ICM: Sunyaev-Zel'dovich Effects

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Outline

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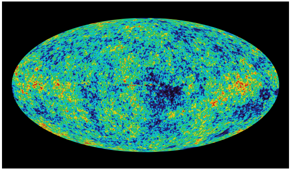
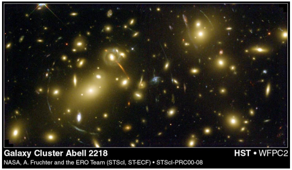
Introduction

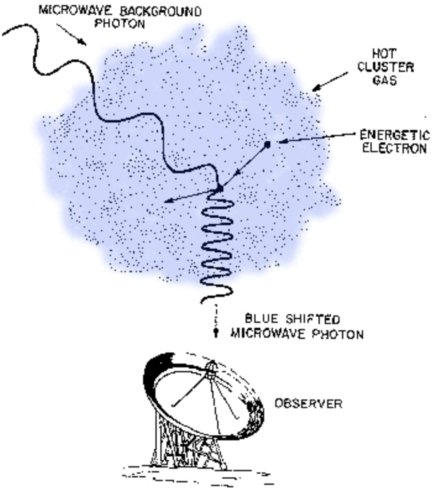
- SZ-Effect connects two important cosmological entities





- The effect was first described by Sunyaev & Zel'dovich (1970,1972)
- Former (hypothetical) work was done by Weymann (1966) and Sunyaev & Zel'dovich (1969). Impact of hot intergalactic gas on the CMB.
- Today the SZ-Effect has become a powerful cosmological tool

SZ-Mechanism: Inverse Compton-Scattering



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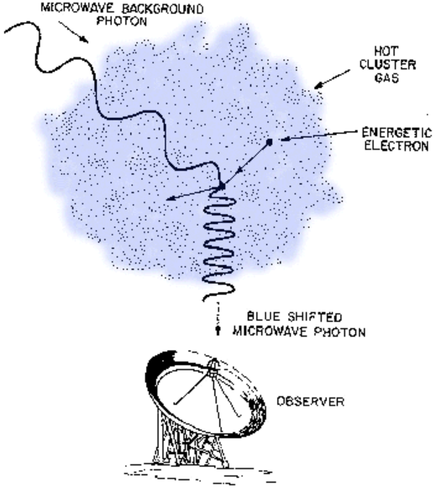
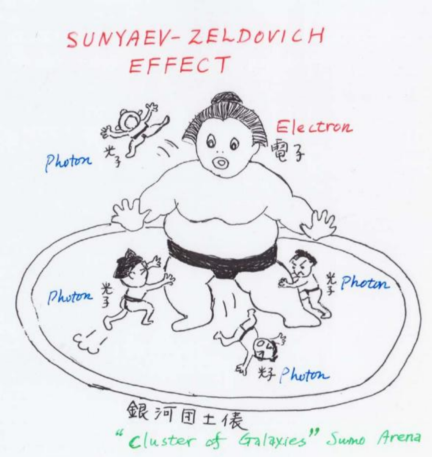
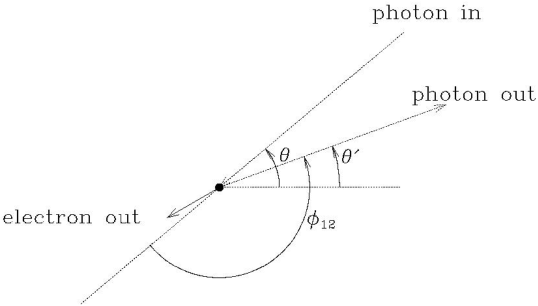




Definitions



$$\mu = \cos\Theta, \quad \mu' = \cos\Theta', \quad \beta = \frac{v_e}{c}, \quad x = \frac{h\nu}{kT_{CMB}}, \quad x_e = \frac{h\nu}{kT_e}$$

$$\tau_e \equiv \int n_e \sigma_T dl, \quad y = \tau_e \boldsymbol{\Theta}, \quad \boldsymbol{\Theta} = \frac{kT_e}{m_e c^2}$$

