4.4 MHD shocks

4.4.1 Single fluid shocks

- So far, we have considered the properties of shocks in the case where there is no magnetic field in the fluid. We now explore what happens when we relax this assumption. To begin with, we assume that ideal MHD applies everywhere but within the shock-front itself, and that the velocities of the ions and the neutrals are the same (i.e. we can treat them as a single fluid).

- We can use the MHD version of the fluid equations to derive jump conditions relating conditions in the pre-shock gas to those in the post-shock gas. In the absence of a magnetic field, the conditions that we arrive at simply require that the flux of mass, momentum and energy is conserved across the shock. In the MHD case, however, we also need to consider what happens to the magnetic field, and so it is useful to look at the full derivation.

- We start by noting that the continuity equation has the same form regardless of whether or not a magnetic field is present. Therefore, the associated shock jump condition in the MHD case is the same as in the hydrodynamical case:

$$\rho_1 v_{1,\perp} = \rho_2 v_{2,\perp}. \tag{422}$$

- The momentum equation, in the form that we derived it in lecture 2, is given by

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \frac{1}{4\pi} (\vec{B} \cdot \nabla) \vec{B} - \frac{1}{8\pi} \nabla \left(|\vec{B}|^2 \right). \tag{423}$$

  In component form, this can be written as

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j \partial_j v_i \right) = -\partial_i p + \frac{1}{4\pi} (B_j \partial_j B_i) - \frac{1}{8\pi} (\partial_i |\vec{B}|^2). \tag{424}$$

- The left-hand side of this equation can be rewritten as

$$\begin{aligned} \rho \left(\frac{\partial v_i}{\partial t} + \rho v_j \partial_j v_i \right) &= \frac{\partial}{\partial t} (\rho v_i) - v_i \frac{\partial \rho}{\partial t} + \partial_j \rho v_j v_i - v_i \partial_j \rho v_j, & (425) \\ &= \frac{\partial}{\partial t} (\rho v_i) + \partial_j \rho v_j v_i - v_i \left(\frac{\partial \rho}{\partial t} + \partial_j \rho v_j \right) & (426) \end{aligned}$$

  The final term on the right-hand side of this expression is simply v_i times the continuity equation, and hence is zero. Therefore, the momentum equation becomes

$$\frac{\partial}{\partial t} (\rho v_i) + \partial_j \rho v_j v_i = -\partial_i p + \frac{1}{4\pi} (B_j \partial_j B_i) - \frac{1}{8\pi} (\partial_i |\vec{B}|^2). \tag{427}$$

- The magnetic pressure term on the right-hand side can be written as

$$\frac{1}{4\pi}\left(B_j \partial_j B_i\right) = \frac{1}{4\pi}\left(\partial_j B_j B_i - B_i \partial_j B_j\right) \tag{428}$$

Since the magnetic field satisfies $\nabla \cdot \vec{B} = 0$, we have $\partial_j B_j = 0$, and hence can write the momentum equation as

$$\frac{\partial}{\partial t}\left(\rho v_i\right) + \partial_j \rho v_j v_i = -\partial_i p + \frac{1}{4\pi}(\partial_j B_j B_i) - \frac{1}{8\pi}(\partial_i |\vec{B}|^2). \tag{429}$$

Finally, collecting terms together and using the identity $\partial_j \equiv \partial_i \delta_{ij}$, where δ_{ij} is the Kronecker delta, we arrive at the following form for the MHD momentum equation:

$$\frac{\partial}{\partial t}\left(\rho v_i\right) + \partial_j \left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right) = 0, \tag{430}$$

where T_{ik} is the **Maxwell stress tensor**

$$T_{ij} = \frac{1}{4\pi}\left(B_i B_j - \frac{1}{2}|\vec{B}|^2 \delta_{ij}\right). \tag{431}$$

- Now consider a small cylindrical volume with cross-sectional area A oriented perpendicular to the shock, with one end in the pre-shock gas and the other in the post-shock gas. From the above form of the momentum equation, we see that

$$\int_V \frac{\partial}{\partial t}\left(\rho v_i\right) \mathrm{d}V = -\int_V \partial_j \left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right) \mathrm{d}V, \tag{432}$$

where V is the volume of the cylinder. If we allow the volume of the cylinder to tend to zero, then the integral of the time derivative term vanishes, and we have simply

$$\int_V \partial_j \left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right) \mathrm{d}V = 0. \tag{433}$$

