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Observational Tests of the Shock Heating Theory for Late-type Stellar Chromospheres ★

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Summary. We combine recent predictions of the positions of stellar temperature minima with a theory of the formation of Ca^+ resonance lines, and thus present observable tests of the shock wave heating theory of stellar chromospheres. Although the trend in the predicted line widths agrees with the trend in the observations, the quantitative agreement is only satisfactory for a solar-type star with $\log g = 4$ and $T_{eff} = 6000$ K. The theoretical minima of giant stars are located much deeper and the minima of cool dwarf stars are located much higher than the observations suggest. We discuss possible explanations of this disparity.

Key words: stellar chromospheres — shock heating — Ca II lines

1. Introduction

In a recent paper Ulmschneider et al. (1977) have predicted the position of the temperature minimum separating photosphere and chromosphere in the atmospheres of late-type stars. The predictions are based on a theory which proposes that chromospheric heating is due to shock dissipation of acoustic waves generated in the subphotospheric convection zone. It is widely believed that this theory is qualitatively correct for the Sun (Ulmschneider and Kalkofen, 1977; Jordan, 1977), although Praderie and Thomas (1976) and Cram (1977) have expressed some reservations. By comparing the theoretical predictions with observations of stellar temperature minima we can check the shock heating theory for a range of atmospheric conditions, and thereby remove quantitative uncertainties in the theory due to poorly-understood scaling factors. Confirmation of the theory would provide strong support for the convection theory and radiation gasdynamic theory underlying the shock heating models; a failure of the theory *could* indicate the need for important revisions in the study of the solar and stellar atmospheres.

Schmitz and Ulmschneider (1977) have proposed a test of the models of Ulmschneider et al. (1977), on the basis of the predicted wavelengths of the minimum continuum brightness temperature produced as a mapping of the electron temperature minimum. Observations of these minima, either in the UV (120–200 nm) or the IR (10–10^4 μm) would provide a reliable test of the theory, but unfortunately the necessary observations must be made from space, and the data are not yet available.

In the visible spectra of late-type stars the most sensitive indicators of the presence of a chromosphere—and hence a temperature minimum—are the cores of the resonance lines of Ca^+. Although uncertainties remain in the theory of the formation of these lines, it is now fairly well established that there is a simple relationship between the shape of the outer edges of the emission cores in H and K and the structure of the temperature minimum in the stellar atmosphere. Observations of the line cores are presently available, and here we use these observations to check the predictions of the shock heating theory.

2. Comparison between Theory and Observation

The emission cores of the Ca II H and K lines are indicators of the presence of a stellar chromosphre (Praderie, 1973). Any property of the emission that is related to the depth of the chromosphere may be used to test the predictions of Ulmschneider et al. (1977). Wilson and Bappu (1957) recognized the two alternative processes which could determine the width of the emission cores: "either the widths are a manifestation of the Doppler effect, i.e. motions, presumably of a turbulent nature, or they are due to abundance broadening as a result of large optical thickness". In the two decades since Wilson and

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Bappu wrote, models based primarily on the Doppler effect have been favoured (Wilson, 1957; Hoyle and Wilson, 1958; Schatzman, 1958; Athay and Skumanich, 1968; Fosbury, 1973; Scharmer, 1976), but only in one case has an explicit line formation mechanism been proposed (Athay and Skumanich, 1968). Recently, Thomas (1973) and Ayres et al. (1975) have advanced a self-consistent variant of the abundance broadening model, based on a non-LTE description of K line formation. This theory provides a link between the width of the K line and the depth of the temperature minimum.

In essence, the model assumes that the K_1 dips in the K line profile are the mapping of a local minimum in a frequency-independent line source function, and that this local minimum is to be identified with the temperature minimum of the star (Jefferies and Thomas, 1960). According to this model the K_1 dips are formed right at the temperature minimum, so that the wavelength position of K_1 ($\Delta\lambda_1 = |\lambda_0 - \lambda_{K_1}|$, where λ_0 is the line centre) is determined by the condition that the monochromatic optical depth to the temperature minimum is unity:

$$\int_{\infty}^{h^*} \rho(h)\kappa_0(h)H\left[a(h), \frac{\Delta\lambda_1}{\Delta\lambda_D(h)}\right]dh = 1\,. \tag{1}$$

