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Thermal Conductivity in Stellar Atmospheres I. Without Magnetic Field

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The coefficient of thermal conductivity for pure hydrogen, pure helium and a gas mixture appropriate for a stellar atmosphere has been computed in the temperature range 3000 to 100000 °K and the range of gas pressure from 10^{-3} to 10^{6} dyn/cm^2. The translational as well as reactive contributions to the coefficient are given. The error is less than 10% (except for pure He) in the neutral region but may be more than 35% in the ionizing region. In the fully ionized region there is complete agreement with Spitzer and Härm (1953).

Key words: thermal conductivity — stellar atmosphere — hydrogen — helium

I. Introduction

Advances in the theory of acoustic wave propagation and in the understanding of the chromosphere-corona transition region of stellar atmospheres make a new computation of the coefficient of thermal conductivity highly desirable. Earlier work has been done by Edmonds (1957) and Oster (1957). However recent theoretical developments especially by Devoto (1966, 1967 b) and much better knowledge of the interparticle potentials make it now possible to compute the coefficient with much greater precision. Recent work on pure hydrogen, see Brezing (1965) and on pure helium see Devoto and Li (1968).

For the fully ionized region Landshoff (1949, 1951), Spitzer and Härm (1953), Spitzer (1962) have shown that because of the long range Coulomb interaction the coefficient of thermal conductivity may be computed via the solution of the Fokker-Planck equation, that is, by dropping in the Boltzmann equation all terms representing close encounters. This procedure is very accurate in a fully ionized gas but looses its validity at low ionization where close encounters, because of the short range charged – neutral and interatomic potentials become more important.

For the electrically neutral region the procedure to compute the coefficient of thermal conductivity usually is to take the Chapman-Enskog-Burnett theory (Chapman and Cowling, 1952; Hirschfelder, Curtiss and Bird, 1964; Muckenfuss and Curtiss, 1958). There are two approximations made in this theory which uses the full Boltzmann equation. Firstly the departures from the equilibrium distribution are considered small such that the Enskog series expansion may be cut off after the first perturbation term. The resulting system of equations is secondly solved in terms of a Sonine polynomial expansion, which is usually carried to the first or second order. This theory has been applied to a great variety of gas mixtures and was found to be very accurate especially at low temperatures. However applied to an ionizing gas the theory did not agree with the results of the fully ionized limit. The reason for this disagreement was found quite recently by Ahtye (1965) to be due to the fact that the Sonine polynomial expansion was apparently not carried far enough. Subsequently Devoto (1967 a) has shown that a fourth order approximation did ensure agreement with the Spitzer and Härm (1953) results in the case of argon to better than 1%. This fact could be verified for hydrogen also (see Fig. 3).

Thus a fourth order Chapman-Enskog-Burnett theory should be quite accurate over the whole range from the neutral to the ionized region.

2. Equations

According to the Chapman-Enskog-Burnett formalism the coefficient of thermal conductivity is given by

$$\lambda = \lambda_R + \lambda_{TR}\,, \tag{1}$$

where λ_R is the reactive conductivity arising from the transport of internal energy and energy of ionization and λ_{TR} the translational conductivity due to transport of purely kinetic energy.

a) For the reactive thermal conductivity Devoto (1967a) has given the approximate expression for a pure ionizing gas.

$$\lambda_R = (\Delta h)^2 \frac{n m_A}{2 \varrho k T^2} \frac{n_E n_A}{n_E + n_A} D_{AMB} , \qquad (2)$$

where $\Delta h = h_E + h_I - h_A$ is the enthalpy difference per particle, D_{AMB} the ambipolar diffusion coefficient, ϱ the density, n the number density, T the temperature, k the Boltzmann constant, E labels the electron, A the atom and I the ion, m_A being the atomic mass. D_{AMB} is given to close approximation by

$$D_{AMB} = 2 D_{IA} , \qquad (3)$$

where D_{IA} is the multicomponent diffusion coefficient. As was found in the case of argon (Devoto, 1967a) and helium (Devoto and Li, 1968) it was found for hydrogen that setting

$$D_{IA} = \mathscr{D}_{IA} , \qquad (4)$$

the binary diffusion coefficient

$$\mathscr{D}_{IA} = \frac{3}{16 n} \left(\frac{2 \pi k T}{m_{IA}} \right)^{1/2} \frac{1}{\bar{Q}_{IA}^{(1,1)}} , \qquad (5)$$

where m_{IA} is the reduced mass and $\bar{Q}_{IA}^{(1,1)}$ the averaged cross-section (see Section 3) of the IA system, resulted in an error of less than 1%.