Inverse -Compton Scattering

- The Thomson-limit ($E_{e_-} \approx E'_{e_-}$) yields:

$$p(\mu)d\mu = \left(2\gamma^4 (1-\beta\mu)^3\right)^{-1} d\mu$$

$$\phi(\mu', \mu)d\mu' = \frac{3}{8}\left(1 + \mu^2\mu'^2 + \frac{1}{2}(1-\mu^2)\left(1-\mu'^2\right)\right) d\mu'.$$

- The frequency of the scattered photon is then:

$$\nu'' = \nu\left(1 + \beta\mu'\right)(1-\beta\mu)^{-1}.$$

- We define for describing the frequency shift:

$$s = \log\left(\nu''/\nu\right)$$

- Probability for a scattering process with shift s by electron with velocity βc:

$$P(s,\beta) = \frac{3}{16\gamma^4\beta}\int_{\mu_1}^{\mu_2}\left(1+\beta\mu'\right)\left(1 + \mu^2\mu'^2 + \frac{1}{2}(1-\mu^2)\left(1-\mu'^2\right)\right)(1-\beta\mu)^{-3} d\mu$$

The Intensity Spectrum

- Electrons are distributed in velocity-space:

$$P_1(s) = \int_{\beta_{lim}}^{1} p_e(\beta) d\beta P(s, \beta)$$

- With the Planckian spectrum of the CMB $I_0(\nu)$ and the spectrum after scattering:

$$I(\nu) = \int_{-\infty}^{\infty} P_1(s) I_0(\nu_0) ds,$$

$$\Rightarrow \Delta I(\nu) = \frac{2h}{c^2} \int P_1(s) ds \left(\frac{v_0^3}{e^{h\nu_0/kT_{\text{CMB}}} - 1} - \frac{v^3}{e^{h\nu/kT_{\text{CMB}}} - 1} \right).$$

- Taking into account multiple scattering processes (to 1st order):

$$\Delta I(\nu) = \frac{2h}{c^2} \tau_e \int P_1(s) ds \left(\frac{v_0^3}{e^{h\nu_0/kT_{\text{CMB}}} - 1} - \frac{v^3}{e^{h\nu/kT_{\text{CMB}}} - 1} \right).$$

- Translated to occupation number representation:

$$\Delta n(x) = \tau_e \int_{-\infty}^{\infty} [n(xe^s) - n(x)] P_1(s) ds$$

Non-Relativistic Limit

- In the non-relativisic limit the occupation-number distribution is given by the Kompaneets equation

$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e}x_e^4\left(\frac{\partial n}{\partial x_e} + n + n^2\right)$$

$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e}x_e^4\frac{\partial n}{\partial x_e} \qquad \text{for small } x_e$$

- Translating this again to intensity representation yields the non-relativistic scattering kernel

$$P_k = \frac{1}{\sqrt{4\pi y}} exp\left(-\frac{(s+3y)^2}{4y}\right)$$

- One finds the simple analytical expression for the thermal SZ-effect:

$$\Rightarrow \Delta I(\nu) = \frac{2(kT_{\text{CMB}})^3}{(hc)^2} y g(x)$$

with the spectral function

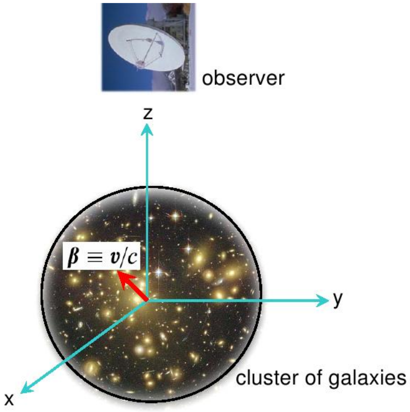
$$g(x) = \frac{x^4 e^x}{(e^x-1)^2}\left[\frac{x(e^x+1)}{e^x-1} - 4\right] \qquad \text{(crossover frequency of } \sim 217 \text{ GHz).}$$