Here ρ is the density, κ_0 line centre opacity per gram, H the Voigt function with parameter a, $\Delta\lambda_D$ the Doppler width, and h the geometrical height above the photosphere. An asterisk denotes quantities measured at the temperature minimum. If we assume (1) all Calcium is in the ground state of Ca^+, (2) the damping parameter is determined by radiation damping, and (3) the K_1 feature is formed in the Lorentz wings of the Voigt profile, we can show that

$$\Delta\lambda_1 = \frac{\lambda_0^2}{c}\left[\frac{A_{\mathrm{Ca}}}{\mu m_H}\cdot\frac{\sqrt{\pi}e^2}{mc}\cdot f\cdot\frac{A_{UL}}{4\pi\sqrt{\pi}}\right]^{1/2} m^{*1/2} \tag{2}$$

$$= 1.23 m^{*1/2} (\text{Å};\ m^* \text{ in gm cm}^{-2})\,, \tag{2a}$$

where

$$m^* = \int_{\infty}^{h^*} \rho(h)dh\,. \tag{3}$$

m^* is the mass column density above the temperature minimum, and the numerical factor in (2a) is derived with $A_{\mathrm{Ca}} = 2.0 \times 10^{-6}$ and $f = 0.66$. A recent solar model (Vernazza et al., 1976) gives $m^* = 10^{-1.3}$ gm cm^{-2}, leading to a predicted value of $\Delta\lambda_1 = 280$ mÅ, compared with the observed solar value (Beckers et al., 1976) of 290 mÅ. As we shall see below, this quantitative agreement must be regarded as accidental: we are concerned here more with the *trends* in the theory.

The calculations of Ulmschneider et al. (1977) predict values of m^* for stars with various values of T_{eff} and $\log g$, and we may use the theory outlined above to relate these predictions to the observable quantity $\Delta\lambda_1$. The theoretical values of m^* found by Ulmschneider et al.

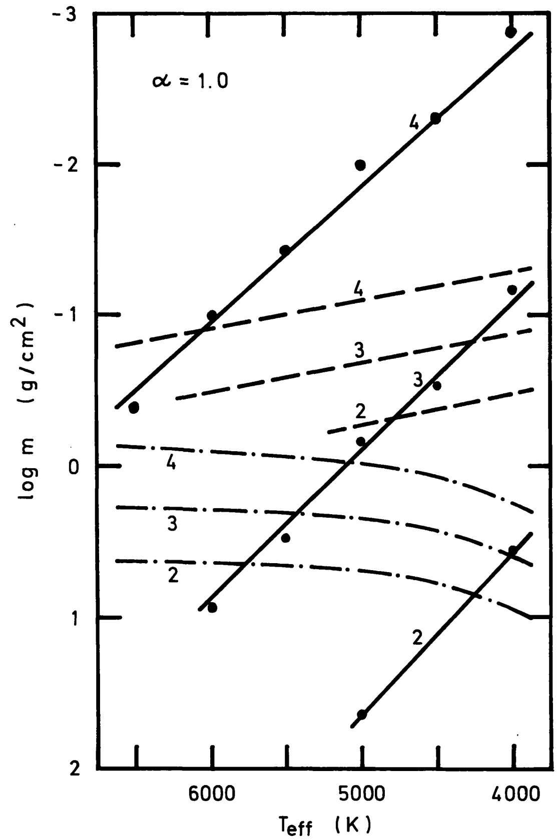

Fig. 1. Theoretical values of the mass column density at the height of shock formation, as predicted by Ulmschneider et al. (1977) are shown as dots. Fit to these theoretical values of m^* (Eq. 4) is shown drawn. Heights of temperature minima inferred from observations of $\Delta\lambda_1$ are shown dashed. Heights of the level where $\tau_{5000} = 0.01$ are shown dash-dot. The curves are labeled with $\log g$

are shown as points in Figure 1. The values are closely matched by the interpolation formula

$$\log m^* = 11.14 \log T_{\mathrm{eff}} - 1.66 \log g - 36.237\,, \tag{4}$$

when α, the ratio of mixing length to pressure scale height, is 1.0 (for $\alpha = 1.5$ the coefficients in (4) are unchanged, and the constant term becomes -35.869). This fit to the theoretical results of Ulmschneider et al. holds with an error $\Delta(\log m^*) < 0.2$ while $\log m^*$ ranges from -2.9 to $+1.6$. Combining (4) and (3) we recover the predicted variation of $\Delta\lambda_1$:

$$\log \Delta\lambda_1 = 5.57 \log T_{\mathrm{eff}} - 0.83 \log g - 18.03\,. \tag{5}$$

