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*SaWLens 1.4: Non-parametric lensing reconstruction*

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# Overview

- 1 The theory behind the algorithm  
JM et al. (2009) arXiv:0806.1967
- 2 The implementation
- 3 A very recent example: CL0024+1654

## SaWLens 1.4: A practical manual

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November 11, 2009

The manual is available together  
with the source code:

<https://www.ita.uni-heidelberg.de/svn/sawlens>

## *What we want from the method*

- Combine multiple observational constraints into a joint reconstruction method in a consistent way.
- The method should be fully non-parametric.
- It should perform well compared to specialised routines in their regimes.
- The demands regarding CPU-time and resources should stay on an acceptable level.

## *The basic idea: Bartelmann et al. (1996)*

- Divide the reconstruction field into a grid.
- Reconstruction quantity is the lensing potential  $\psi$ .
- Find a relation between the observed quantities and  $\psi$ .
- Define  $\chi^2$ -functions on the data grid.

$$\chi^2(\psi) = \chi_1^2(\psi) + \chi_2^2(\psi) + \chi_3^2(\psi) + \dots$$

- Minimise the overall  $\chi^2$  with respect to the potential.
- Possible constraints: shear, flexion, magnification, critical curve position, multiple image positions, velocity dispersions, X-ray temperature, SZ, ...

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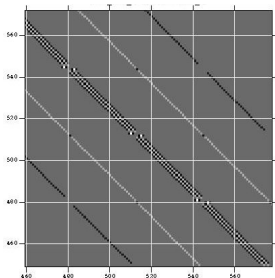
## Numerics on a grid: Finite differencing

We need to relate every observed quantity to the  $\psi$ -values on the grid, e.g. the convergence:

1	-1	1/2				1/2	-1/2	1/2	+
-1/2							-1		
1/2							1/2		
									1/2
+							1/2	-1	-1/2
			1/3	-1/6	1/3				1/2
			-1/6	-2/3	-1/6				
			1/3	-1/6	1/3				
+						+			+

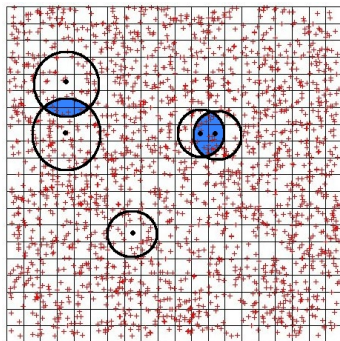
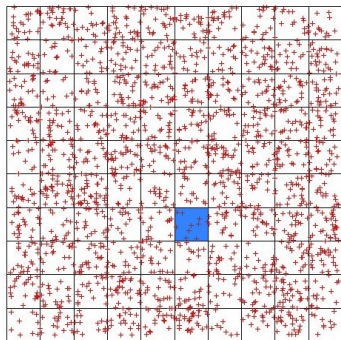
$$\kappa = \frac{1}{2} (\psi_{,11} + \psi_{,22})$$

$$\kappa_i = \mathcal{K}_{ij} \psi_j$$



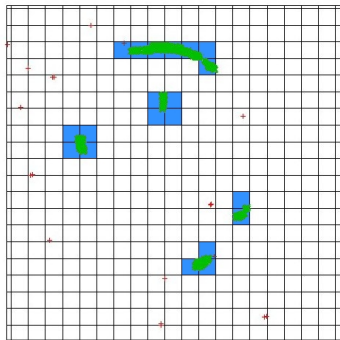
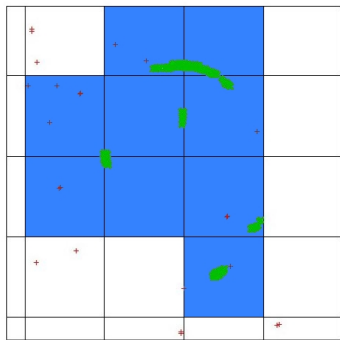
## One example: weak lensing

$$\chi_w^2(\psi) = \sum_{i,j} \left( \varepsilon - \frac{Z(z)\gamma(\psi)}{1 - Z(z)\kappa(\psi)} \right)_i C_{ij}^{-1} \left( \varepsilon - \frac{Z(z)\gamma(\psi)}{1 - Z(z)\kappa(\psi)} \right)_j$$