Kinematic SZ-Effect

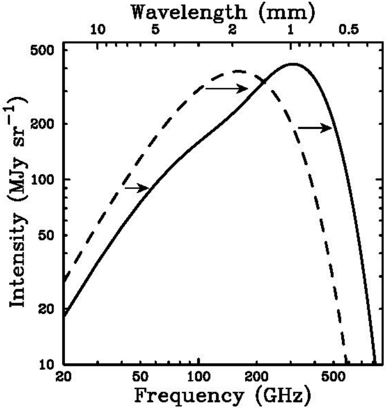
- The kinematic SZ-Effect also takes into account the cluster velocity wrt the CMB rest frame.
- In the non-relativistic limit the kinematic SZE is a simple temperature-amplitude distortion:

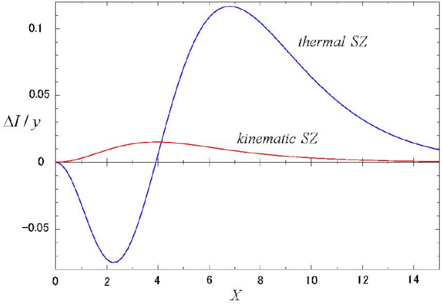
$$\frac{\Delta T_{CMB}}{T_{CMB}} = -\tau_e \beta_{\mathrm{pec}}.$$

- Relativistic corrections take into account the Lorentz boost of the electrons and contain higher order terms in β_{pec}
- For detecting the kinematic effect the crossover frequency is



Thermal and Kinematic SZE





Note:

Left panel: Effect of a cluster with 1000 times the usual mass.

Relativistic Corrections

- Relativistic corrections become important at the Wien side of the spectrum.
  Of special interest are changes to the crossover frequency (near the transition to Wien spectrum).
- Also important are the rel. corrections in clusters with high-temperature ICM.
- kSZE becomes frequency dependent
- Different approaches are possible:
  1. Relativistic Maxwellian distribution of the electrons (Rephaeli 1995).
  2. Generalised Kompaneets equation (Nozawa et al. 1998).
  3. LT of the photon transfer equation (Sazonov & Sunyaev 1998).
- Shimon & Rephaeli (2003) showed that all approaches are equivalent and derived an "elegant" analytic formula including multiple scattering and kinematic effects.
- Going to higher order in τ (Itoh & Nozawa 2004)

As can be easily seen...

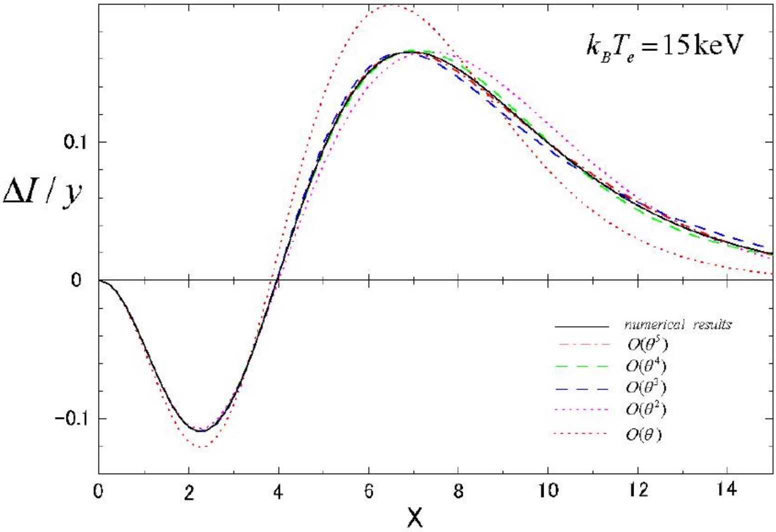
$$\Delta n_{\mathrm{t}} = \tau(\Theta F_1 + \Theta^2 F_2 + \Theta^3 F_3 + \Theta^4 F_4 + \Theta^5 F_5 + \Theta^6 F_6 + \Theta^7 F_7 + \Theta^8 F_8) + \tau^2(\Theta^2 F_9 + \Theta^3 F_{10} + \Theta^4 F_{11} + \Theta^5 F_{12}).$$

