Triple unification of inflation, dark matter and dark energy

May 9, 2008

- Leonard Susskind, The Anthropic Landscape of String Theory (2003)
- A. Liddle, A. Ureña-López, Inflation, dark matter and dark energy in the string landscape (2006)
- A. Liddle, C. Pahud, A. Ureña-López, Triple unification of inflation, dark matter and dark energy using a single field (2008)

Overview

1. Classical and modern cosmology
2. Dark energy, dark matter and inflation
3. Combining inflation and dark energy
4. Combining inflation and dark matter

Classical Cosmology

General relativity

Expanding universe

Big Bang nucleosynthesis

CMB

Modern Cosmology

We try to understand

Modern Cosmology

We try to understand

Structure formation

Modern Cosmology

We try to understand

Structure formation

Dark matter

Modern Cosmology

We try to understand

Structure formation

Dark matter

Dark energy

Modern Cosmology

We try to understand

Structure formation

Dark matter

Dark energy

Inflation

Modern Cosmology

We try to combine

Dark matter

Dark energy

Inflation

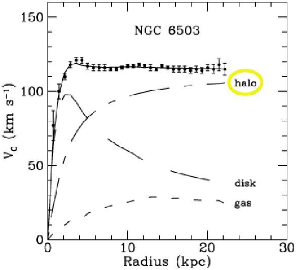
while retaining

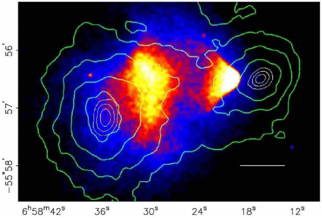
Hot Big Bang standard ΛCDM

Dark Matter: Why do we need it?

- Rotation curves of spiral galaxies
- Lensing results, especially "Bullet Cluster" (Clowe et al., 2006)
- Determining Ω_m and Ω_b or ratio
  - CMB
  - X-Ray
  - SZ-Effect
  - Nucleosynthesis

  $\Rightarrow \Omega_m \simeq 0.25 \quad \Omega_b \simeq 0.05$
- Machos cannot account for missing mass (microlensing)
- Also neutrinos are ruled out (mass to low)



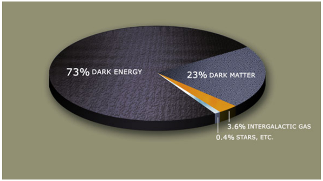


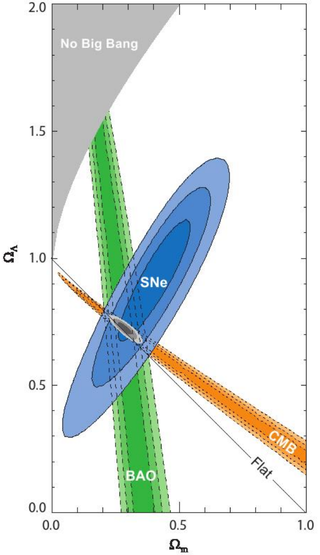
Dark Matter: What do we know about it?

- We can observe it
- But we have no idea what it really consists of
- Different categories
  - Hot dark matter; ultrarelativistic particles, collides with structure formation on small scales
  - Cold dark matter; non-relativistic particles, no detected candidate
  - Warm dark matter; relativistic, mixture of both
- Dark matter candidates based on extensions to the standard model
  - WIMPS
  - sterile neutrinos
  - axion
- Many experiments are set up to detect dark matter or directly produce it

Dark Energy: Why do we need it?

- CMB power spectrum would look different
- CMB observations favor a flat universe
  $\Rightarrow$ There is something missing
- Supernova observations (Supernova Cosmology Project 2008)
- Structure formation





Dark Energy: What do we know about it?

- Unfortunately even less then about dark matter
- It might be just a cosmological constant Λ

$$G_{\mu\nu} = 8\pi G \left[T_{\mu\nu} + T_{\mu\nu}^{\text{vac}} \right], \quad T_{\mu\nu}^{\text{vac}} = -\frac{\Lambda}{8\pi G} g_{\mu\nu}$$

$\Rightarrow$ fine tuning problem (later more about that)

- Implementation of Dark Energy with a dynamical scalar field is also possible (quintessence)

$$L_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V^{'}(\varphi) = 0$$

$\Rightarrow$ The potential remains arbitrary and also coupling to e.g. DM is possible.

Inflation: Why do we need it?