## Another example: strong lensing

$$\chi_s^2(\psi) = \sum_i \frac{\left( (1 - Z(z)\kappa(\psi))^2 - (Z(z)\gamma(\psi))^2 \right)_i^2}{\sigma_i^2}$$



## Obtaining a non-parametric result

$$\frac{\partial \chi^2(\psi)}{\partial \psi_l} \stackrel{!}{=} 0$$

Using  $\gamma_i = \mathcal{G}_{ik}\psi_k$ ,  $\kappa_i = \mathcal{K}_{ik}\psi_k$   
and  $\frac{\partial}{\partial \psi_l} \mathcal{K}_{ik}\psi_k = \mathcal{K}_{ik}\delta_{kl}$

$$\mathcal{B}_{lk}\psi_k = \mathcal{V}_l$$

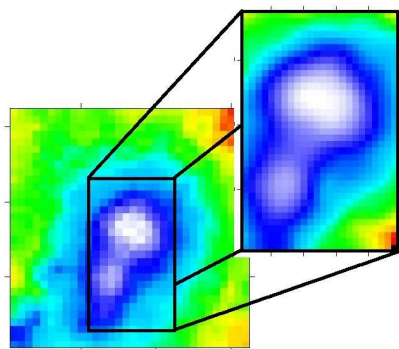
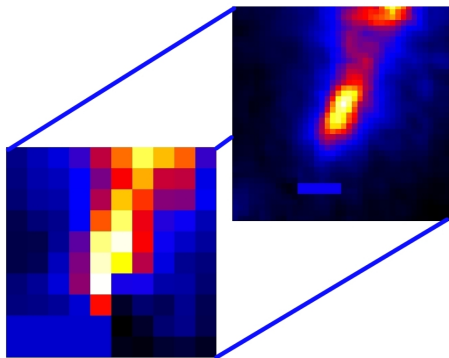
With  $\psi_k$  being the final result as the  
potential on every grid position.

$$\begin{aligned} \mathcal{B}_{lk} = & \sum_{i,j} \mathcal{F}_{ij}^1 Z_i Z_j \\ & \cdot \left[ \varepsilon_i^1 \varepsilon_j^1 \mathcal{K}_{ik} \mathcal{K}_{jl} + \varepsilon_i^1 \varepsilon_j^1 \mathcal{K}_{jk} \mathcal{K}_{il} + \varepsilon_i^1 \mathcal{K}_{ik} \mathcal{G}_{jl}^1 + \varepsilon_j^1 \mathcal{G}_{jk}^1 \mathcal{K}_{il} \right. \\ & \left. + \varepsilon_j^1 \mathcal{K}_{jk} \mathcal{G}_{il}^1 + \varepsilon_j^1 \mathcal{G}_{ik}^1 \mathcal{K}_{jl} + \mathcal{G}_{ik}^1 \mathcal{G}_{jl}^1 + \mathcal{G}_{jk}^1 \mathcal{G}_{il}^1 \right] \\ & + \sum_{i,j} \mathcal{F}_{ij}^2 Z_i Z_j \\ & \cdot \left[ \varepsilon_i^2 \varepsilon_j^2 \mathcal{K}_{ik} \mathcal{K}_{jl} + \varepsilon_i^2 \varepsilon_j^2 \mathcal{K}_{jk} \mathcal{K}_{il} + \varepsilon_i^2 \mathcal{K}_{ik} \mathcal{G}_{jl}^2 + \varepsilon_j^2 \mathcal{G}_{jk}^2 \mathcal{K}_{il} \right. \\ & \left. + \varepsilon_j^2 \mathcal{K}_{jk} \mathcal{G}_{il}^2 + \varepsilon_j^2 \mathcal{G}_{ik}^2 \mathcal{K}_{jl} + \mathcal{G}_{ik}^2 \mathcal{G}_{jl}^2 + \mathcal{G}_{jk}^2 \mathcal{G}_{il}^2 \right] \\ & + \sum_m \frac{4(\det \mathcal{A})_m}{\sigma_m^2} Z_m^2 \\ & \cdot \left[ \mathcal{K}_{mk} \mathcal{K}_{ml} - \mathcal{G}_{mk}^1 \mathcal{G}_{ml}^1 - \mathcal{G}_{mk}^2 \mathcal{G}_{ml}^2 \right] + \sum_n \eta_n \mathcal{K}_{nk} \mathcal{K}_{nl} \quad (\text{A.17}) \end{aligned}$$

