1/2	-1/2	1/2	+
-2	1	-1/2				-1/2	-1/2	-1/2	
3/2							3/2		
-1/2							-1/2		
									+
1/4								1/4	
-1/4	-1/2	1/4					1/4	-1	1/4
1/4	1/2	-1/4					-1/4	1	-1/4
-1/4 -					-			-1/4	-

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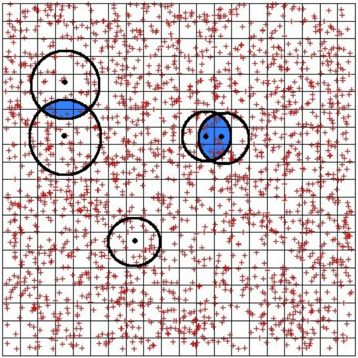

1		-3/2	1/2		-1/4	5/4		-5/4	1/4 -
-3	3					-3/2		3/2	
3/2	-3/2					3/4		-3/4	
									-
3/2	-3/2					3/4		-3/4	
-7/2	9/2	-3/2	1/2		-1/4	-1		1	1/4
3/2	-3/2					3/4		-3/4	
+				+					-

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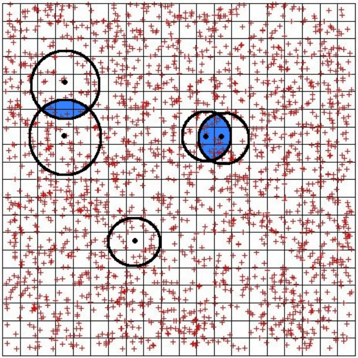
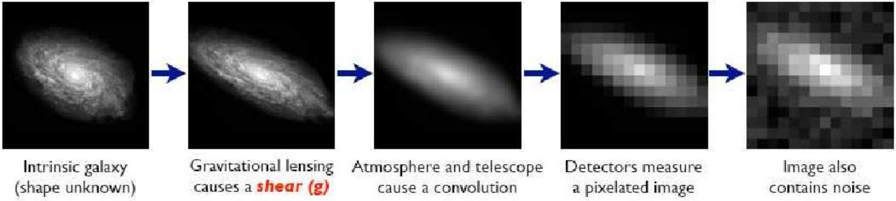


1	-3	3/2				3/2	-7/2	3/2	+
	3	-3/2				-3/2	9/2	-3/2	
-3/2							-3/2		
1/2							1/2		
-1/4							-1/4		+
5/4	-3/2	3/4				3/4	-1	3/4	
-5/4	3/2	-3/4				-3/4	1	-3/4	
1/4 -				-			1/4		-

Measuring galaxy shapes in the real world: A big problem!



(Handbook for the GREAT08 Challenge, Bridle et al. 2008)

PSF correction scheme	Shear measurement method: **Passive** (Measure a number from the data)	Shear measurement method: **Active** (Fit a model to the data)
Subtraction (Subtract a number from the data)	KSB+ (various) Reglens (RM) RRG* K2K* Ellipto*	BJ02 (MJ, MJ2)
Deconvolution (Invert the PSF convolution)	Shapelets (JB)	Lensfit Shapelets (KK) BJ02 (RN) im2shape*

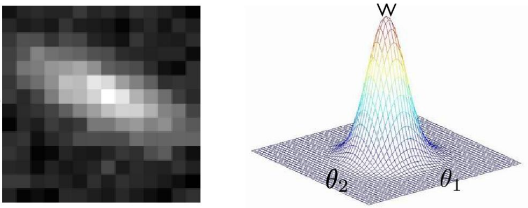
The moments approach

$$M = \int d^2\theta \; I(\theta) W(\theta) \qquad s = 0 \qquad (16)$$

$$D_i = \int d^2\theta \; I(\theta) W(\theta) \theta_i \qquad s = 1 \qquad (17)$$

$$Q_{ij} = \int d^2\theta \; I(\theta) W(\theta) \theta_i \theta_j \qquad s = 0, 1 \qquad (18)$$

$$Q_{ij \ldots n} = \int d^2\theta \; I(\theta) W(\theta) \theta_i \ldots \theta_n \qquad s = \ldots \qquad (19)$$



(Figure stolen from C. Heymans, I'm sure she stole it somewhere else)

KSB and Holics (Kaiser et al. 1995, Okura et al. 2008)

$$\varepsilon = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

But, this does neither account for the weight function, nor for the KSB correction.