In order to find an expression for the reactive thermal conductivity valid for a gas mixture we note that from Eqs. (2) and (5) we may write using the relations $\varrho = m_A (n_A + n_I)$, $n_I = n_E$ and $m_{IA} = m_A/2$

$$\lambda_R = (\Delta h)^2 \frac{6}{16} \left(\frac{\pi}{k m_A T^3} \right)^{1/2} \frac{x_I x_A}{\bar{Q}_{IA}^{(1,1)}} , \qquad (6)$$

where $x_I = n_I/(n_A + n_I)$ and $x_A = n_A/(n_A + n_I)$ are the relative concentrations of the ions and atoms.

Equation (5) may now be easily generalized for a second stage of ionization (A labeling now the ion and I the higher ion) and by multiplying with the fraction of the relative abundance ($\varepsilon_i / \Sigma \varepsilon_i$ see Table 3) for a gas mixture. For the enthalpy only the translational and ionization energies were taken into account.

b) The translational thermal conductivity computed with the fourth order approximation in the Sonine polynomial expansion is given by Devoto (1966). Because this involves a large amount of different averaged cross-sections, most of which are not yet computed for the relevant interatomic potentials, Devoto (1967b) showed that the expressions may be considerably simplified by using the fact that during a binary electron-neutral and electron-ion collision only the momentum of the electron is appreciably altered, while the electron velocity, the velocity and momentum of the heavy particles stay close to their original value. Devoto finds that to a good approximation one may write

$$[\lambda_{TR}]_4 = [\lambda_E]_3 + [\lambda_H]_2 , \qquad (7)$$

where $[\lambda_E]_3$ is the electron contribution and $[\lambda_H]_2$ the contribution of the heavy particles which need only be carried to the second order.

c) The electron contribution to the translational conductivity $[\lambda_E]_3$ may be written (Devoto, 1967b)

$$[\lambda_E]_3 = \frac{75 n_0^2 k}{8} \left(\frac{2 \pi k T}{m_0} \right)^{1/2} \frac{1}{q^{11} - (q^{12})^2 / q^{22}} , \qquad (8)$$

where m_0 is the mass of the electron and the quantities q^{mp} are given by

$$\begin{aligned} q^{11} &= 8 \sqrt{2}\, n_0^2 \bar{Q}_{00}^{(2,2)} + 8 \sum_{j=1}^{5} n_0 n_j \\ &\quad \cdot \left(\frac{25}{4} \bar{Q}_{0j}^{(1,1)} - 15 \bar{Q}_{0j}^{(1,2)} + 12 \bar{Q}_{0j}^{(1,3)} \right) , \\ q^{12} &= 8 \sqrt{2}\, n_0^2 \left(\frac{7}{4} \bar{Q}_{00}^{(2,2)} - 2 \bar{Q}_{00}^{(2,3)} \right) \\ &\quad + 8 \sum_{j=1}^{5} n_0 n_j \\ &\quad \cdot \left(\frac{175}{16} \bar{Q}_{0j}^{(1,1)} - \frac{315}{8} \bar{Q}_{0j}^{(1,2)} + 57 \bar{Q}_{0j}^{(1,3)} - 30 \bar{Q}_{0j}^{(1,4)} \right) \\ q^{22} &= 8 \sqrt{2}\, n_0^2 \left(\frac{77}{16} \bar{Q}_{00}^{(2,2)} - 7 \bar{Q}_{00}^{(2,3)} + 5 \bar{Q}_{00}^{(2,4)} \right) \\ &\quad + 8 \sum_{j=1}^{5} n_0 n_j \\ &\quad \cdot \left(\frac{1225}{64} \bar{Q}_{0j}^{(1,1)} - \frac{735}{8} \bar{Q}_{0j}^{(1,2)} + \frac{399}{2} \bar{Q}_{0j}^{(1,3)} \right. \\ &\qquad \left. - 210 \bar{Q}_{0j}^{(1,4)} + 90 \bar{Q}_{0j}^{(1,5)} \right) . \end{aligned} \qquad (9)$$

The averaged cross-sections $\bar{Q}_{ij}^{(l,s)}$ are given in Section 3 and the n_i are the number densities of the six constituents e, H, He, H^+, He^+, He^{++}. Index 0 labels the electron, 1 the H atom, 2 the He atom, 3 the H^+ ion, 4 the He^+ ion and 5 the He^{++} ion.