- Flatness Problem: Why is the universe so old?

$$\Omega - 1 = \frac{K}{a^2 H^2}$$

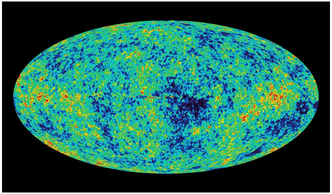
$$|1-\Omega| \propto t^{2/3} \quad \text{flat, rad-dom.}$$

$$|1-\Omega| \propto t \quad \text{flat, matter-dom.}$$

$$\Rightarrow \quad |\Omega(t_{nuc} - 1)| \lesssim 10^{-16}$$

$\Rightarrow$ extreme fine tuning necessary

- Horizon problem: Why is the universe so smooth?
  - Temperature of the CMB
  - Homogeneity necessary for nucleosynthesis



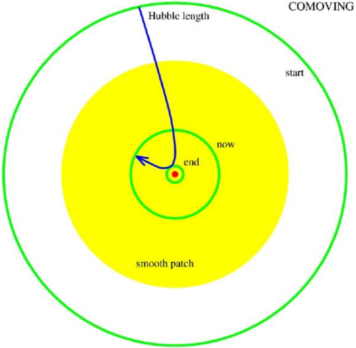
Inflation: How scalar fields can help

- What inflation means:

$$\text{Inflation} \Longleftrightarrow \ddot{a} > 0$$

$$\text{Inflation} \Longleftrightarrow \frac{d}{dt}\left(\frac{1}{aH}\right) < 0$$

$$\text{Inflation} \Longleftrightarrow \rho + 3P < 0$$



- For a scalar φ field:

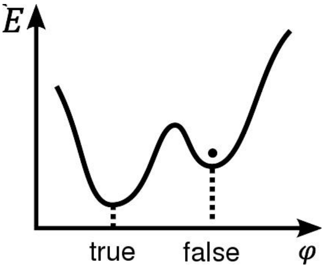
$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$\Rightarrow \dot{\varphi}^2 < V(\varphi) \quad \text{for inflation to occur}$$

Described by slow-roll conditions:

$$\epsilon(\varphi) \ll 1, \ |\eta(\varphi)| \ll 1; \quad \epsilon(\varphi) = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta(\varphi) = M_{pl}^2 \frac{V''}{V}$$

Inflation: "side effects"



- Vacuum fluctuations of the inflaton deliver a fully consistent explanation for the primordial density perturbations.
- In the future it might be possible to fully constrain the inflaton potential.
- One can give many talks on that topic.

Unification of dark energy and inflation

- Obviously scalar fields are the key for the unification of inflation, dark energy and dark matter
- Let us assume a scalar field with the following potential:

$$V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2$$

  where V_0 is an arbitrary constant.

- This gives us the following Lagrangian:

$$L_\varphi = \frac{1}{2}g_{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 - V_0,$$

- We want to interpret the cosmological constant as vacuum fluctuations of a scalar field.

The fine tuning problem

- After canonical quantization of the field, the Hamiltonian reads:

$$H_\varphi = \int \tilde{d}k \;\, \omega \;\, a^\dagger(\mathbf{k}) a(\mathbf{k}) + (\mathcal{E}_0 - V_0) V.$$

with the total zero-point energy per unit volume:

$$\mathcal{E}_0 = \frac{1}{2}(2\pi)^{-3} \int d^3k \;\, \omega = \frac{1}{2}(2\pi)^{-3} \int d^3k \;\, \left(\mathbf{k}^2 + m^2\right)^{1/2}$$

- The integral above is divergent, so one has to introduce an ultraviolet cutoff E_c and we find:

$$\mathcal{E}_0 \propto E_c^4 \;\; \Rightarrow \;\; \Lambda_{\mathrm{vac}} = \frac{\mathcal{E}_0}{8\pi G}; \quad E_c = M_{pl} \;\; \Rightarrow \;\; \Lambda_{\mathrm{vac}} \sim 10^{38} \mathrm{GeV}^2$$

- Observations: $\Lambda_{\mathrm{obs}} = \Lambda_{\mathrm{vac}} + \Lambda_0 \simeq 10^{-83} \mathrm{GeV}^2$ (not zero!)
- We need a 120 orders of magnitude accurate renormalisation Λ_0!