Applying Gauss' theorem then yields

$$\int_S \left(\rho v_j v_i + p\delta_{ij} - T_{ij}\right) n_j \mathrm{d}A = 0, \tag{434}$$

where $\vec{n}$ is the vector normal to the area element $\mathrm{d}A$, and S is the closed surface of the cylinder. Since we can make the length of the cylinder as short as we like, the only surfaces we need consider are the two ends. We therefore find that

$$A\left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_1 - A\left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_2 = 0, \tag{435}$$

where the subscripts denote that the contents of the brackets are evaluated in the pre-shock and post-shock gas, respectively. From this, our desired jump condition follows trivially:

$$\left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_1 = \left[\rho v_j v_i + p\delta_{ij} - T_{ij}\right]_2. \tag{436}$$

- To translate this from component notation back into something more useful, note that we can locally decompose the fluid velocity into two components, $v_\perp$ and $v_\parallel$, where $v_\perp$ is oriented perpendicular to the magnetic field and $v_\parallel$ is ordered parallel to it. Similarly, $\vec{B}$ can also be decomposed into perpendicular and parallel components, $B_\perp$ and $B_\parallel$.
- If we let both i and j represent the perpendicular component, then $i = j$ and the jump condition tells us that

$$\rho_1 v_{1,\perp}^2 + p_1 - \frac{1}{8\pi}\left(B_{1,\perp}^2 - B_{1,\parallel}^2\right) = \rho_2 v_{2,\perp}^2 + p_2 - \frac{1}{8\pi}\left(B_{2,\perp}^2 - B_{2,\parallel}^2\right). \tag{437}$$

Alternatively, if we let i represent the perpendicular component and j represent the parallel component, then $i \neq j$ and the momentum jump condition yields

$$\rho_1 v_{1,\perp} v_{1,\parallel} - \frac{1}{4\pi} B_{1,\perp} B_{1,\parallel} = \rho_2 v_{2,\perp} v_{2,\parallel} - \frac{1}{4\pi} B_{2,\perp} B_{2,\parallel}. \tag{438}$$

- A similar analysis applied to the constraint that $\nabla \cdot \vec{B} = 0$ gives us another jump condition for the magnetic field

$$B_{1,\perp} = B_{2,\perp}, \tag{439}$$

and allows us to simplify the first of the momentum jump conditions to

$$\rho_1 v_{1,\perp}^2 + p_1 + \frac{1}{8\pi} B_{1,\parallel}^2 = \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2. \tag{440}$$

We therefore see that if the flow of the gas is perfectly parallel to the field lines, so that the shock is oriented perpendicularly to them and $B_\parallel = 0$, then the momentum jump condition that we obtain is the same as in the hydrodynamical case. This makes sense on physical grounds – in this scenario, the field exerts no net force on the gas, so it is not surprising that the jump conditions remain unaltered. We also see that when the parallel component of the field is non-zero, then our jump condition for the momentum in the perpendicular direction *does* depend on the magnetic field, which provides an additional source of pressure.
- A further jump condition on the velocity and the magnetic field comes from the induction equation

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{B} \times \vec{v}\right) = 0. \tag{441}$$

If we consider the same small cylindrical volume as before and require the time derivative of the magnetic field to vanish within it, then it follows that

$$\int_V \nabla \times \left(\vec{B} \times \vec{v}\right) \mathrm{d}V = 0. \tag{442}$$

This can be converted to the following surface integral

$$\int_S \vec{n} \times \left(\vec{B} \times \vec{v}\right) \mathrm{d}S = 0, \tag{443}$$

where $\vec{n}$ is the normal to the surface, and the vector identity

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} \tag{444}$$

then allows us to write this as

$$\int_S \left[(\vec{n} \cdot \vec{v})\vec{B} - (\vec{n} \cdot \vec{B})\vec{v} \right] \mathrm{d}S = 0. \tag{445}$$

- As before, we can choose our volume V and surface S so that the only parts of the surface integral that we need to worry about are the ends of the cylinder, and hence can simply take $\vec{n}$ to be perpendicular to the shock front. We therefore have

$$\int_S \left[v_{\perp}\vec{B} - B_{\perp}\vec{v} \right] \mathrm{d}S = 0. \tag{446}$$

From this, we obtain the jump condition

$$v_{1,\perp}B_{1,\parallel} - B_{1,\perp}v_{1,\parallel} = v_{2,\perp}B_{2,\parallel} - B_{2,\perp}v_{2,\parallel}. \tag{447}$$

(Taking the other component of the vector simply yields the trivial result that $v_{\perp}B_{\perp} - B_{\perp}v_{\perp} = 0$, and hence tells us nothing new).

