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Thermal Conductivity in Stellar Atmospheres II, without Magnetic Field

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Summary. In this second paper in a series we calculate on basis of the Chapman Enskog Burnett theory the coefficient of thermal conductivity for a gas mixture of stellar abundances assuming departures from local thermodynamic equilibrium (non-LTE). In addition, using the elementary kinetic theory, simple approximation formulas for the coefficient of thermal conductivity are given that allow evaluation under arbitrary ionization conditions and arbitrary element abundance. The deviation of the approximation from the rigorous calculation in both LTE and non-LTE cases is at most ten percent.

Key word: Thermal conductivity

1. Introduction

In the previous paper of this series (Ulmschneider, 1970; henceforth called Paper I) on basis of the Chapman Enskog Burnett theory the coefficient of thermal conductivity λ was computed for a few gas mixtures of astrophysical interest assuming LTE ionization. For many applications however i.e. in the chromosphere-corona transition layer of a star the assumption of LTE is not valid. In these cases the ionization occurs under conditions departing by various degree from thermodynamic equilibrium. In the transition layer the equilibrium between the gas constituents is governed primarily by the balance of collisional ionization and spontaneous radiative recombination (Zirin, 1966, p. 141). For this reason a recomputation of the coefficient of thermal conductivity using an appropriate ionization law is necessary. This is carried out in Section 2.

For application in hydrodynamic stellar atmosphere calculations the presentation of the coefficient of thermal conductivity in form of figures like in Section 2 and in Paper I, due to the rapid variation of the coefficient, is relatively cumbersome to use. Besides, different degrees of departures from LTE and different sets of gas mixtures are usually desired. For this reason we decided to approximate our rigorous calculations with a formula based on the elementary kinetic theory. This approximation formula is derived in Section 3. The maximum deviation of this approximation from the rigorous calculation occured at the start of the hydrogen ionization region and was at most 10% both in the cases of LTE and non-LTE ionizations. The approximation formula can therefore be taken to evaluate λ in situations of arbitrary departure from LTE and for any mixture of H and He.

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2. Thermal Conductivity under Conditions of Non-LTE Ionization

The Chapman Enskog Burnett theory used for the purpose of calculation of the coefficient of thermal conductivity λ has been described in detail in Paper I. For a recomputation of λ it is thus sufficient to describe the different mode of ionization. The corona like ionization equilibrium is a balance of collisional ionization and spontaneous radiative recombination (Zirin, 1966, p. 147). Let $N(i)$ be the number density of an ion of stage i. Then

$$\frac{N(i+1)}{N(i)} = \frac{C(i)}{A(i)}, \tag{1}$$

where

$$C(i) = 2.7\zeta_i T^{-3/2} e^{-y_i} y_i^{-2} N_e \tag{2}$$

is the collisional ionization rate given by Jefferies (1968, p. 123) based on cross sections of Seaton (1964) and where

$$A(i) = 1.24\,10^{-13} n_i \left(\frac{E_H}{kT}\right)^{1/2} \frac{E_i}{E_H} y_i e^{y_i} E_1(y_i) N_e \tag{3}$$

is the radiative recombination rate given by House (1964) based on calculations by Elwert (1952). Here ζ_i is the number of electrons in the outer shell of ion i, T the kinetic temperature, k the Boltzmann constant, E_i and E_H are the ionization potentials of H and ion i. N_e is the

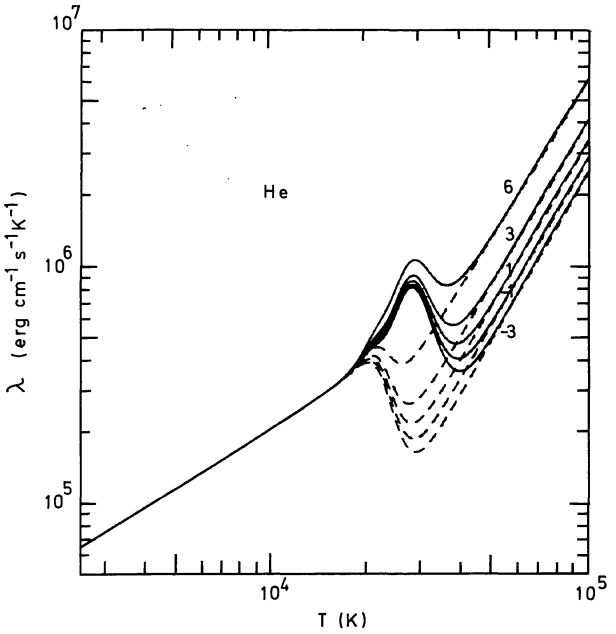

Fig. 1. Thermal conductivity λ of pure He as a function of temperature T with the logarithm of the gas pressure as a parameter (drawn). The translational contribution is shown dashed. The calculations are for a coronalike ionization law