d) The heavy particle contribution $[\lambda_H]_2$ may be computed in standard fashion (Devoto, 1966).

$$[\lambda_H]_2 = - \frac{75 k (2 \pi k T)^{1/2}}{8 |\hat{q}|} \begin{vmatrix} q_{11}^{00} \cdots q_{15}^{00} & q_{11}^{01} \cdots q_{15}^{01} & 0 \\ \vdots \qquad \vdots & \vdots \qquad \vdots & \vdots \\ q_{51}^{00} \cdots q_{55}^{00} & q_{51}^{01} \cdots q_{55}^{01} & 0 \\ q_{11}^{10} \cdots q_{15}^{10} & q_{11}^{11} \cdots q_{15}^{11} & n_1 \\ \vdots \qquad \vdots & \vdots \qquad \vdots & \vdots \\ q_{51}^{10} \cdots q_{55}^{10} & q_{51}^{11} \cdots q_{55}^{11} & n_5 \\ 0 \cdots 0 & n_1/\sqrt{m_1} \cdots n_5/\sqrt{m_5} & 0 \end{vmatrix} \qquad (10)$$

where $|\hat{q}|$ is the determinant of Eq. (10) without the last row and last column. The n_i are number densities, m_i the masses and the quantities q_{ij}^{mp} are given by

$$q_{ij}^{00} = 8 \sum_{l=1}^{5} \frac{n_l\, m_i^{1/2}}{(m_i + m_l)^{1/2}}\, \bar{Q}_{il}^{(1,1)}$$

$$\cdot \left\{ n_i \left(\frac{m_l}{m_j}\right)^{1/2} (\delta_{ij} - \delta_{jl}) - n_j \frac{(m_l\, m_j)^{1/2}}{m_i} (1 - \delta_{il}) \right\},$$

$$q_{ij}^{01} = 8\, n_i \left(\frac{m_i}{m_j}\right)^{3/2} \sum_{l=1}^{5} \frac{n_l\, m_l^{3/2}}{(m_i + m_l)^{3/2}}$$

$$\cdot \left(\frac{5}{2} \bar{Q}_{il}^{(1,1)} - 3\, \bar{Q}_{il}^{(1,2)}\right) (\delta_{ij} - \delta_{jl}), \qquad q_{ij}^{10} = \frac{m_j}{m_i} q_{ij}^{01},$$

$$q_{ij}^{11} = 8\, n_i \left(\frac{m_i}{m_j}\right)^{3/2} \sum_{l=1}^{5} \frac{n_l\, m_l^{1/2}}{(m_i + m_l)^{5/2}}$$

$$\cdot \left\{ \left(\frac{5}{4} (6\, m_j^2 + 5\, m_l^2)\, \bar{Q}_{il}^{(1,1)} - 15\, m_l^2\, \bar{Q}_{il}^{(1,2)} + 12\, m_l^2\, \bar{Q}_{il}^{(1,3)}\right)\right.$$

$$\left. \cdot (\delta_{ij} - \delta_{jl}) + 4\, m_j\, m_l\, \bar{Q}_{il}^{(2,2)} (\delta_{ij} + \delta_{jl}) \right\}. \tag{11}$$

δ is the Kronecker delta and the averaged cross-sections are given in Section 3.

3. Averaged Cross-Sections

Given an interparticle potential $\varphi_{ij}(r)$ then the cross-sections $Q_{ij}^{(l)}(E)$ may be computed (Hirschfelder, Curtiss, Bird, 1964, p. 525) via

$$Q_{ij}^{(l)}(E) = 2\pi \int_0^\infty (1 - \cos^l \chi(b,E))\, b\, db\,, \qquad l = 1, 2, 3, \ldots \tag{12}$$

where b is the impact parameter, E the initial kinetic energy of the colliding particles i, j in their center of mass coordinates. The deflection angle χ is given by

$$\chi(b, E) = \pi - 2b \int_{r_m}^{\infty} \frac{dr/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{\varphi_{ij}(r)}{E}}}\,, \tag{13}$$

where r_m is the distance of closest approach and φ_{ij} the interparticle potential. The averaged cross-sections may now be obtained by

$$\bar{Q}_{ij}^{(l,s)}(T) = \pi\sigma^2\, \Omega_{ij}^{(l,s)*} = \frac{2(l+1)}{(s+1)!\,(2l + 1 - (-1)^l)} \int_0^\infty$$

$$\cdot\, e^{-u}\, u^{s+1}\, Q_{ij}^{(l)}(u\, kT)\, du\,, \tag{14}$$

where $u = E/kT$ and σ is a characteristic distance defined by the potential.