- Finally, it is possible to write the energy equation for the flow in conservative form as[10]

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \epsilon + \frac{B^2}{8\pi}\right) + \nabla \cdot \left[\rho\vec{v}\left(h + \frac{1}{2}v^2\right) + \frac{1}{4\pi}(\vec{B} \times \vec{v}) \times \vec{B}\right] = 0 \tag{448}$$

from which a final jump condition follows:

$$\rho_1 v_{1,\perp}\left(h_1 + \frac{1}{2}v_1^2\right) - \frac{1}{4\pi}CB_{1,\parallel} = \rho_2 v_{2,\perp}\left(h_2 + \frac{1}{2}v_2^2\right) - \frac{1}{4\pi}CB_{2,\parallel}, \tag{449}$$

where $C = B_{1,\perp}v_{1,\parallel} - B_{1,\parallel}v_{1,\perp} = B_{2,\perp}v_{2,\parallel} - B_{2,\parallel}v_{2,\perp}$ is conserved through the shock.

- The full set of jump conditions for an MHD shock is therefore

$$\rho_1 v_{1,\perp} = \rho_2 v_{2,\perp}, \tag{450}$$

$$B_{1,\perp} = B_{2,\perp}, \tag{451}$$

$$\rho_1 v_{1,\perp}^2 + p_1 + \frac{1}{8\pi}B_{1,\parallel}^2 = \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi}B_{2,\parallel}^2, \tag{452}$$

$$\rho_1 v_{1,\perp}v_{1,\parallel} - \frac{1}{4\pi}B_{1,\perp}B_{1,\parallel} = \rho_2 v_{2,\perp}v_{2,\parallel} - \frac{1}{4\pi}B_{2,\perp}B_{2,\parallel}, \tag{453}$$

$$v_{1,\perp}B_{1,\parallel} - B_{1,\perp}v_{1,\parallel} = v_{2,\perp}B_{2,\parallel} - B_{2,\perp}v_{2,\parallel}, \tag{454}$$

$$\rho_1 v_{1,\perp}\left(h_1 + \frac{1}{2}v_1^2\right) - \frac{1}{4\pi}CB_{1,\parallel} = \rho_2 v_{2,\perp}\left(h_2 + \frac{1}{2}v_2^2\right) - \frac{1}{4\pi}CB_{2,\parallel}. \tag{455}$$

[10] In the interests of brevity, we leave proof of this statement as an exercise for the reader.

- It is clear from the form of these jump conditions that MHD shocks are considerably more complicated than their hydrodynamical equivalents. On reflection, however, this should not be surprising. In a hydrodynamical flow, we have only a single characteristic velocity, the sound speed $c_{\rm s}$, at which signals can propagate. In an MHD flow, however, there are *three* characteristic velocities: the sound speed, and also $v_{\rm F}$ and $v_{\rm S}$, the speeds of the fast and slow MHD waves. Shocks are associated with the jump of the flow velocity from above the phase velocity of a given wave to below, and hence there are *six* possible types of MHD shock.

- The first type of MHD shock is known as a **fast shock**. In this case, we have $v_{1,\perp} > v_{\rm F}$ and $v_{\rm F} > v_{2,\perp} > v_{{\rm A},\perp}$, where $v_{{\rm A},\perp}$ is the Alfven velocity in the perpendicular direction. The flow therefore jumps from above to below the velocity of the fast MHD wave, but remains faster than the Alfven velocity.

- Another type of MHD shock is a **slow shock**, where $v_{{\rm A},\perp} > v_{1,\perp} > v_{\rm S}$ and $v_{2,\perp} < v_{\rm S}$. In this case, both pre-shock and post-shock velocities are slower than the Alfven velocity.

- Finally, there are four different types of **intermediate shock**:

$$\begin{aligned} v_{1,\perp} > v_{\rm F}, &\qquad v_{{\rm A},\perp} > v_{2,\perp} > v_{\rm S}, \\ v_{1,\perp} > v_{\rm F}, &\qquad v_{2,\perp} < v_{\rm S}, \\ v_{\rm F} > v_{1,\perp} > v_{{\rm A},\perp}, &\qquad v_{{\rm A},\perp} > v_{2,\perp} > v_{\rm S}, \\ v_{\rm F} > v_{1,\perp} > v_{{\rm A},\perp}, &\qquad v_{2,\perp} < v_{\rm S}. \end{aligned}$$

  In all four of these shocks, the pre-shock flow is super-Alfvenic (i.e. faster than the Alfven velocity) and the post-shock flow is sub-Alfvenic.