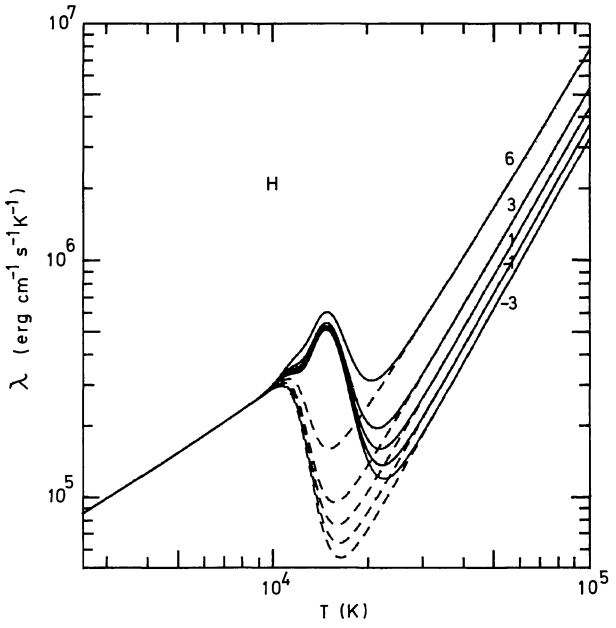

Fig. 2. Thermal conductivity λ of pure H as a function of temperature T with the logarithm of the gas pressure as a parameter (drawn). The translational contribution is shown dashed. The calculations are for a coronalike ionization law

number density of electrons, n_i the main quantum number of the ground level of ion i and y_i is given by

$$y_i = \frac{E_i}{kT}. \tag{4}$$

$E_1(y_i)$ is the exponential integral function. For H, He, He^+ and 11 other elements tabulated in Paper I, relative abundances were computed using Equation (1). With the equation of charge conservation the number densities of all constituents of the gas mixture can be determined if the temperature T and the electron, P_e or gas pressure P are given. With the densities the coefficient of thermal conductivity λ may be computed using Equations (6) to (11) of Paper I. The results are shown in Figures 1 to 4 for pure H, pure He and mixtures based on the Goldberg Müller Aller (1960) abundance (see Paper I) consisting primarily of H and 10% He as well as H and 15% He. Because the coronalike ionization is independent of pressure, the value of λ in the H and He ionization regions is seen to differ considerably from the results for the LTE ionization (Paper I).

3. Approximation Formula for the Thermal Conductivity

a) Translational Thermal Conductivity

Approximate formulas for the translational thermal conductivity λ_{TR} have already been given by Sutherland (1895) and by Wassiljewa (1904) and may be derived from the Chapman Enskog Burnett theory (Brokaw, 1958, 1965). Based on the elementary kinetic theory (Hirschfelder et al., 1964, p. 8f) we may therefore write

$$\lambda_{TR} = \sum_{i=1}^{N} \lambda_i, \tag{5}$$

where

$$\lambda_i = A_i T^{1/2} N_i \Big/ \left(\sum_{k=1}^{N} N_k Q_{ik} C_{ik} \right). \tag{6}$$

Here N_j is the number density of a particle of type j and Q_{ik} the cross section for collisions between particles i and k. A_i is a numerical constant and C_{ik} is a slowly varying function depending primarily on the mass ratio of the particles i and k. In the application to astrophysically important mixtures of H and He we found that the C_{ik} could savely be replaced by constants. The summation in Equation (5) (see Paper I) extends over H, He, H^+, He^+, He^{++}, e and 11 additional elements that are abundant in a stellar atmosphere. As noted in Paper I these 11 elements, because of their low abundance compared to H and He, serve only as source of free electrons but otherwise do not contribute to Equation (5). The contribution of the ions H^+, He^+ and He^{++} in Equation (5) is at most 4% of the electron contribution, λ_e and thus was absorbed in λ_e by adjusting the constants A_e and C_{ek}. Similarly as in Paper I we neglect the collisions between H and He^+, H and He^{++}, and between He and He^{++}. Thus only the H, He and electron contributions are left in Equation (5). The remaining unknowns A_i, C_{ik} were determined from a fit to the rigorous solution. For the H,