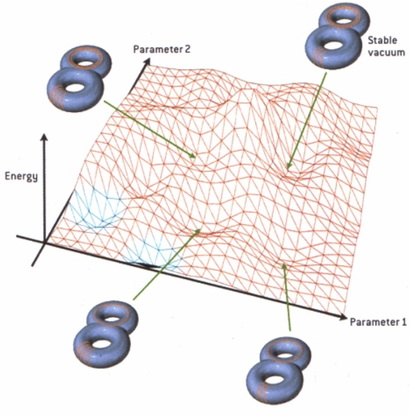
String theory

- String theory has most likely no unique solution.
- In the beginning there were five (six) different theories, which turned out to be a continuum of solutions to one master theory.
- The space of this solutions is called "the moduli space of supersymmetric vacua".
- Moving in this space means varying dynamical moduli.
- While moving in ordinary space this moduli behave like massless scalar fields.
- All these solutions are supersymmetric, include massless scalar fields with vanishing vacuum energy

  ⇒ They do not describe our world.

The landscape

- There have to be other solutions, off the coast of supermoduli-space.
- The space of all these string theory vacua is called the landscape.
- Supermoduli-space is just the supersymmetric part with vanishing potential.
- With respect to compactification schemes from eleven to four dimensions it is very likely to find an astronomical number of vacua with non-vanishing cosmological constant.



"The Landscape" (Picture from *Scientific American*)

Probability

- Now the probability for an island in the landscape for our observed universe is extremely high.
  Weinberg (1987), Bousso and Polchinski (2000), Kachru, Kallosh, Linde and Trivedi (2003)
- Totally footed on anthropic principle.
- The number of possible vacua should be measured in googols or googolplexes.

Interlude

$$1\ \text{googol} = 10^{100}, \quad 1\ \text{googolplex} = 10^{\text{googol}}$$

- Name invented by a nine-year-old boy and is often misspelled as google.
- $70! \sim 1.2$ googol, $\quad 1$ googol $<$ Shannon number
- Google™ calls it headquarters Googleplex
- D. Adams, The Hitchhiker's Guide To The Galaxy:
  "Googleplex Star Thinker in the Seventh Galaxy of Light and Ingenuity"
- A googolplex is by far not the largest number ever used in a mathematical proof!

Unification of dark matter and inflation

- Again we assume a harmonic inflation potential

$$V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2$$

- Equation of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$
$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$$

- Obviously this is an harmonic oscillator.
- Friction term is given by the expanding universe.

CDM by coherent inflaton oscillations

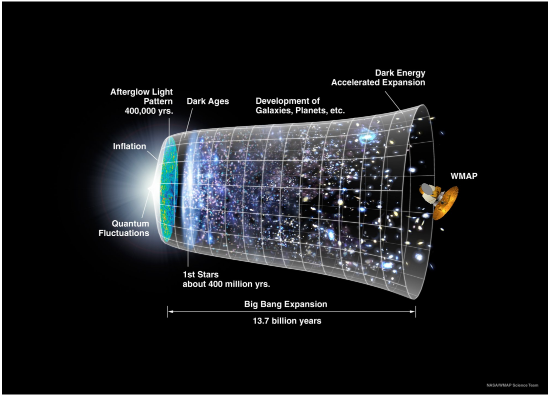
- Initially, while $m \ll H$ is frozen by the friction of the expanding universe and drives inflation. (Slow-roll condition).
- Once H falls below m the inflaton field starts oscillating.
- Remember:

$$\begin{aligned} \rho_\varphi &= \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \\ \bar{\rho}_\varphi &= \left\langle \dot{\varphi}^2 \right\rangle_t \\ \Rightarrow \; \dot{\bar{\rho}}_\varphi + 3H\bar{\rho}_\varphi &= 0 \end{aligned}$$

- The time-averaged energy-density of the oscillating inflaton field behaves exactly like CDM

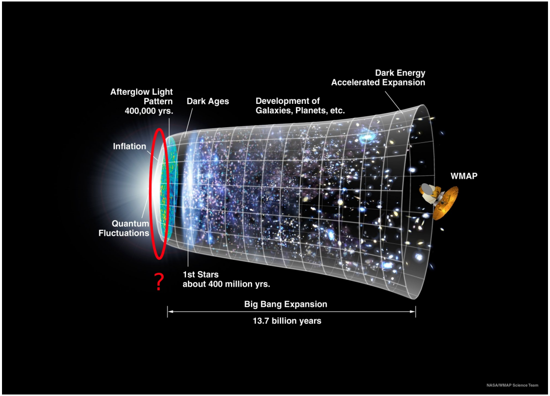
$$\bar{\rho}_\varphi \propto a^{-3}$$

Hot Big Bang ΛCDM



By now, we are just living on a very cold island with non-vanishing cosmological constant and an oscillating inflaton field, which behaves like CDM

Hot Big Bang ΛCDM



What happened here?