- A comprehensive analysis of the behaviour of all of these different types of MHD shock is beyond the scope of this lecture course.[11] Here, we restrict our discussion to a couple of simple but informative cases.

- If $B_{\perp} = 0$, then the shock jump conditions reduce to

$$\rho_1 v_{1,\perp} = \rho_2 v_{2,\perp}, \tag{456}$$

$$\rho_1 v_{1,\perp}^2 + p_1 + \frac{1}{8\pi} B_{1,\parallel}^2 = \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2, \tag{457}$$

$$v_{1,\parallel} = v_{2,\parallel} \tag{458}$$

$$v_{1,\perp} B_{1,\parallel} = v_{2,\perp} B_{2,\parallel}, \tag{459}$$

$$\rho_1 v_{1,\perp} \left(h_1 + \frac{1}{2} v_1^2 \right) + \frac{1}{4\pi} B_{1,\parallel}^2 v_{1,\perp} = \rho_2 v_{2,\perp} \left(h_2 + \frac{1}{2} v_2^2 \right) + \frac{1}{4\pi} B_{2,\parallel}^2 v_{2,\perp}. \tag{460}$$

  Using these, it is possible to show that in this case, the only type of shock that is physically possible is a fast shock, with $v_{1,\perp} > \left(v_{{\rm A},\perp,1}^2 + c_{{\rm s},1}^2 \right)^{1/2}$. The compression ratio produced by this shock can be written as

$$\frac{\rho_2}{\rho_1} = \frac{2(\gamma+1)}{D + \left[D^2 + 4(\gamma+1)(2-\gamma)\mathcal{M}_{{\rm A},1}^{-2} \right]^{1/2}}, \tag{461}$$

[11] See e.g. Draine & McKee (1993, ARA&A, 31, 373) for a more detailed discussion

where

$$D = (\gamma - 1) + \left(2\mathcal{M}_1^{-2} + \gamma\mathcal{M}_{A,1}^{-2}\right), \tag{462}$$

and $\mathcal{M}_{A,1} \equiv v_{1,\perp}/v_{A,1}$ is the Alfven Mach number of the shock. In the limit where $\mathcal{M} \gg 1$ and $\mathcal{M}_A \gg 1$, $D \to (\gamma - 1)$ and the compression ratio becomes

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}, \tag{463}$$

just as for a purely hydrodynamical shock. For weaker shocks, however, it is clear that we get less compression when the magnetic field is strong and $\mathcal{M}_{A,1}$ is small than when the magnetic field is weak or absent and $\mathcal{M}_{A,1}$ is very large. This is a consequence of the additional resistance to compression provided by the magnetic pressure in this scenario.

- The other important special case is when $B_{\parallel,1} = 0$. In this case, the shock jump conditions become

$$\begin{aligned}
\rho_1 v_{1,\perp} &= \rho_2 v_{2,\perp}, &(464)\\
B_{1,\perp} &= B_{2,\perp}, &(465)\\
\rho_1 v_{1,\perp}^2 + p_1 &= \rho_2 v_{2,\perp}^2 + p_2 + \frac{1}{8\pi} B_{2,\parallel}^2, &(466)\\
\rho_1 v_{1,\perp} v_{1,\parallel} &= \rho_2 v_{2,\perp} v_{2,\parallel} - \frac{1}{4\pi} B_{2,\perp} B_{2,\parallel}, &(467)\\
-B_{1,\perp} v_{1,\parallel} &= v_{2,\perp} B_{2,\parallel} - B_{2,\perp} v_{2,\parallel}, &(468)\\
\rho_1 v_{1,\perp}\left(h_1 + \frac{1}{2}v_1^2\right) &= \rho_2 v_{2,\perp}\left(h_2 + \frac{1}{2}v_2^2\right) - \frac{1}{4\pi} B_{1,\perp} B_{2,\parallel} v_{1,\parallel}. &(469)
\end{aligned}$$

  Note that in this case, we cannot automatically assume that $B_{2,\parallel} = 0$, as $B_\parallel$ is not necessarily conserved through the shock.