For any type of interparticle potential φ_{ij} the averaged cross-sections may be computed with a computer program described by Smith and Munn (1964). However as the φ_{ij} are not yet known very accurately and because their knowledge is only needed in a certain region around the thermal energy range it is usually attempted to fit a simple potential to the true curve. The cross-sections may then be interpolated out of computed tables for these potentials. This reduces the amount of numerical work tremendously. Two potentials were found to be particularly useful in our work, the exponential repulsive potential (Monchick, 1959)

$$\varphi(r) = \varepsilon\, e^{-r/\sigma}\,, \tag{15}$$

and the Morse potential (Smith and Munn, 1964)

$$\varphi(r) = \varepsilon \left(e^{-\frac{2C}{\sigma}(r - r_e)} - 2\, e^{-\frac{C}{\sigma}(r - r_e)}\right), \tag{16}$$

where ε is the energy scale, σ the distance scale, C a parameter and $r_e = \sigma(1 + \ln(2)/C)$. We had to extrapolate the Smith and Munn (1964) tables for the Morse potential. Luckily this extrapolation is comparatively unambiguous because firstly $\log \Omega^{(1,1)*}$ versus $\log C$ and $\log \Omega^{(2,2)*}$ versus $\log C$ have to be smooth curves but also $A^* = \Omega^{(2,2)*}/\Omega^{(1,1)*}$ versus $\log C$.

In Table 1 we have given the values of $\Omega^{(2,2)*}$, A^*, B^*, and C^* which have been obtained this way and augment Table 4 with $C = 1.0$ of Smith and Munn (1964). The actual interpolation for different values C may then follow the procedure prescribed by these authors.

Table 1. *Collision integral extrapolations for the Morse potential with $C = 1.0$*

T^*	$\Omega^{(2,2)*}$	A^*	B^*	C^*
1	3.15	1.26	1.30	0.568
2	1.165	1.32	1.34	0.618
4	0.5143	1.39	1.50	0.651

A) Electron Contribution

There are two types of collisions involved, electron-atom and Coulomb collisions.

1. Electron-Atom Collisions

For electron-hydrogen and electron-helium collisions the potential is usually given in the form of phase shifts from which the cross-section of Eq. (12) may be computed (Hirschfelder, Curtiss, Bird, 1964,

p. 675) by

$$Q^{(1)}_{0i}(E) = \frac{4\pi}{\varkappa^2} \sum_{l=0}^{\infty} (l+1) \sin^2 (\delta_{l+1} - \delta_l) \,. \qquad (17)$$

Here δ_l is the phase shift of the radial wave function for angular momentum l and $\varkappa = (2 m_0 E)^{1/2}/\hbar$ is the wave number.

a) H-e *Cross-Section*

Because the phase shifts are only computed up to $l = 2$ we cut off the series in Eq. (17) after the second term. For S wave scattering ($l = 0$) we use the work of Schwartz (1961), for $l = 1$ that of Armstead (1968) and for $l = 2$ that of Gailitis (1965). Owing two the different spin positions in the collision we get triplet $S = 1$ and singlet $S = 0$ phase shifts for each of which we have to compute the cross-section via Eq. (17). The mean cross-section may then be obtained by

$$Q^{(1)}(E) = \left(\frac{1}{4} Q^{(1)}_{\text{singlet}}(E) + \frac{3}{4} Q^{(1)}_{\text{triplet}}(E)\right) . \qquad (18)$$

The averaged cross-section may then be computed with Eq. (14) using 1/2 of a Gauss-Laguerre 64 point method as suggested by Smith and Munn (1964).

b) He-e *Cross-Section*

Here we use the phase shifts for $l = 0, 1, 2$ of LaBahn and Callaway (1966) and apply the same procedure as above. The result was found in perfect agreement with Devoto and Li (1968).

2. Coulomb Cross-Sections

Here as in the work of Spitzer and Härm (1953) a screened Coulomb potential is used, the averaged cross-sections of which have been computed approximately analytically by Liboff (1959) and Devoto (1967a)

$$\bar{Q}^{(1,s)}_{ij} = \frac{4\pi}{s(s+1)} b_0^2 \left(\ln \Lambda - \frac{1}{2} - 2\bar{\gamma} + \psi(s)\right) ,$$

$$s = 1, 2, 3, \ldots ,$$

$$\bar{Q}^{(2,s)}_{ij} = \frac{12\pi}{s(s+1)} b_0^2 (\ln \Lambda - 1 - 2\bar{\gamma} + \psi(s)) ,$$