and the result vector

$$\begin{aligned} \mathcal{V}_l = & \sum_{i,j} \mathcal{F}_{ij}^1 \left[ \varepsilon_i^1 \varepsilon_j^1 \mathcal{K}_{jl} + \varepsilon_i^1 Z_j \mathcal{G}_{jl}^1 + \varepsilon_i^1 \varepsilon_j^1 Z_i \mathcal{K}_{il} + \varepsilon_j^1 Z_i \mathcal{G}_{il}^1 \right] \\ & + \sum_{i,j} \mathcal{F}_{ij}^2 \left[ \varepsilon_i^2 \varepsilon_j^2 \mathcal{K}_{jl} + \varepsilon_i^2 Z_j \mathcal{G}_{jl}^2 + \varepsilon_i^2 \varepsilon_j^2 Z_i \mathcal{K}_{il} + \varepsilon_j^2 Z_i \mathcal{G}_{il}^2 \right] \\ & + \sum_m \frac{4(\det \mathcal{A})_m}{\sigma_m^2} Z_m \mathcal{K}_{ml} + \sum_n \eta_n k_n^l \mathcal{K}_{nl}, \quad (\text{A.18}) \end{aligned}$$

*A convenient trick: Iterations*



# SaWLens 1.4

## General info

- Written in C++
- ~ 12000 lines
- MPI-implementation
- CUDA on the way
- 3 independent steps

## Requirements

- At least ~ 8 cores
- ~ 1 GB of main memory per core
- negligible disk space per reconstruction: < 6 GB

## Features

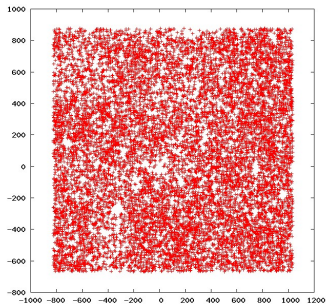
- WL: Ellipticity, Flexion
- SL: Ccurve, Msystems
- Field masking

## Dependencies

- GSL
- MPI
- ATLAS
- LAPACK
- CFITSIO
- CCfits
- LibAstroC++

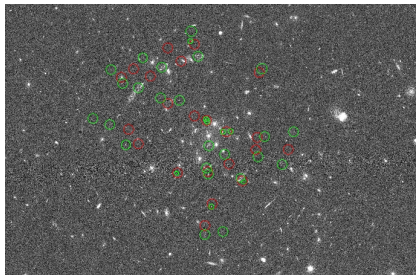


## CL0024: input catalogues



*Ellipticity catalogue*

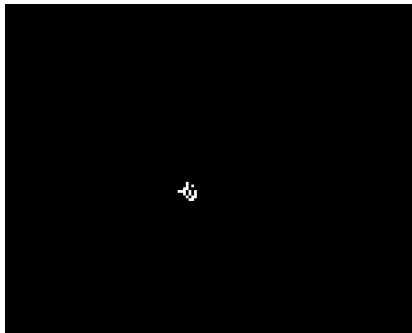
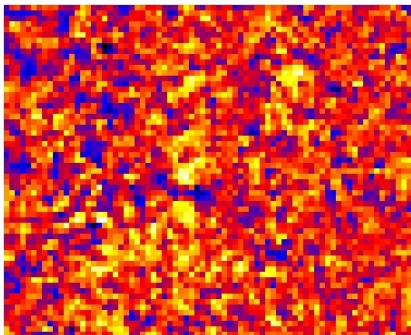
x-pos	y-pos	$\gamma_1$	$\gamma_2$	weight
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*Critical curve catalogue*

x-pos	y-pos	redshift
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# Step 1: Data grids (Runtime ~ 1hr)



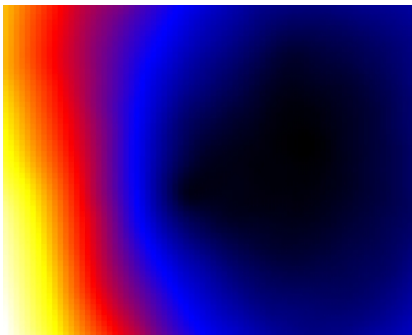
fv: Summary of ellip\_75.fits in /home/jmerten/files/talks/roma\_clustermeeting/71209/