- One possible solution consistent with these jump conditions has $B_{2,\parallel} = 0$. In this case, the jump conditions simplify further, becoming the same as for a purely hydrodynamical shock, and the magnetic field plays no role in determining the shock properties. This "hydrodynamical" solution exists whenever $v_1 > c_{s,1}$, and produces a compression ratio, temperature ratio, etc. that are the same as in the absence of a magnetic field. This hydrodynamical solution is a fast shock if $c_{s,1} > v_{A,1}$, and can be either a fast, slow or intermediate shock if $c_{s,1} < v_{A,1}$.

- However, a second solution exists in the $B_{\parallel,1} = 0$ case in which $B_{\parallel,2} \neq 0$. In this case, known as a **switch-on shock**, both the magnetic field and the velocity acquire post-shock components parallel to the shock front despite having no such components in the pre-shock flow. In order for the switch-on solution to exist, the pre-shock velocity must satisfy

$$v_{crit} > v_{1,\perp} > v_{A,1,\perp} > c_{s,1}, \tag{470}$$

  where

$$v_{crit} = \left[\frac{(\gamma + 1)v_{A,1,\perp}^2 - 2c_{s,1}^2}{\gamma - 1}\right]^{1/2}. \tag{471}$$

For $\gamma = 5/3$, switch-on solutions exist only for a small range of Alfvenic Mach numbers, $1 < \mathcal{M}_{\rm A} <$, but for softer equations of state ($\gamma < 5/3$), the allowed range of $\mathcal{M}_{\rm A}$ can be much broader.

- Similarly, it is possible to show that there is also a class of solutions for which $B_{\parallel,1} \neq 0$ but $B_{\parallel,2} = 0$. These are known as **switch-off shocks**, and are a type of slow shock.

4.4.2 Multi-fluid shocks

- In a partially charged fluid, magnetic forces act only on the charged particles and not on the neutral particles. If there are many charged particles, and collisions between them and the neutral particles occur frequently, then this is a good approximation. In this case, collisions will rapidly redistribute momentum between the charged and the neutral components of the plasma, and the end result is the same as if the magnetic forces acted on both fluids.

- If the fractional ionization of the gas becomes very small, however, then this approximation begins to break down, as the coupling between the charged particles and the neutrals is no longer strong enough to maintain both components at the same velocity.

- To analyze this situation, we assume that the velocity of the ions and the electrons in the gas is equal, and focus on the behavior of the ions, since they carry most of the momentum in the charged component of the plasma.

- In the rest frame of the neutrals, and in the absence of any significant gravitational or pressure forces, the ions will feel two main forces: a magnetic force per unit volume

$$f_{\rm mag} = \frac{1}{4\pi} \left(\nabla \times \vec{B}\right) \times \vec{B}, \tag{472}$$

and a drag force due to collisions between the ions and the neutrals,

$$f_{\rm drag} = \gamma \rho_{\rm n} \rho_{\rm i} \left(\vec{v}_{\rm i} - \vec{v}_{\rm n}\right), \tag{473}$$

where γ is the drag coefficient and the other quantities have their obvious meanings.

- Equating these and solving for the relative velocity $\vec{v}_{\rm d}$ between the ions and the neutrals yields

$$\vec{v}_{\rm d} \equiv \vec{v}_{\rm i} - \vec{v}_{\rm n} = \frac{1}{4\pi\gamma\rho_{\rm n}\rho_{\rm i}} \left(\nabla \times \vec{B}\right) \times \vec{B}. \tag{474}$$

- To estimate when this is likely to become important, we use our usual trick of approximating the derivative as $1/L$, where L is a characteristic length scale, and obtain

$$v_{\rm d} \equiv |\vec{v}_{\rm d}| \sim \frac{B^2}{4\pi\gamma\rho_{\rm n}\rho_{\rm i}L} = \frac{v_{\rm A}^2}{\gamma\rho_{\rm i}L}, \tag{475}$$

where we have assumed that $\rho_{\rm i} \ll \rho_{\rm n}$.

- In conditions typical of e.g. a dense prestellar core inside a molecular cloud, we have $\gamma = 3.5 \times 10^{13}\ \mathrm{cm^3\,g^{-1}\,s^{-1}}$, $\rho_\mathrm{i} \simeq 2 \times 10^{-26}\ \mathrm{g\,cm^{-3}}$ and $L \sim 0.1$ pc. We therefore have

$$v_\mathrm{d} \simeq \frac{v_\mathrm{A}^2}{2\ \mathrm{km\,s^{-1}}}. \tag{476}$$

  In these conditions, $v_\mathrm{A} \sim 0.1\ \mathrm{km\,s^{-1}}$, and so $v_\mathrm{d} \ll v_\mathrm{A}$; i.e. motion of the neutrals relative to the ions occurs slowly even in these dense regions.

