Astrophysical Fluid Dynamics

Assignment #5: due November 27

1 Analytical solution of the Bondi problem.

Recall that we have seen in the lectures that the Bondi accretion problem can be reduced to solving the following equation:

$$\frac{\tilde{v}^2}{2} + \frac{1}{(\gamma - 1)\tilde{r}^{2(\gamma-1)}\tilde{v}^{(\gamma-1)}} - \frac{2}{\tilde{r}} = C \tag{1}$$

The case $\gamma = 1.5$ is the only case of Bondi accretion that can be solved analytically.

(a) Solve for $\tilde{r} = \tilde{r}(\tilde{v})$ in this case.

(b) Show that the critical solution can be written as

$$\tilde{r} = \frac{4}{\tilde{v}^{1/2}(1 + \tilde{v}^{1/2})(1 + \tilde{v})} \tag{2}$$

(c) Construct a diagram that sketches solutions $\tilde{v} = \tilde{v}(\tilde{r})$ for various values of C and discuss its properties.

2 Solar mass loss

Spectral line observations show that the temperature of the plasma at the basis of the Solar corona is $T \sim 2 \times 10^6\,\mathrm{K}$ and the number density of electrons is $n_\mathrm{e} \sim 10^8\,\mathrm{cm}^{-3}$. Assuming that the Solar corona is made by a fully ionised plasma of pure hydrogen

(a) Show that the velocity of the isothermal Parker wind close to the stellar surface is given approximately by

$$v \simeq c_\mathrm{s} e^{3/2} \left(\frac{r_\mathrm{s}}{r}\right)^2 \exp\left(-\frac{2r_\mathrm{s}}{r}\right), \tag{3}$$

where c_s is the sound speed and r_s is the sonic radius.

(b) Estimate the Solar mass loss $\dot{M}$.

Is your estimate reliable if you have a factor of 2 uncertainty on the coronal temperature? What value of $\dot{M}$ would you have obtained if you used the Sun photospheric temperature $T \simeq 6000\,\mathrm{K}$ rather than the coronal temperature?