$$\begin{aligned}\Delta n/\tau = & -\beta_{\mathrm{c}}\mu_{\mathrm{c}}\left(\frac{1}{2}A_1 + \Theta F_{13} + \Theta^2 F_{14} + \Theta^3 F_{15} + \Theta^4 F_{16}\right) \\ & + \beta_{\mathrm{c}}^2 P_2(\mu_{\mathrm{c}})(F_{17} + \Theta F_{18} + \Theta^2 F_{19} + \Theta^3 F_{20} + \Theta^4 F_{21}) \\ & + \beta_{\mathrm{c}}^2\left(\frac{1}{3}F_1 + \Theta F_{22} + \Theta^2 F_{23} + \Theta^3 F_{24} + \Theta^4 F_{25}\right),\end{aligned}$$

$$\begin{aligned}F_1 &= 2A_1 + \frac{1}{4}A_2, \\ F_2 &= 5A_1 + \frac{47}{8}A_2 + \frac{21}{20}A_3 + \frac{7}{160}A_4, \\ F_3 &= \frac{15}{4}A_1 + \frac{1023}{32}A_2 + \frac{217}{10}A_3 + \frac{329}{80}A_4 + \frac{11}{40}A_5 + \frac{11}{1920}A_6, \\ F_4 &= -\frac{15}{4}A_1 + \frac{2505}{32}A_2 + \frac{3549}{20}A_3 + \frac{14253}{160}A_4 + \frac{9297}{560}A_5 + \frac{12059}{8960}A_6 + \frac{1}{21}A_7 + \frac{1}{1680}A_8, \\ F_5 &= \frac{135}{64}A_1 + \frac{30375}{512}A_2 + \frac{62391}{80}A_3 + \frac{614727}{640}A_4 + \frac{124389}{320}A_5 + \frac{355703}{5120}A_6 + \frac{2071}{336}A_7 + \frac{1879}{6720}A_8 \\ &\quad + \frac{11}{1792}A_9 + \frac{11}{215040}A_{10},\end{aligned}$$

$$x_0 = 3.830016(1 + 1.120594\Theta + 2.078258\Theta^2 - 80.748072\Theta^3 + 1548.250996\Theta^4 + 0.800424\tau\Theta + 1.183073\tau\Theta^2).$$

Importance of rel. Corrections (Itoh et al. 2003)



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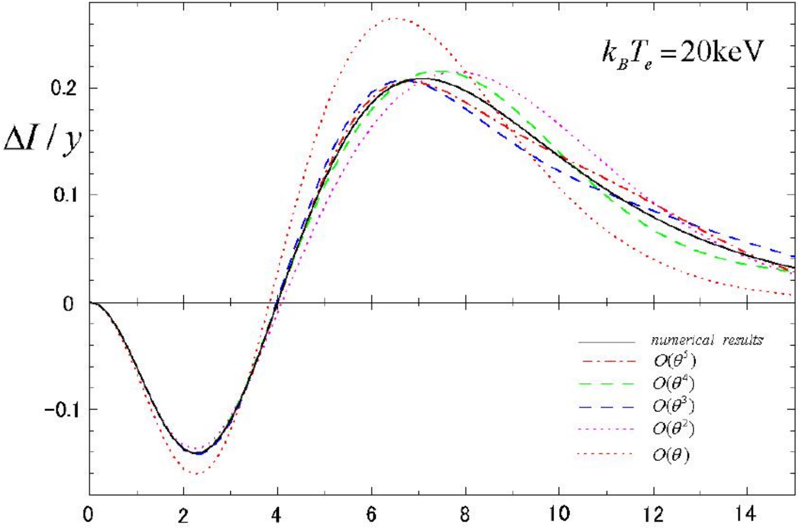