The empirical dependence of $\Delta\lambda_1$ on T_{eff} and g has been derived by Ayres et al. (1975), who give

$$\log \Delta\lambda_1 = (1.4 \pm 0.2) \log T_{\mathrm{eff}} - (0.27 \pm 0.04) \log g - 4.58\,. \tag{6}$$

from observations of 7 stars, and by Cram et al. (1977b), who give

$$\log \Delta\lambda_1 = (1.1 \pm 0.2) \log T_{\mathrm{eff}} - (0.20 \pm 0.02) \log g - 3.76\,, \tag{7}$$

from observations of 30 stars. Both of these relationships are based on the work of Reimers (1973), who exhibited a functional relationship between the visual magnitude M_V and ($T_{\rm eff}$, g) for late-type stars. The two independent determinations are statistically consistent within the observational errors, and separate measurements of the same stars show excellent agreement. The scatter of the observations about the regression line is of the order of 0.1 in $\log \Delta\lambda_1$, for $\log \Delta\lambda_1$ ranging from -0.5 to $+0.4$: this scatter may be due to observational errors in measuring $\Delta\lambda_1$, or errors in the inferred values of $T_{\rm eff}$ and g, or intrinsic scatter implying that $T_{\rm eff}$ and g are not sufficient to accurately define $\Delta\lambda_1$ (for example, there may be a dependence on the stellar age or metal abundance; Neckel, 1974).

In Table 1 we compare the theoretical and observational values of $\log \Delta\lambda_1$ for various values of $T_{\rm eff}$ and g. While the *trend* in $\Delta\lambda_1$ with changes in $T_{\rm eff}$ and g is the same for theory and observation, the *quantitative* agreement is satisfactory only for the solar-type star $T_{\rm eff} = 6000$ K, $\log g = 4$. The agreement for this value is not unexpected, because the theoretical prediction of m^* for the Sun agrees well with the value given by semi-empirical models (Ulmschneider and Kalkofen, 1977). However, for no other stellar models is the agreement acceptable, and for the model $T_{\rm eff} = 5000$ K, $\log g = 2$ the error in m^* is almost 2.0 in the logarithm. This corresponds to a difference between an observed value $\Delta\lambda_1$ of order 0.8 Å, and a predicted value of order 8.0 Å. We conclude that either the theory for the formation of the K line is grossly in error, or the theoretical temperature minimum in the model giant star is too deep by 2 orders of magnitude in m^*.

Table 1. Theoretical (first number) and observational (second) values of $\log (\Delta\lambda_1$, Å) for the Ca II K line, for various values of $T_{\rm eff}$ and $\log g$. The theoretical values are obtained using Equation (2b) and values of m^* from Ulmschneider et al. (1977, Table 1); the observational values are derived from Equation (7)

$T_{\rm eff}$(K)	$\log g$		
	2	3	4
6500	—	—	−0.09, −0.37
6000	—	0.56, −0.20	−0.41, −0.40
5500	—	0.33, −0.25	−0.63, −0.45
5000	0.91, −0.09	0.01, −0.29	−0.91, −0.49
4500	—	−0.17, −0.34	−1.06, −0.54
4000	0.37, −0.20	−0.49, −0.40	−1.35, −0.60

3. Discussion

3.A) Uncertainties in the Theory for the Formation of the K Line

The primary assumption underlying the model for the formation of the K line is the validity of the non-LTE theory proposed by Jefferies and Thomas (1960). Studies of the Sun provide some support for this theory, but also raise some doubts. Recent theoretical models based on non-LTE theory predict line profiles that agree well with the observed spatially unresolved line profile at various points across the solar disk, particularly when the assumption of complete redistribution (i.e. a frequency-independent line source function) is relaxed (Vardavas and Cram, 1974; Shine et al. 1975a; Ayres and Linsky, 1976). (We comment below on the modifications required to account for the effects of partial redistribution.) The agreement between theory and observation could be construed as confirmation of the non-LTE theory, but such a conclusion would be premature in the light of spatially resolved observations of the solar K line.