$$s = 1, 2, 3, \ldots \qquad (19)$$

where $b_0 = Z_i Z_j e^2/2\,kT$ is the average closest impact parameter, e the electronic charge in cgs units, Z_i the charge number, $\bar{\gamma} = 0.5772$ Euler's constant, $\Lambda = 2d/b_0$, $d = \sqrt{kT/8\pi\, n_0 e^2}$ the Debye length valid for shielding of both ions and electrons, $\psi(s) = \sum_{n=1}^{s-1} 1/n$, $\psi(1) = 0$. Following Devoto (1967a) we take only the leading logarithmic term in the bracket at the right hand side of Eq. (19).

B) *Heavy Particle Contribution*

Table 2 lists the collisions of heavy particles which we have to consider.

1. The collisions labeled C are Coulomb collisions which may be treated using Eq. (19).
2. The collisions labeled X occur exceedingly rare because the relevant zones of ionization are well separated. They will be disregarded in our computations (see Section 4).
3. The collisions labeled N are neutral-neutral collisions.

Table 2. *Types of heavy particle collisions considered. N are neutral-neutral, I ion-neutral, C Coulomb collisions. X are collisions which occur exceedingly rare*

	H	He	H$^+$	He$^+$	He^{++}
H	*N*	*N*	*I*	*X*	*X*
He	*N*	*N*	*I*	*I*	*X*
H$^+$	*I*	*I*	*C*	*C*	*C*
He$^+$	*X*	*I*	*C*	*C*	*C*
He^{++}	*X*	*X*	*C*	*C*	*C*

a) H-H *Collision*

Averaged cross-sections of these collisions are given by Vanderslice *et al.* (1962). We use however the new computations of Kolos and Wolnjewicz (1965). For the attractive $^1\Sigma$ potential we fitted a Morse potential Eq. (16) with

$$\varepsilon = 9.573 \text{ eV}, \quad C = 1.4124, \quad \sigma = 0.5098 \text{ Å}.$$

For the repulsive $^3\Sigma$ potential we fitted an exponential repulsive potential Eq. (15) with $\varepsilon = 57.511$ eV and $\sigma = 0.3671$ Å.

b) He—He *Collisions*

There is still a considerable uncertainty about the $^1\Sigma$ potential (Gilbert and Wahl, 1967). We have chosen an exponential repulsive Eq (15), fit to the values of Philipson (1962) with $\varepsilon = 191.47$ eV and $\sigma = 0.2578$ Å reported by Devoto and Li (1968). This potential fit agrees well with new computations of Gilbert and Wahl (1967) while the thermal conductivity computed with it agrees well with measurements of Collins, Greif and Bryson (1965).

c) He—H *Collisions*

The $^2\Sigma$ ground state is repulsive. An exponential repulsive fit Eq. (15) has therefore been done to the values of Michels and Harries (1963) with $\varepsilon = 89.523$ eV and $\sigma = 0.3082$ Å.

4. The collisions labeled *I* are ion-neutral collisions.

If the colliding particles are of the same element charge transfer has to be taken into account as an additional interaction process. Cross-sections Q_{TR} for this are usually much larger than elastic cross-sections. Mason, Vanderslice and Yos (1959) have shown that charge transfer does not enter $Q^{(l)}$ of Eq. (12) for l even. For l odd they find

$$Q^{(l)} = 2\,Q_{TR}\,. \tag{20}$$

Dalgarno (1958) has suggested on basis of theory that Q_{TR} may be given over a fairly large range by approximately

$$Q_{TR} = \frac{1}{2}(A - B\ln g)^2 \tag{21}$$

where A and B are constants and g is the relative speed of the colliding particles.

This may be integrated analytically (Devoto, 1967a) to give the averaged collision cross-sections.

$$\begin{aligned}\bar{Q}^{(l,s)} = {} & A^2 - A\,B\,x + \left(\frac{Bx}{2}\right) + \frac{B\zeta}{2}(Bx - 2A)\\ & + \frac{B^2}{4}\left(\frac{\pi^2}{6} - \sum_{n=1}^{S+1}\frac{1}{n^2} + \zeta^2\right)\\ & + \frac{B}{2}(B(x+\zeta) - 2A)\ln\tfrac{T}{M} + \left(\frac{B}{2}\ln\frac{T}{M}\right)^2\end{aligned} \tag{22}$$

where $x = \ln(4R)$, R the universal gas constant, M the molecular weight of the gas, $\zeta = \sum_{n=1}^{s+1}\frac{1}{n} - \bar{\gamma}$ and $\gamma = 0.5772$ is the Euler-Mascheroni constant. For A and B given in Å we have $Q^{(l,s)}$ in A^2.