Weighting scheme correction

$$\delta e = f^{sh}(Q_{ij}, Q_{ijk}, Q_{ijkl}, W, \frac{\partial W}{\partial \theta^2})$$

PSF correction

$$\delta e = f^{sm}(Q_{ij}, Q_{ijk}, Q_{ijkl}, W, \frac{\partial W}{\partial \theta^2}, \frac{\partial^2 W}{\partial^2 \theta^2})$$

$$\zeta = \frac{Q_{111} + Q_{112} + i(Q_{112} + Q_{22})}{Q_{1111} + 2Q_{1122} + Q_{2222}}$$

$$\mathcal{F} \sim \left\langle \frac{\zeta}{9/4 - 3(TrQ)^2/\xi} \right\rangle$$

$$\delta = \frac{Q_{111} - 3Q_{112} + i(3Q_{112} - Q_{222})}{Q_{1111} + 2Q_{1122} + Q_{2222}}$$

$$\mathcal{G} = \frac{4}{3}\langle\delta\rangle$$

Weighting scheme correction

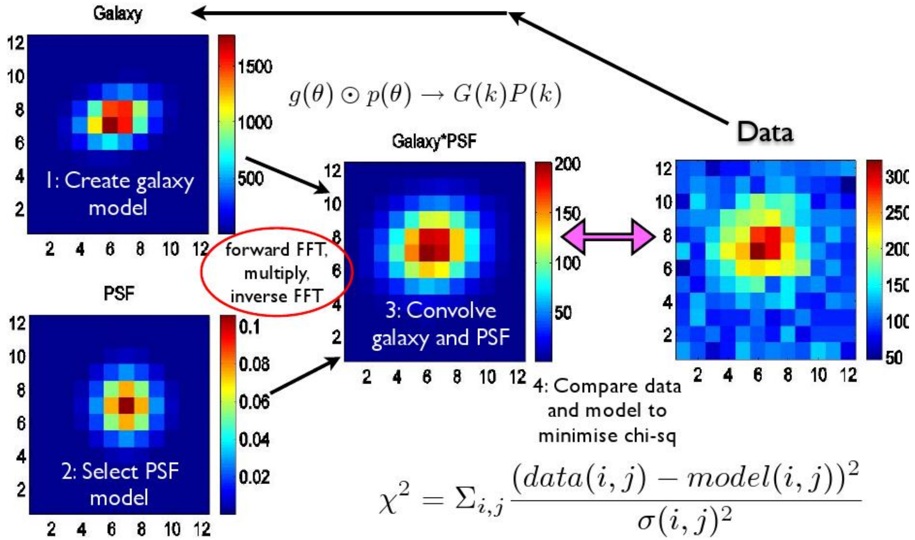
$$\Delta\zeta, \Delta\delta = f(Q_{ijkl}, Q_{ijklmn}, W, \frac{\partial W}{\partial r})$$

PSF correction

$$\Delta\zeta, \Delta\delta = f(M, tr(Q_{ij}), Q_{ijk}, Q_{ijkl}, Q_{ijklmn}, W, \frac{\partial W}{\partial r}, \frac{\partial^2 W}{\partial r^2}, \frac{\partial^3 W}{\partial r^3})$$

But, this does not take into account shear x flexion terms.

Alternatives: Model Fitting (e.g. Kitching et al. 2003)



$$\chi^2 = \Sigma_{i,j} \frac{(data(i,j) - model(i,j))^2}{\sigma(i,j)^2}$$