When the solar K line is observed with very high spatial, spectral, and temporal resolution, it is seen that the doubly-peaked emission core is a statistical effect produced by averaging over fine structures which emit time-dependent, highly asymmetrical line profiles (Zirin, 1966, p. 288; Liu, 1972). After studying such high quality spectra, Pasachoff (1970, 1971) concluded that the doubly peaked emission core in the mean profile is produced by averaging over individual fine structures which emit either a single red or a single blue peak, but almost never both at the same point. He suggested that these line profiles were produced as intrinsically narrow emission lines superimposed on a broad photospheric absorption. The emission lines were formed in rising and falling elements, tentatively identified with spicules. Pasachoff's model is a well-defined (most are not so explicit) member of the class of Doppler broadening models often used in discussions of K line formation. If the model is valid it vitiates the non-LTE models and rules out our use of $\Delta\lambda_1$ as a measure of m^*.

We are reluctant to accept the Doppler broadening model of Pasachoff, for two main reasons. First, we know from observations of the solar IR and UV continua that the chromosphere is *very* thick in the K line ($\tau_0 > 10^3$). This ensures that abundance broadening will be important and strongly suggests that the K-line emission (unexplained by Pasachoff's model) is due to coupling between the non-LTE line source function and the chromospheric temperature rise. Second, studies of the temporal changes of K line profiles in high quality spectra have revealed patterns of evolution from singly to doubly-peaked profiles (and vice versa) which cannot be easily explained by the Doppler shift model, but which can be explained in terms of the effects of wave-like chromospheric velocity fields on the non-LTE line profile (Durrant et al., 1976; Cram et al., 1977a). It appears that the non-LTE model can explain both the unresolved and the resolved solar K line, when it is suitably extended to include the chromospheric velocity field.

Thus, we are inclined to accept the basic model for the formation of the K line as outlined in Section 1; however, we should consider the validity of several assumptions before accepting Equation (1):

(i) Frequency-independent Source Function

Quantitative comparisons of CRD (complete redistribution, frequency-independent line source function) and PRD (partial redistribution, frequency-dependent source function) line profiles for a range of stellar models are not yet available. However, Ayres et al. (1975) express the view that with PRD "the scaling with m^* is essentially unchanged from the CRD results". The conclusion is supported by comparisons of PRD and CRD line profiles for the Sun (Shine et al. 1975a, Fig. 3), Arcturus (Ayres and Linsky, 1975, Fig. 7) and two models exhibited by Shine et al. (1975b).

(ii) Chromospheric Velocity Fields

We have seen that the solar K line fine structure can be explained by combining non-LTE theory with chromospheric velocity fields. In plane-parallel homogeneous models designed to explain unresolved observations, these velocity fields are treated as micro- and macroturbulent broadening. We have little confidence in this approach, because the resolved observations show that most small-scale line profiles are not particular realizations of a micro- or macro-turbulent ensemble (for example, these models do not account for any asymmetric profile). However, the effect of the chromospheric velocity field is most obvious in the K_{232} parts of the line profile, and the K_1 part of the line profile is not so strongly affected by these motions. This follows because the K_1 feature is formed far from the Doppler core, so that the line absorption coefficient is only a weak function of the velocity gradient. The argument is supported by studies of the cross-correlation coefficient between $\Delta\lambda_{1V}$ and $\Delta\lambda_{1R}$, which show that these dips tend to move together or apart in unison (Grossman-Doerth et al., 1974). This implies that the fluctuations in $\Delta\lambda_1$ observed in the solar K line are due to changes in m^* along the line of sight, and not directly to the Doppler effect. Even if there is a Doppler contribution to the stellar K_1 widths, it could only increase the line width and worsen the discrepancy we have found.