Index	Extension	Type	Dimension	View
<input type="checkbox"/> 0	Primary	Image	75 X 61	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 1	mean_ellip2	Image	75 X 61	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 2	ellip1_sd	Image	75 X 61	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 3	ellip2_sd	Image	75 X 61	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 4	overlap	Image	4575 X 4575	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 5	ellip1_covariance	Image	4575 X 4575	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 6	ellip2_covariance	Image	4575 X 4575	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 7	field_mask	Image	75 X 61	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table

fv: Summary of ccurve\_150.fits in /home/jmerten/science/cluster\_project/CL0024/elli

Index	Extension	Type	Dimension	View
<input type="checkbox"/> 0	Primary	Image	150 X 121	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 1	pixel_relevance	Image	150 X 121	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 2	error	Image	150 X 121	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 3	redshift_info	Image	150 X 121	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table
<input type="checkbox"/> 4	field_mask	Image	150 X 121	<input type="checkbox"/> Header <input type="checkbox"/> Image <input type="checkbox"/> Table

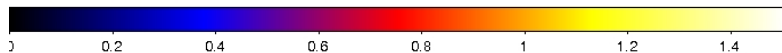
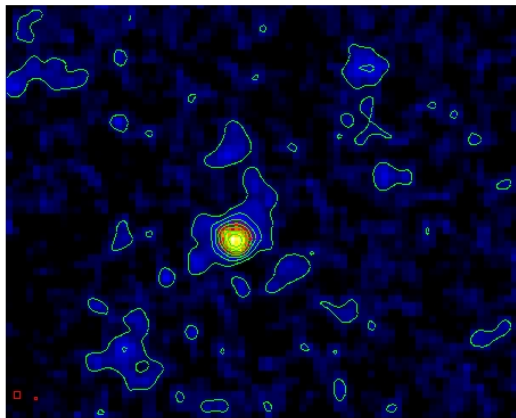
## Step2: Joint reconstruction (Runtime $\sim$ 3hrs)



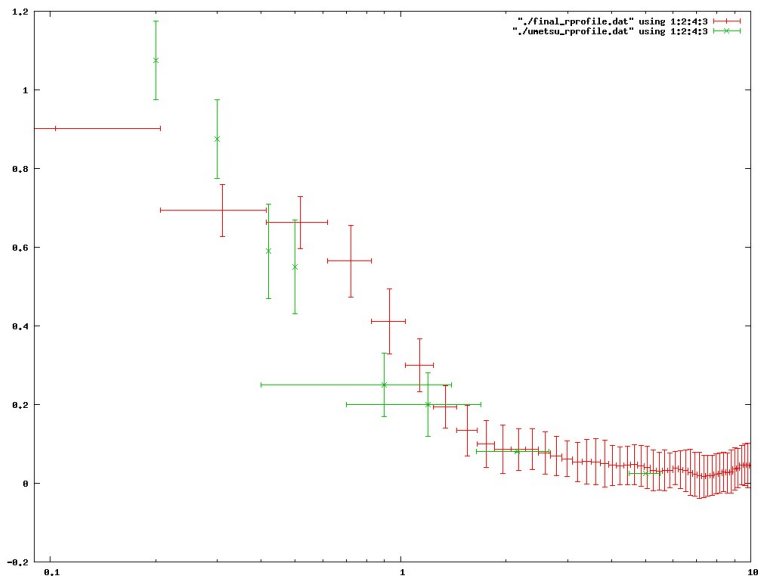
fv: Summary of rec\_newreg\_75.fits in /home/jmerten/science/cluster\_project/CL0

Index	Extension	Type	Dimension	View		
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<input type="checkbox"/> 1	convergence	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 2	shear1	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 3	shear2	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 4	f1	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 5	f2	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 6	g1	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 7	g2	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 8	jacdet	Image	75 X 61	Header	Image	Table
<input type="checkbox"/> 9	field_mask	Image	75 X 61	Header	Image	Table

*Step 3 (optional): Highly resolved core (Runtime ~ 1min)*



### Step 3 (optional): Highly resolved core (Runtime ~ 1min)



# *Work in progress and what is missing*

## 1 Physics

- ▶ Improved usage of strong-lensing constraints
- ▶ Flexion implementation has to be tested carefully
- ▶ Inclusion of cluster-member dynamics
- ▶ Incorporation of parametric strong-lensing models
- ▶ X-Ray constraints

## 2 Numerics

- ▶ CUDA-implementation
- ▶ Documentation
- ▶ User interface