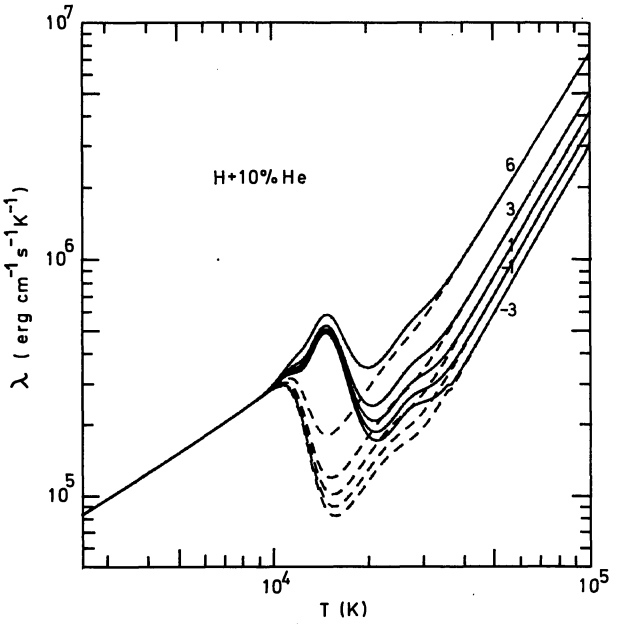

Fig. 3. Thermal conductivity λ as a function of temperature T of a mixture having the Goldberg-Müller-Aller (1960) abundance and 10% He with the logarithm of the gas pressure as a parameter (drawn). The translational contribution is shown dashed. The calculations are for a coronoalike ionization law

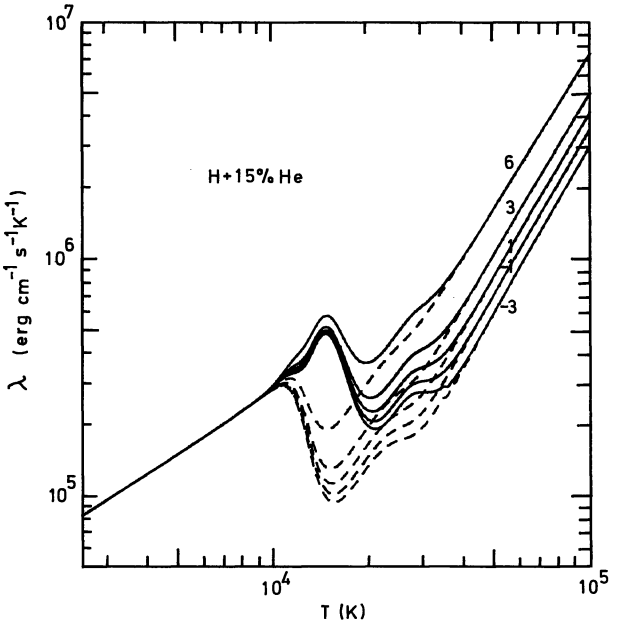

Fig. 4. Thermal conductivity λ as a function of temperature T of a mixture having the Goldberg-Müller-Aller (1960) abundance and 15% He with the logarithm of the gas pressure as a parameter (drawn). The translational contribution is shown dashed. The calculations are for a coronalike ionization law

He, e contributions in erg cm^{-1} s^{-1} K^{-1} we find

$$\lambda_H = 2.4107\,10^{-12}\,T^{1/2}/[Q_{H,H} + (0.85Q_{H,H^+}N_{H^+} + Q_{H,e}N_e + 1.2305Q_{H,He}N_{He})/N_H], \quad (7)$$

$$\lambda_{He} = 1.0486\,10^{-17}T^{1/2}/[Q_{He,He} + (Q_{He,He^+}N_{He^+} + 1.2305Q_{H,He}N_H + 1.3539Q_{He,H^+}N_{H^+} + 0.0723Q_{He,e}N_e)/N_{He}] \quad (8)$$

$$\lambda_e = 6.1159\,10^{-11}T^{1/2}/[Q_{e,e} + (0.4838Q_{e,H^+}N_{H^+} + 0.2478Q_{e,He^+}N_{He^+} + 0.2460Q_{e,He^{++}}N_{He^{++}} + Q_{e,He}N_{He} + Q_{e,H}N_H)/N_e]. \quad (9)$$

Here the Coulomb cross sections have been taken from Spitzer (1962),

$$Q_{ik} = E_1 T^{-2} \ln(E_2 T^2 P_e^{-1/2}), \quad (10)$$

where P_e is the electron pressure in dyn/cm^{2}s and E_1, E_2 are constants given in Table 1. For the remaining cross sections we fitted the relation

$$Q_{ik} = B_0 + B_1 \ln T + B_2 (\ln T)^2 \quad (11)$$

to the averaged cross sections $\bar{Q}_{ik}^{(1,1)}$, taken from Paper I. In the temperature range from 3000 to 60 000 K this fit was better than 4%. The values of B_0, B_1 and B_2 from these fits are given in Table 2.