(iii) Occupation of the Ground State of Ca^+

We have assumed that all Calcium is in the ground state of Ca^+, and that the Ca abundance is the same for all stars. Clearly, variations in Ca abundance cannot account for the factor of 10^2 correction we require to bring theory and observation together. The ionization potential of Ca° is only 6.11 eV and it seems unlikely that significant formation of neutral Ca will occur in the stars we are considering (see Linsky, 1968, Table 4-II). There will of course be significant depletion of Ca^+ where the stellar chromospheric temperature rises above 10^4 K (I.P. of $Ca^+ = 11.87$ eV), but we would not expect this to occur in the first few scale heights above the temperature minimum, where most of the absorption occurs. Note that formation of either Ca° or Ca^{++} will tend to reduce $\Delta\lambda_1$, but there seems no reason to expect the 10^2 depopulation of Ca^+ required to account for our problem.

(iv) Line Absorption Profile in the Wings

We have assumed that the K_1 dip is formed in the radiation damping part of the Lorentz wings of the Voigt profile. For any reasonable value of the Doppler width the Lorentz form is valid, but for large values of m^* the hydrogen density may be so large that Van der Waals broadening may compete with radiative broadening. This would tend to increase the theoretical predictions of $\Delta\lambda_1$.

We conclude that uncertainties in our theory for the formation of the K line are unable to account for the disagreement between the theoretical predictions and the observed values of $\Delta\lambda_1$. The greatest uncertainty is due to the assumption of CRD, but it seems to be unlikely that a PRD model would reduce the theoretical width by as much as a factor of two. This would still leave a disparity of a factor of 30 between theoretical and observational values of m^*.

3.B) *Uncertainties in the Theoretical Temperature Minima*

We focus on two main sources of uncertainty in the theoretical prediction of temperature minima:

(i) Calculation of Acoustic Noise Generation

The acoustic fluxes used by Ulmschneider et al. (1977) were predicted by Renzini et al. (1977) on the basis of the Lighthill method for studying the generation of sound by turbulence, and a mixing-length model of stellar convection. According to this theory the generated acoustic flux is given by

$$\pi F_m \sim v^8/\bar{v}^5 \tag{8}$$

(in the notation of Renzini et al.). We may relate F_m to the height of shock formation by noting that the hydrostatic equilibrium equation is

$$P(\tau) = gm(\tau)\,. \tag{9}$$

Renzini et al. (1977) have shown that an estimate of the position of shock formation in the absence of radiation damping is given by

$$\rho^* v_s^{*2} = \pi F_m\,. \tag{10}$$

This may be combined with the perfect gas equation and Equation (9) under the assumption that gas pressure (rather than turbulent or radiation pressure) is responsible for supporting the atomosphere to give

$$m^* \sim F_m T_{\mathrm{eff}}^{1/2} g^{-1} . \tag{11}$$

This gives, with the scaling law (Renzini et al.)

$$\pi F_m \sim \alpha^{8/3} g^{-1} T_{\mathrm{eff}}^{15} , \tag{12}$$

the relation

$$m^* \sim \alpha^{8/3} g^{-2} T_{\mathrm{eff}}^{15.5} , \tag{13}$$

which is in reasonable agreement with the more accurate result (4).

There are, of course, many uncertainties underlying this prediction. The essence of Lighthill's method is retained in modern studies of noise emission from jet engines (Mani, 1976), and for these flows there is good agreement between theoretical and observational scaling laws. Stein (1968) has discussed the modifications that must be made to Lighthill's method to account for stratification, gravity, and finite source volume, and there does not appear to be any major objection to a careful application of Lighthill's method to the study of wave generation in stellar atmospheres. However, both in laboratories and in stellar atmospheres there is great uncertainty about the description of the turbulence which enters the source terms in Lighthill's theory.

Calculations of acoustic noise generation in stellar atmospheres are invariably based on a mixing-length model of convection. As can be seen from the work of Renzini et al. (1977) and others, the acoustic energy generation is extremely sensitive to the mean turbulent velocity $\bar{v}$ in a region that extends about one scale height below the top of the convection zone, the level where $\tau = 1$. It is precisely this region where radiative energy exchange, partial Hydrogen ionization, strong stratification, and other factors can act most strongly to modify the mixing-length model of convection. The value of $\bar{v}$ is therefore uncertain, and consequently there is a large uncertainty in the predicted acoustic flux. Furthermore, as noted by Ulmschneider and Kalkofen (1977), the height of shock formation depends not only on the magnitude of the flux but also on the period of the acoustic waves, because radiative damping acts more strongly on short period waves. The assumptions and simplifications underlying the calculations of Renzini et al. (1977) could thus have an important effect on the predicted heights of shock formation, and a more detailed study based on an integration of the spatial and temporal turbulence spectra as discussed by Stein (1968) could remove some of these problems. At present it appears that a 10^2 error in m^* could be accounted for by uncertainties in the predicted acoustic energy emission. There is a clear need for refinement of this section of the theory.