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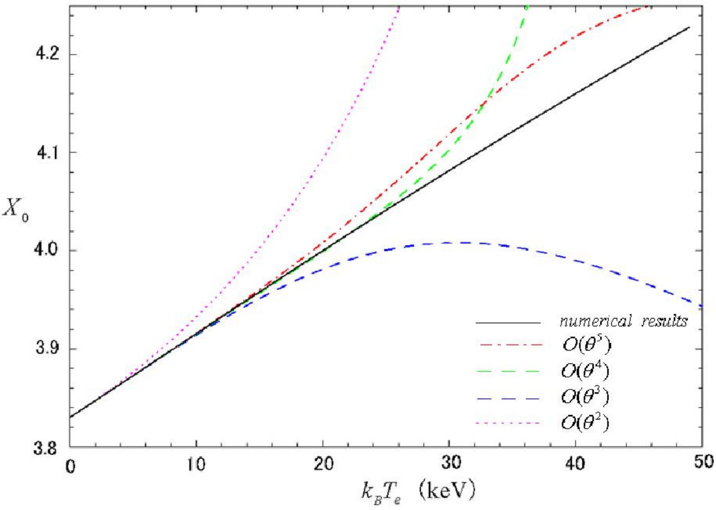
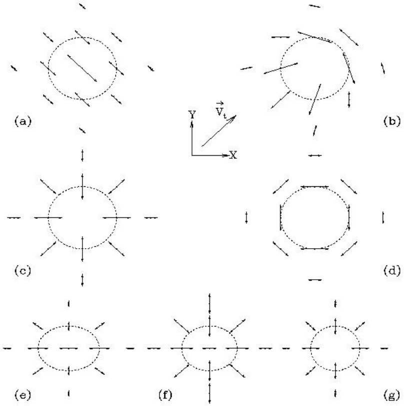


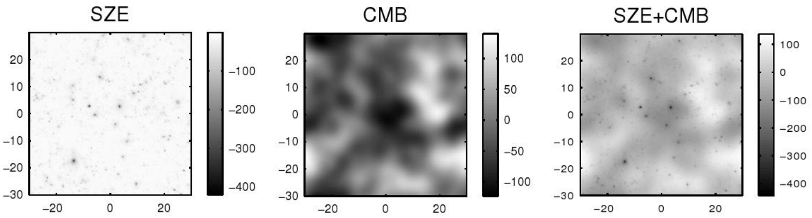
Polarisation

- Incident unpolarized radiaton will become linearly polarized if it has a finite quadrupole moment
- Polarisation effects are much weaker than effects on the total intensity and will not be measureable in the near future



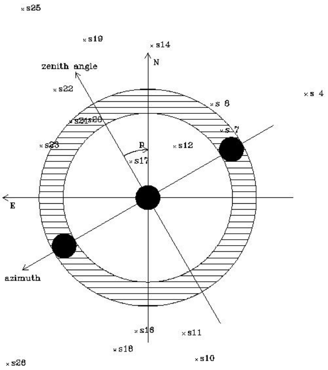
Systematics in Observations

- The SZE signal is relatively weak: $\sim 100\mu K$ at a given frequency
- The measurement is completely dominated by systematics
  $\Rightarrow$ All SZE measurements are differential
- The most important sources of systematical errors are given by:
  1. Emission of the Earth's atmosphere $\sim 3K$
  2. Ground noise
  3. Radio sources (including grav. lensing)
  4. Detector calibration
  5. CMB-anisotropies (makes measuring kSZE difficult, one needs to use rel. corrections)



Single Dish Observations

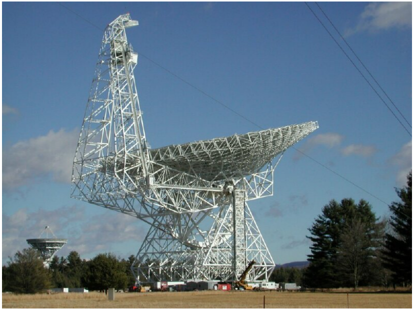
- Pioneering work by Birkinshaw with the OVRO 40m telescope (1978, 1991)
- To control atmoshperic and ground noise:
  1. Position-switching schemes (dish and secondary mirror)
  2. Beam-switching schemes
- Use of bolometric detectors
  1. High sensitivity
  2. Array operation for element differencing
  3. Multiple-band observation
- External radio-source monitoring