Table 1. Parameters E_1, E_2 for the Coulomb cross sections of Equation (10)

Cross section	E_1 (cm^2 K^2)	E_2 (dyn$^{1/2}$ cm^{-1} K^{-2})
$Q_{e,e}$, Q_{e,H^+}, Q_{H^+,H^+}	$2.1933\,10^{-6}$	$1.3715\,10^{-4}$
Q_{e,He^+}	$7.9321\,10^{-6}$	$1.0228\,10^{-4}$
$Q_{e,He^{++}}$	$1.7547\,10^{-5}$	$6.8799\,10^{-5}$
$Q_{He^+,He^{++}}$	$3.1734\,10^{-5}$	$5.1007\,10^{-5}$

Table 2. Parameters B_0, B_1 and B_2 for the cross sections of Equation (11)

Cross section	B_0 (cm^2)	B_1 (cm^2)	B_2 (cm^2)
$Q_{H,H}$	$+6.4539\,10^{-15}$	$-8.2913\,10^{-16}$	$+2.3651\,10^{-17}$
Q_{H,H^+}	$+2.5585\,10^{-14}$	$-2.0315\,10^{-15}$	$+4.0371\,10^{-17}$
$Q_{H,e}$	$+1.4204\,10^{-14}$	$-1.7941\,10^{-15}$	$+4.8634\,10^{-17}$
$Q_{He,He}$	$+4.0748\,10^{-15}$	$-5.7824\,10^{-16}$	$+2.0593\,10^{-17}$
Q_{He,He^+}	$+1.2985\,10^{-14}$	$-1.0798\,10^{-15}$	$+2.2483\,10^{-17}$
$Q_{He,e}$	$-3.9942\,10^{-15}$	$+1.0713\,10^{-15}$	$-6.1205\,10^{-17}$
$Q_{He,H}$	$+5.2065\,10^{-5}$	$-7.8033\,10^{-16}$	$+2.9357\,10^{-17}$
Q_{He,H^+}	$+2.0006\,10^{-14}$	$-3.7342\,10^{-15}$	$+1.7502\,10^{-16}$

b) Reactive Thermal Conductivity

The total conductivity λ can be written (Paper I) as

$$\lambda = \lambda_{TR} + \lambda_R \quad (12)$$

where λ_R is the reactive thermal conductivity. The enthalpy difference Δh per particle (Paper I, Eq. 2) is given by

$$\Delta h = 5/2\ kT + E_i \quad (13)$$

where E_i is the ionization energy of the particle that is being ionized. We have from Equation (6) of Paper I

$$\lambda_R = 5.2080\,10^{-12} T^{-3/2}[\alpha_H X_H X_{H^+}(63114+T)^2/Q_{H,H^+} + 0.5\alpha_{He} X_{He} X_{He^+}(114120+T)^2/Q_{He,He^+} + 0.5\alpha_{He} X_{He^+} X_{He^{++}}(252560+T)^2/Q_{He^+,He^{++}}]. \quad (14)$$

Here the concentrations α are given by

$$\alpha_H = (N_H + N_{H^+})/(N_H + N_{H^+} + N_{He} + N_{He^+} + N_{He^{++}}),$$
$$\alpha_{He} = (N_{He} + N_{He^+} + N_{He^{++}})/(N_H + N_{H^+} + N_{He} + N_{He^+} + N_{He^{++}}), \quad (15)$$

while the relative abundances X are given by

$$X_H = N_H/(N_H + N_{H^+}),$$
$$X_{He} = N_{He}/(N_{He} + N_{He^+} + N_{He^{++}}) \quad \text{etc.} \quad (16)$$

c) Discussion

In the approximation formula for the coefficient of thermal conductivity just described, the constants A_i and C_{ik} have been fitted such that the rigorous calculations assuming both LTE and coronalike ionization were closely reproduced. The error of this fit is at most 10% in the ionization regions and much less in other regions. Because the errors in the rigorous calculation have been estimated (see Paper I) to be considerably larger, the approximation can be taken as an almost perfect fit.

The approximation formula moreover can be used for any intermediate type of ionization law and for arbitrary mixtures of H and He. The range of validity may savely be extended to temperatures higher than 10^5 K because the approximation formula goes over into the Spitzer (1962) formula in the limit of complete ionization. At low temperatures the formation of molecular hydrogen terminates the region of validity of our approximation formula. A Fortran subroutine evaluating λ has been written and tested by the authors and may be obtained upon request.

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