(ii) Location of the Temperature Minimum

A systematic error may be introduced into the predictions of Ulmschneider et al. (1977) by the identification of the height of shock formation with the height of the temperature minimum. As noted by Ulmschneider et al. the temperature minimum is expected to lie above the height of shock formation for those models in which the shocks are formed in the radiation damping zone, because the dissipated wave energy can be efficiently carried away by a very small increase in the temperature. Thus it is unlikely that mechanical energy dissipation will produce a significant temperature rise until the radiation damping zone is passed, and this suggests that there may be an upper limit to m^* imposed by the condition that the temperature can rise only for sufficiently small optical depths. To illustrate the importance of this effect we compare in Figure 1 the column densities to the levels $\tau = 0.01$ with the heights of shock formation in the models studied by Ulmschneider et al. Since $\tau = 0.01$ probably represents an upper limit to the optical thickness above the radiation damping zone, it is clear that a large decrease in the predicted value of m^* for the models $(T_{\mathrm{eff}}, \log g) = (5000, 2)$, $(5500, 3)$, and $(6000, 3)$ is indicated. This change would tend to improve the agreement between theory and observation.

A systematic correction to m^* might also be required for those models in which the shocks are formed at very small optical depths, where Ulmschneider et al. were forced to extrapolate the tabulated models. As emphasized by Cram (1977), a reliable prediction of m^* requires a detailed study of the interplay between mechanical heating, line blanketing and non-LTE effects. It is possible that line blanketing would lead to a more rapid temperature decrease than that used by Ulmschneider et al., and this could lead to deeper heights of shock formation. There may also be a radiatively induced temperature reversal (Cayrel, 1963) which would lead to a temperature minimum whose position is essentially unrelated to the height of shock formation.

It is clear that the reliability of the predictions of Ulmschneider et al. (1977) could be improved by a more detailed study of the interplay between the various processes that determine the position of the temperature minimum. Such a study is indeed possible, in contrast to the study of acoustic noise generation where there is little hope for an improved theory for convection in the near future.

4. Conclusion

We have combined theoretical predictions of the positions of stellar temperature minima with a theory for the formation of Ca II resonance lines to provide an observable test of the shock heating theory of stellar chromospheres. We have found:

(i) Both theory and observation show that the K line width parameter $\Delta\lambda_1$ increases with increasing T_{eff} and with decreasing g.

(ii) The quantitative agreement for a solar-type model is satisfactory.

(iii) There is a large discrepancy between theory and observation for models in which the shock formation heights occur for optical depths greater than $\tau = 0.01$. It is very probable that a detailed calculation of the position of the temperature minimum taking account of the radiative damping of the developing shock would considerably reduce the discrepancy.

(iv) There is a large discrepancy between theory and observation for the cool dwarf models in which the shocks form above $\tau = 10^{-5}$. There is a pressing need for improved models for the outermost layers of such stars, and such models could bring theory and observation closer together.

Our study has confirmed the value of the Ca^+ resonance lines as diagnostic probes of stellar chromospheres, and a combination of observations made in these lines with observations of the radiation temperature minima in the UV and IR (Schmitz and Ulmschneider, 1977) would provide a great deal of important information on the transition from photosphere to chromosphere in late-type stars. Note that if we accept the validity of the shock heating theory the method we have used here is *in principle* sensitive to a 3% error in subphotospheric velocities, since we can measure $\Delta\lambda_1$ with an error of 10%.

We have shown that the uncertainties of the theoretical predictions are intimately connected with uncertainties in the theory of convection in stars on the one hand, and with approximations made in the treatment of the structure of the shock heated atmosphere on the other. It is comparatively easy to improve the second aspect, and with improved models it might be possible to confirm or refute the shock heating theory and to eventually proceed to an improvement of convection theory.

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