- In these conditions, and if the conductivity of the plasma remains high, we can continue to write the induction equation in the ideal MHD form

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{B} \times \vec{v}_\mathrm{i}\right) = 0, \tag{477}$$

  where now we clarify that the velocity appearing in this expression is the velocity of the *ions*. In terms of the velocity of the neutrals, the equation instead becomes

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{B} \times \vec{v}_\mathrm{n}\right) = \nabla \times \left\{ \frac{\vec{B}}{4\pi\gamma\rho_\mathrm{n}\rho_\mathrm{i}} \times \left[\vec{B} \times \left(\nabla \times \vec{B}\right)\right] \right\}. \tag{478}$$

- If the coupling between the ions and the neutrals is very strong, then the term on the right-hand side of this expression is very small and can be neglected. In this case, the field evolution with respect to the ions is the same as with respect to the neutrals. When this is not the case, however, then the term on the right-hand side corresponds to the diffusion of the field with respect to the neutrals, a process termed **ambipolar diffusion**.

- If L is the typical length scale over which the magnetic field varies, then we can write the ambipolar diffusion timescale as

$$t_\mathrm{AD} \sim \frac{L}{v_\mathrm{d}}. \tag{479}$$

  In many circumstances, L is large and v_d is small, and hence the am bipolar diffusion timescale is very long. For example, in the prestellar core we considered above, $L = 0.1$ pc, $v_\mathrm{d} \sim 5 \times 10^{-3}\ \mathrm{km\,s^{-1}}$, and so $t_\mathrm{AD} \sim 20$ Myr.

- However, in MHD shocks, the magnetic field strength and configuration can change dramatically over very short length scales. Consequently, ambipolar diffusion can become very important in determining the structure of the shocks, just as molecular viscosity plays an important role in shocks even when it is irrelevant elsewhere in the flow.

- One of the most important consequences of the weak coupling between neutrals and ions in the ambipolar diffusion regime is what it implies for the propagation of MHD waves through the gas. If the ions and neutrals are only weakly coupled, then the Alfven velocity in the ions is given by

$$v_{\rm A,i} = \frac{B}{\sqrt{4\pi\rho_{\rm i}}}, \tag{480}$$

where $\rho_{\rm i}$ is the mass density of the ions. In a weakly ionized gas, $\rho_{\rm i} \ll \rho_{\rm n}$, the mass density of the neutrals, and so the Alfven velocity is much higher than it would be if the ions and the neutrals were strongly coupled.

- To give a few numbers for context: in a typical GMC, in the strongly-coupled regime, $v_{\rm A} \sim 2\,{\rm km\,s^{-1}}$, while in the weakly-coupled regime, $v_{\rm A,i} > 100\,{\rm km\,s^{-1}}$.

- The large size of the Alfven velocity in the ions in the weakly-coupled case means that from the point of view of the ions, almost all of the disturbances in the flow are sub-alfvenic, with propagation velocities $v < v_{\rm A,i}$ (which is also the approximate velocity at which the fast mode waves propagate). Consequently, shocks do not form in the ionized component – its properties remain continuous.

- The behavior of the neutral component depends on how strongly it is coupled to the ionized component, and how strong the shock is. Collisions between neutrals and ions dissipate energy, and in a weakly coupled shock, this dissipation occurs over an extended region. If the resulting dissipative heating is small, then it can be balanced by radiative cooling in the gas, allowing the gas temperature to remain small, and the flow of the neutrals to remain supersonic throughout the "shock". In this case, both components are continuous and we refer to this structure as a **C-type shock**.

- If radiative cooling cannot keep up with ion-neutral dissipation, on the other hand, either because the shock is very strong or because the ion-neutral coupling is not sufficiently weak, then the neutral gas will undergo a supersonic to subsonic transition in a collisional sub-shock, with a thickness of the same order as the neutral particle mean free path. In this case, we refer to the resulting structure as a **J-type shock**.

- Note that in both cases, the behavior of the flow variables far upstream or far downstream of the shock is the same as a single-fluid MHD radiative shock. The weak coupling only affects the behavior of the shock region itself, and cannot influence the conservation of momentum, mass etc. through the shock.