a) H—H$^+$ *Collisions*

The $^2\Sigma$ groundstate of H H$^+$ is attractive. We have fitted a Morse potential [Eq. (16)] with $\varepsilon = 3.52$ eV, $C = 0.843$ and $\sigma = 0.5806$ Å to the values of Bates and Reid (1968). For the charge transfer cross-section [Eq. (21)] there are recent theoretical calculations by Parcell and May (1967) and Smith (1967). These computations are however in some disagreement with the experimental results of Fite *et al.* (1960) and Fite *et al.* (1962). Therefore it was decided to do a numerical fit to these experimental points with Eq. (21) from which $A = 29.03$ Å and $B = 1.27$ Å was found (see Fig. 1).

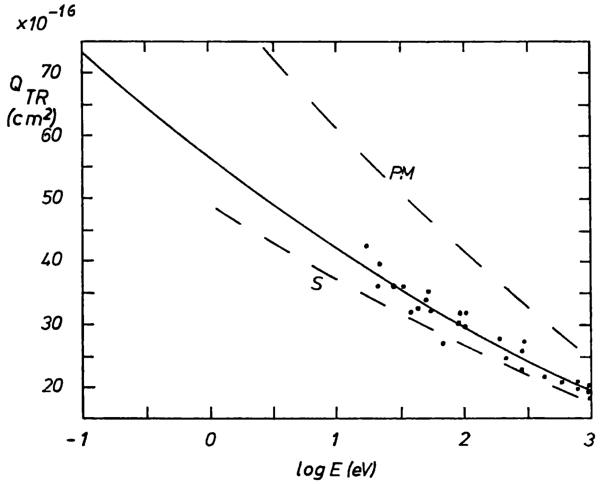

Fig. 1. Charge transfer cross-section Q_{TR} for H$^+$—H collisions Points are experimental values of Fite *et al.* (1960) and Fite *et al.* (1962). PM and S label theoretical values of Parcell and May (1967) and Smith (1967) respectively. Drawn curve is the fit used in our work

b) He—He$^+$ *Collisions*

The elastic cross-sections arise from two equally probable portions of the $^2\Sigma$ state of He He$^+$. For the gerade portion a Morse potential fit [Eq. (16)] with $\varepsilon = 2.7$ eV, $C = 1.52$ and $\sigma = 0.728$ Å and for the ungerade portion an exponential repulsive fit [Eq. (15)] with $\varepsilon = 179.7$ eV and $\sigma = 0.344$ Å was given by Devoto and Li (1968) on basis of yet unpublished work of Gilbert and Wahl (1967). The total averaged cross-sections may be computed from

$$\bar{Q}^{(l,s)} = \frac{1}{2}(\bar{Q}_g^{(l,s)} + \bar{Q}_u^{(l,s)})\,. \tag{23}$$

For the charge transfer cross-section Devoto and Li (1968) have fitted Eq. (21) with $A = 20.46$ Å and $B = 0.947$ Å to the values given by Dalgarno (1958) at 0.1 eV and Cramer and Simons (1957) at 5 eV.

c) He—H$^+$ *Collisions*

The ground state $^1\Sigma$ of He H$^+$ is attractive. There are two recent calculations of this potential by Wolniewicz (1965) and Peyerimhoff (1965) the first one being probably the better according to Michels (1966). After subtraction of E_∞ Wolniewicz (1965) is between 5 to 6% higher. However his computations go only to $r = 1$ Å. Thus we use Peyerimhoffs values in our work. A Morse potential fit [Eq. (16)] with $\varepsilon = 1.944$ eV, $C = 1.460$ and $\sigma = 0.5222$ Å was obtained. Charge transfer needs not to be considered in this collision because of the high binding energy of the electron to the He atom.

4. Ionization

For the stellar atmosphere we take the Goldberg-Müller-Aller (1960) abundances by number of 12 elements. For the He abundance we use the two values 10% and 15% of the H abundance. The values taken for the 13 elements are shown in Table 3.

The ionization was computed in local-thermal equilibrium using the Saha equation (Unsöld, 1955, p. 79) and taking the lowering of the ionization potential into account (Griem, 1964, p. 139).