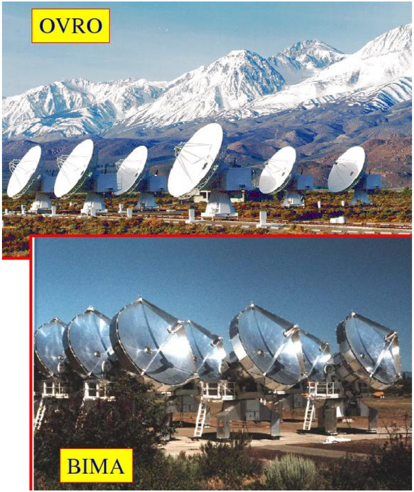
Interferometric Methods

- While single-dish telescopes allow large sky surveys, interferometers allow high resolution SZE maps due to largely improved systematics.
- Radio arrays can be adjusted to observe particular scales by changing their baseline
  Due to interferometry they are only sensitive to spatial frequencies near B/λ.
- Because of this SZEs can be masked by going to large baselines and radio sources can be identified separately.
- Arrays achieve extremely low systematics
  1. Correlations between pairs of telecopes are calculated $n(n-1)/2$ measurements
  2. Interferometers do not respond to constant background levels (const. atmosphere, ground noise, CMB background)
  3. Gradients in the atmosphere become uncorrelated with baselines longer than a few meters

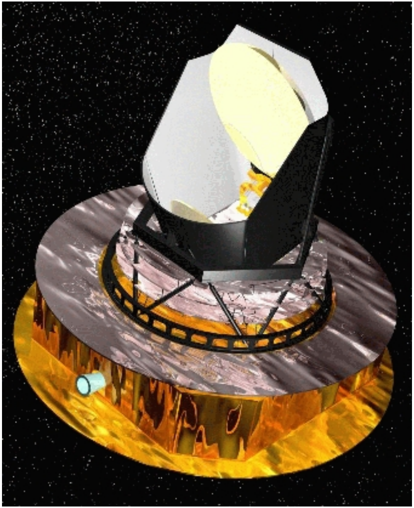
The Big Ones



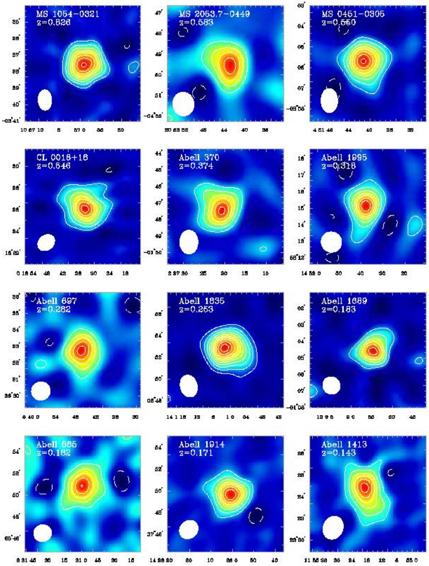
The Useful Ones

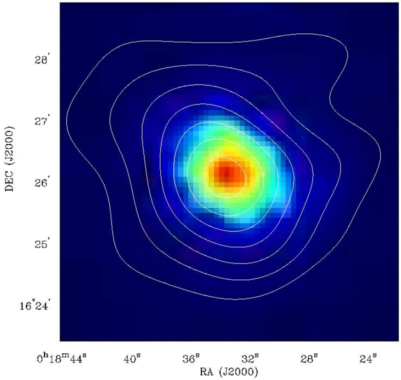


The Special One

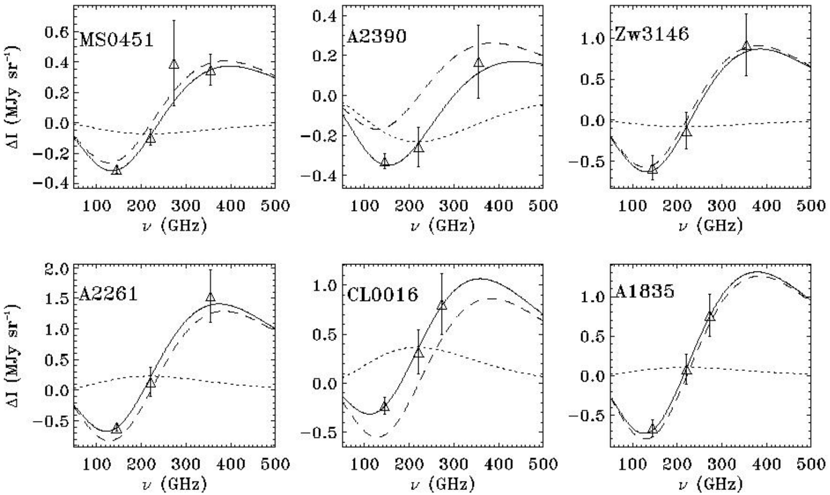


Results (Carlstrom 1999)





