Astrophysical Fluid Dynamics

Assignment #9: due January 15th

1. Consider a blob of gas with density $\rho_{\rm b}$ and pressure $P_{\rm b}$ embedded in an ambient medium with a density ρ and pressure P that are both functions of height. At the initial position of the blob, $\rho = \rho_{\rm i}$ and $P = P_{\rm i}$. Assume that the blob is initially in mechanical and thermal equilibrium with the surrounding gas, so that $\rho_{\rm b} = \rho_{\rm i}$ and $P_{\rm b} = P_{\rm i}$.

   (a) We now displace the blob upwards by a small distance δz. Suppose that the blob remains in pressure equilibrium with its surroundings and that its evolution is adiabatic. Show that in this case, its density is given approximately by

   $$\rho_{\rm b} \simeq \rho_{\rm i} + \frac{\rho_{\rm i}}{\gamma P_{\rm i}} \frac{{\rm d}P}{{\rm d}z} \delta z. \tag{1}$$

   (b) Use the result from part (a) to show that the blob will continue to accelerate upwards unless

   $$\frac{{\rm d}\rho}{{\rm d}z} < \frac{\rho}{\gamma P} \frac{{\rm d}P}{{\rm d}z}. \tag{2}$$

   This is known as the **Schwarzschild criterion for convective stability**, and the associated instability is known as the **convective instability**.

   (c) Show that the Schwarzschild stability criterion can also be written as

   $$\frac{{\rm d}s}{{\rm d}z} > 0, \tag{3}$$

   where s is the entropy of the gas.

2. (a) A spherical gas cloud of mass M and radius R is composed of a monatomic gas with a uniform density ρ and uniform temperature T. Find the value of M for which the total energy of the sphere is zero. Comment on your result. [**Note**: use the convention that gravitational binding energies are negative].

   (b) Now suppose that the gas cloud contracts homogeneously (i.e. maintaining uniform density), and that as it does so, its temperature evolves as $T \propto \rho^n$. As the sphere contracts, the number of Jeans masses that it contains varies with density as

   $$N_{\rm J}(\rho) \equiv \frac{M}{M_{\rm J}(\rho)} \propto \rho^m. \tag{4}$$

   Find an expression for m in terms of n.

   (c) If the evolution of the gas cloud is adiabatic, then $n = 2/3$. How does $N_{\rm J}$ evolve with density in this case? What does this imply for the ability of the cloud to undergo gravitational collapse?