Table 3. *Relative abundances by number of 13 Elements considered*

Powers of ten are in brackets. The elements heavier then He are important only so far as they furnish free electrons at the low temperature range

Element	Abundance ε_i	Element	Abundance ε_i
H	1	Mg	2.5 (−5)
He	0.1, 0.15	Al	1.6 (−6)
C	5.25 (−4)	Si	3.2 (−5)
N	9.55 (−5)	S	2.0 (−5)
O	9.12 (−4)	Ca	1.4 (−6)
Ne	5.02 (−4)	Fe	3.7 (−6)
Na	2.0 (−6)		

A mixture of electrons and 13 elements with their respective stages of ionization gives a tremendous amount of different types of binary collisions to consider. Fortunately because of the low abundance we may neglect all collisions except those of electron, H, He and their ions. In addition we have disregarded any species which has a number density of 10^{-5} of the total number density of the gas. This reduces the order of the determinant in Eq. (10) in almost all cases. The sole purpose of retaining the other elements is that they provide electrons for thermal conduction.

5. Results

Figure 2 shows the thermal conductivity in erg/cm s °K for pure He as function of temperature T with the logarithm of the gas pressure p as parameter. For every value of the gas pressure we have shown the total thermal conductivity (drawn) and the translational contribution (dashed). For comparison the values of Devoto and Li (1968) computed at $p = 10^6$ dyn/cm^2 are given. This illustrates the uncertainty in the basic data. At low temperatures Devoto and Li have taken for the He—He interaction an exponential repulsive [Eq. (15)] fit with $\varepsilon = 196$ eV

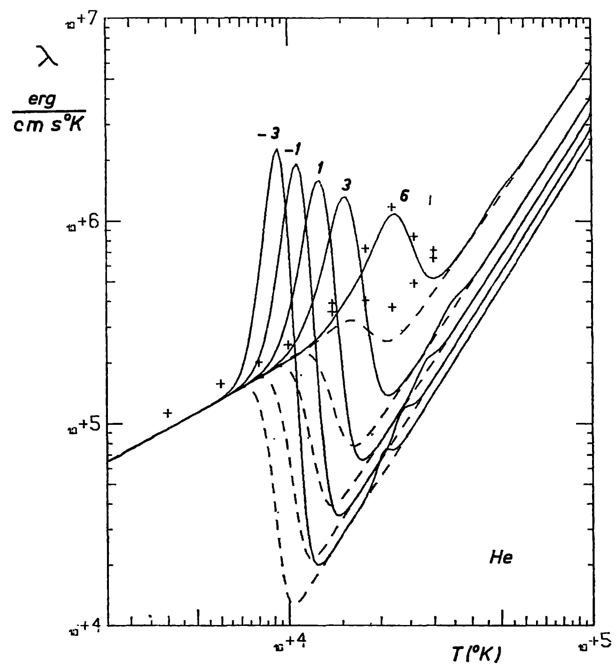

Fig. 2. Thermal conductivity versus temperature of pure He with the logarithm of the gas pressure as parameter (drawn). The translational contribution is shown dashed. The crosses are values given by Devoto and Li (1968)

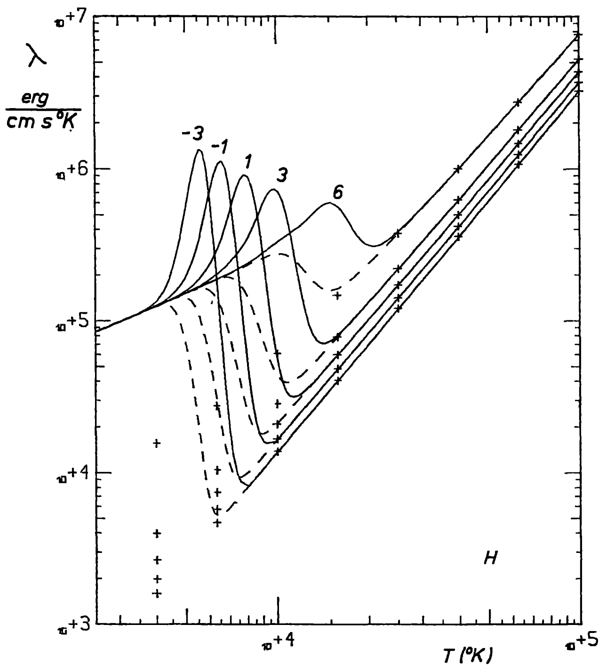

Fig. 3. Thermal conductivity versus temperature of pure hydrogen with the logarithm of the gas pressure as parameter (drawn). The translational contribution is shown dashed. The crosses are values computed with the expressions of Spitzer and Härm (1953)