Results (Benson 2003)



Results (Benson 2003)

TABLE 6
SUMMARY OF RESULTS

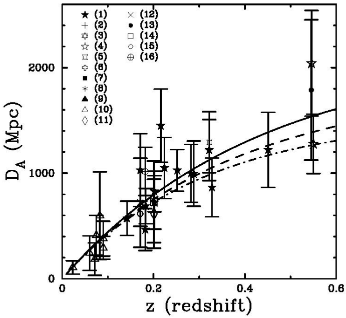
Cluster	Date	ΔR.A. (arcsec)	$y_0 \times 10^4$	v_{pec} (km s^{-1})
A2261	1999 Mar	$6.4^{+18.6}_{-19.5}$	$7.41^{+1.95}_{-1.98}$	-1575^{+1500}_{-975}
A2390	2000 Nov	$-4.8^{+18.1}_{-19.1}$	$1.72^{+1.01}_{-0.76}$	$+1900^{+6225}_{-2650}$
Zw 3146	2000 Nov	$11.4^{+30.9}_{-30.9}$	$3.62^{+1.83}_{-2.52}$	-400^{+3700}_{-1925}
A1835	1996 Apr	$28.0^{+16.0}_{-15.0}$	$7.66^{+1.64}_{-1.66}$	-175^{+1675}_{-1275}
Cl 0016	1996 Nov	$6.2^{+34.5}_{-37.4}$	$3.27^{+1.45}_{-2.86}$	-4100^{+2650}_{-1625}
MS 0451	1996 Nov	$-15.5^{+26.0}_{-24.0}$	$3.20^{+1.61}_{-1.61}$	$+175^{+5750}_{-2625}$
	1997 Nov	$12.0^{+10.0}_{-11.0}$	$2.07^{+0.70}_{-0.72}$	$+1775^{+3900}_{-2150}$
	2000 Nov	$-21.5^{+21.0}_{-19.0}$	$3.17^{+0.86}_{-0.88}$	-300^{+1950}_{-1275}
Combined fit for MS 0451			$2.84^{+0.52}_{-0.52}$	$+800^{+1525}_{-1125}$

SZE and Cosmology: The Hubble Constant

- $\Delta T_{\mathrm{SZE}} \propto \int dl \;\; n_e T_e$
- $S_X \propto \int dl \;\; n_e^2 \Lambda_{\mathrm{eH}}$
  with X-Ray cooling function Λ_{eH}
- By eliminating electron density

  $$\Rightarrow \quad D_A \propto \frac{(\Delta T_0)^2 \Lambda_{\mathrm{eH0}}}{S_{X0} T_{e0}^2} \frac{1}{\Theta_c}$$

- Θ_c is the characteristic scale of the cluster-density model
- What we can measure is the characteristic scale of the cluster in the sky Θ_c^{sky}
- By assuming $\langle \Theta_c / \Theta_c^{\mathrm{sky}} \rangle = 1$ and $\sqrt{\langle n_e^2 \rangle} = \langle n_e \rangle$ we can measure the angular diameter distance of the cluster.



SZE and Cosmology: Gas fraction and Ω_M

- The gas mass of a cluster can be measured directly through tSZ observations if the electron temperature is known.
- The total gravitating mass is obtained through lensing or by the assumption of hydrostatic equilibrium.
- Determining the gas fraction for a large region of the cluster provides a fair estimate for the universal baryon mass fraction

  $$f_B \equiv \frac{\Omega_B}{\Omega_M}$$

- Ω_B can be obtained by Big-Bang nucleosythesis, D/H measurements in Lyman α clouds or CMB primary anisotropies analysis.
- Therefor an estimate for Ω_M with the help of the SZE is also possible.

SZE and Cosmology: Cluster Abundance

- Galaxy Clusters produce a significant SZ signal.
- Cluster detection by SZE is not limited at high redhsift as e.g. X-Ray surveys.
- Thousands of clusters in a wide redshift range should be detected by large SZE surveys (Planck).
- SZE is a powerful tool to observe the high redshift universe.
- source-count against redshift plots are a poweful measure of cosmological parameters