Constraints on inflaton oscillations I

1. The observed dark matter mass per photon

$$\xi_{\mathrm{dm}} \equiv \frac{\rho_{\varphi}}{n_{\gamma}}$$

$$\Rightarrow \left(\frac{m}{m_{\mathrm{pl}}}\right)^{1/2} \frac{\varphi_*^2}{m_{\mathrm{pl}}^2} \simeq 4 \ x \ 10^{-29} \quad \text{(WMAP3)}$$

   where φ_* denotes the field amplitude at $m = H$

2. The amplitude of primordial scalar perturbations

$$\frac{m}{m_{\mathrm{pl}}} \simeq 10^{-6} \quad \text{(WMAP3)}$$

3. If $k > a(t)m$ these modes would be out of phase with the inflaton field and as consequence appear suppressed

$$\Rightarrow \quad m > 10^{-23}\mathrm{eV}$$

Constraints on inflaton oscillations II

- Combining the first to constraints yields:

$$\varphi_* \simeq 10^{-13} m_{\mathrm{pl}}$$

- At the end of inflation φ is still of order m_{pl}, because it was almost static.
- As a result we need a drastic reduction of the energy density of the inflaton field between the end of inflation and t_*
- In addition we also need a surviving relic abundance

$$\frac{\rho_{\mathrm{end}}(\varphi)}{\rho_*(\varphi)} \sim 10^{26}$$

- The question is which mechanism can achieve this reduction.

Post-inflation processes

1. Reheating
   - Describes the decay of the inflaton field into a single particle:

$$\dot{\rho}_\varphi + 3H\dot{\varphi}^2 = -\Gamma_\varphi \dot{\varphi}^2$$

   - Proceeds once $\Gamma_\varphi \ll H$.
   - Problem: leads to complete inflaton decay.
2. Preheating
   - Inflaton is coupled to another scalar field:

$$L_{\text{int}} = g^2 \varphi^2 \chi^2$$

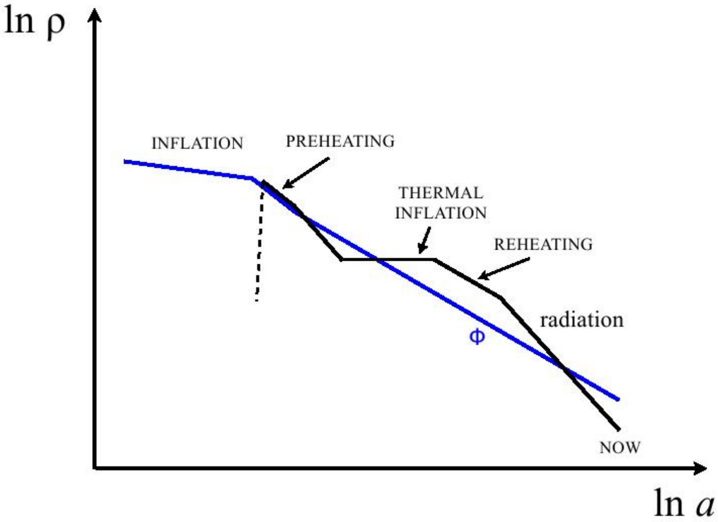
   - Advantage: Preheating stops if $|\varphi| < m/g$.
   - Problem: It stops to early so we would see a radiation-dominated era which is much too short.

$$\frac{\rho_{\text{rad}}}{\rho_\varphi} \sim 1$$

Thermal inflation

- Thermal inflation takes place in addition to inflation caused by a different scalar field.
- Assume a light scalar field ($m \sim 100\text{GeV}$) in thermal equilibrium and call it flaton.
- The field should have a false vacuum at $\phi = 0$.
- Inflationary phase takes place if $T^4 \lesssim V_0$
- Thermal inflation ends once the temperature drops below the flaton mass by spontaneous symmetry breaking and escapes the false vacuum (Higgs mechanism).
- Thermal inflation was proposed earlier in another context.

A possible scenario



Summary

- Dark matter, dark energy and inflation are the crucial ingredients of modern cosmology.
- Unfortunately they are all not very well understood.
- A unification of them is desirable and might be possible through scalar fields, while conserving ΛCDM.
- One should believe in string theory.
- And if so, also in the anthropic principle.
- Dark matter might be just the oscillations of the inflaton.
- Working scenarios are viable but appear very constructed, at least to me.