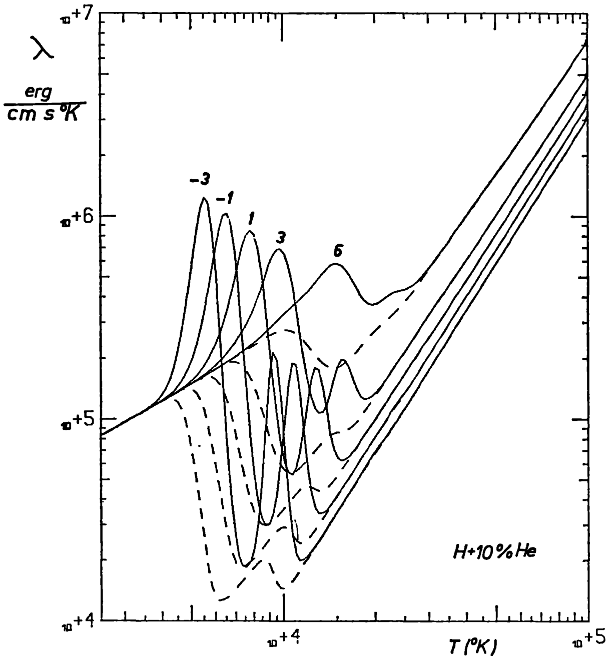

Fig. 4. Thermal conductivity versus temperature of a mixture having the Goldberg-Müller-Aller (1960) abundance and 10% He with the logarithm of the gas pressure as parameter (drawn). The translational contribution is shown dashed

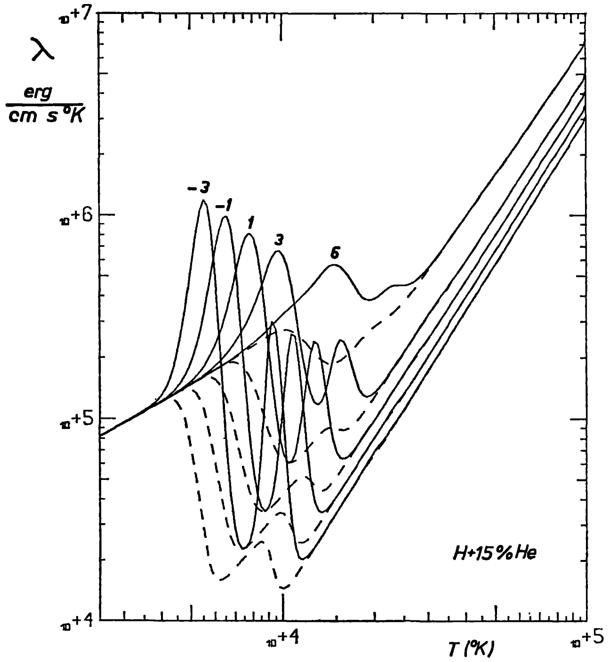

Fig. 5. Thermal conductivity versus temperature of a mixture having the Goldberg-Müller-Aller (1960) abundance and 15% He with the logarithm of the gas pressure as parameter (drawn). The translational contribution is shown dashed

and $\sigma = 0.2368$ Å. This results in an uncertainty of the coefficient of thermal conductivity of $\sim 18\%$ due to the incomplete knowledge of the He–He interaction.

In the ionizing region the difference is much larger mostly because Devoto and Li used the full expressions [Eq. (19)] for the Coulomb average cross-sections. Another difference stems from the different extrapolation of the Morse potential for the He–He$^+$ cross-section. In addition there is the theoretical uncertainty in the computation of the reactive conductivity. This will certainly increase the 35% originating from uncertainty in the averaged cross-sections alone.

Figure 3 shows the thermal conductivity in erg/cm s °K for pure hydrogen. A comparison with Brezing (1965) shows his values in error by about a factor of 3 at low temperatures. A comparison with Vanderslice *et al.* (1962) however shows a good agreement (10%), the difference being due to the different potential fit in our computation. Brezing (1965) has not taken a reactive contribution into account. For the fully ionized region we have made a comparison with Spitzer [1962, Eqs. (5–47)]. His values $K = \varepsilon\, \delta_T K_L$ are shown as crosses. The agreement is essentially complete (0.5%) for the purely electronic contribution $[\lambda_E]_3$, the heavy ion contribution $[\lambda_H]_2$ adds a small amount, such that our total values are 2% above Spitzer's.

Figures 4 and 5 give the thermal conductivity for a stellar atmosphere with 10% He and 15% He respectively.

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