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ACOUSTIC DISSIPATION AND H⁻ RADIATION IN THE SOLAR CHROMOSPHERE. II

W. KALKOFEN
Harvard-Smithsonian Center for Astrophysics
AND
P. ULMSCHNEIDER
Institut für Astronomie und Astrophysik, Würzburg
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ABSTRACT

The relation between mechanical heating and radiative cooling in the solar chromosphere is investigated, and the dependence of the temperature rise on the mechanical dissipation rate and on the microscopic state of H^- ions at the foot of the chromosphere is studied. It is found that H^- is very nearly in local thermodynamic equilibrium (LTE) at the temperature minimum; but near the upper end of the range where H^- dominates the opacity, deviations from LTE should be taken into account in estimating the mechanical energy input from the empirical temperature distribution.

The use of empirical models for calculating the mechanical energy input into the chromosphere is considered. The estimate of mechanical heating implied by the recent work of Praderie and Thomas is found to be too low by about an order of magnitude.

Subject headings: atomic processes — opacities — Sun: atmosphere — Sun: chromosphere

I. INTRODUCTION

The outer layers of the solar atmosphere are believed to be heated by mechanical waves, generated in the hydrogen convection zone, that travel outward and dissipate in the chromosphere and corona (Biermann 1946; Schwarzschild 1948). At least half the mechanical energy reaching the layers that are optically thin in the continuum is estimated to be dissipated in the high-density regions at the foot of the chromosphere just above the temperature minimum. However, two studies by Praderie and Thomas (1972, 1976) have cast doubt on this statement. Their work appears to show that the mechanical energy input by wave heating into the low chromosphere is smaller than usually estimated, and by a substantial factor. If true, their conclusion would imply that the heating of the low chromosphere makes a minor contribution to the total mechanical energy budget of the outer layers of the solar atmosphere.

The purpose of this paper is to investigate the relation between wave heating, the temperature rise in the chromosphere, and the radiative emission from the mechanically heated layers. First we study the radiation emitted from a heated slab with gray opacity when the source function is increased by a known amount. Then we consider the temperature rise caused by wave heating when the opacity is due to H^- ions in statistical equilibrium and compare our result with the special cases studied by Praderie and Thomas. Finally we discuss the use of models of the solar atmosphere for the purpose of calculating the excess radiative emission due to wave heating. In the Appendix we derive an equation for calculating the excess radiative flux from the low chromosphere and give definitions and numerical values of the radiative cooling rate and the photorecombination rate for H^-.

II. THE NET COOLING RATE

We consider the idealized model of the solar atmosphere of Praderie and Thomas (1972, 1976), which consists of a gray, semi-infinite, plane-parallel atmosphere in radiative equilibrium with a mechanically heated chromosphere near the surface. The chromosphere is represented by a finite slab of optical thickness τ_c covering the depth range $0 \le \tau \le \tau_c$. Dissipation of mechanical waves is assumed to increase the integrated source function by the constant amount δB over its value in radiative equilibrium. The problem is to relate the energy dissipated by the waves to the increase in the emitted radiation.

The radiated flux Φ emitted by the slab can be calculated either by integrating over the net cooling rate,

$$\Phi(0, \tau_c) = \int_{\tau_c}^{0} d\tau \frac{d\delta F}{d\tau} = 4\pi \int_{0}^{\tau_c} d\tau(\delta B - \delta J) , \quad (1)$$

where δJ is the mean integrated intensity, or by computing the net flux, $\delta F(\tau)$, from the specific intensity, δI_μ, by integrating over angle,

$$\delta F(\tau) = 2\pi \int_{-1}^{1} d\mu \mu \delta I_\mu(\tau) , \quad (2)$$

and then taking the flux difference,

$$\Phi(0, \tau_c) = \delta F(0) - \delta F(\tau_c) . \quad (3)$$

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Using the latter method we write for the outward and inward directed intensities in the slab:

$$\delta I_\mu{}^+ = \delta B\{1 - \exp[-(\tau_c - \tau)/\mu]\}\,,$$

$$\delta I_\mu{}^- = \delta B[1 - \exp(\tau/\mu)]\,, \quad \tau \le \tau_c\,, \qquad (4)$$

with the convention $\mu \equiv |\mu|$ for the inward intensity $\delta I_\mu{}^-$. The angle integral (2) of I_μ yields the net flux

$$\delta F(\tau) = 2\pi\delta B[E_3(\tau) - E_3(|\tau_c - \tau|)] \qquad (5)$$

for an isolated slab, where $0 < \tau < \tau_c$, as well as for a slab overlying an atmosphere. In the latter case the net flux below τ_c is obtained by observing that $\delta I_\mu{}^+ = 0$ for $\tau \ge \tau_c$ and that $\delta I_\mu{}^-$ decays exponentially with depth from the boundary $\tau = \tau_c$. The radiative flux at the upper and lower boundaries of the slab is given by

$$\delta F(0) = 2\pi\delta B[\tfrac{1}{2} - E_3(\tau_c)]\,, \qquad \delta F(\tau_c) = -\delta F(0)\,. \qquad (6)$$

The total net cooling rate (3) of the slab is therefore

$$\Phi(0, \tau_c) = 4\pi\delta B[\tfrac{1}{2} - E_3(\tau_c)]\,. \qquad (7)$$

The isolated slab loses one half of the energy at the top, $\tau = 0$, and the other half at the bottom, $\tau = \tau_c$. If the slab overlies a semi-infinite atmosphere, the inward flux, $\delta F(\tau_c)$, heats the lower layers, thereby increasing the outward flux from the atmosphere. In the layers $\tau > \tau_c$ this reflected flux exactly compensates for the inward emitted flux. All the emission (7) caused by the mechanical wave dissipation then appears as energy flux lost from the atmosphere at $\tau = 0$.

The source-function increase in the lower layers which drives the additional outward flux is very small. It can easily be estimated from the inward intensity (4). In the two-stream approximation at the Eddington value of the angle, where $\mu = 1/\sqrt{3}$, the inward intensity just below the heated slab is equal to $3^{1/2}\delta B\tau_c$. Since the reflection from the deeper layers contributes an equal amount to the mean intensity and since heating and cooling balance in the steady state, the source function rise is equal to $3^{1/2}\delta B\tau_c$. Thus, the ratio of increases in the source function on either side of the boundary between the atmosphere and the slab is $3^{1/2}\tau_c$; for a very thin chromosphere, the source function in the underlying atmosphere has therefore practically the radiative equilibrium value.

When the slab is optically thin, the exponential integral in equation (7) may be expanded. The radiative energy flux Φ lost from the slab may then be written as

$$\Phi(0, \tau_c) = 4\pi\delta B\tau_c E_2(\tau_c/2)\,, \quad \tau_c \lesssim 0.1\,,$$

and

$$\Phi(0, \tau_c) = 4\pi\delta B\tau_c\,, \qquad \tau_c \ll 1\,. \qquad (8)$$

The expressions indicate that the emitted flux Φ, and therefore the inferred mechanical heating, depends only on the total emission in the slab and is independent of the detailed structure of the increased emission, provided the slab is very thin.

Since the solar temperature minimum lies near $\tau = 10^{-4}$, the chromosphere satisfies the condition $\tau_c \ll 1$, at least for the H^- opacity, suggesting that the model of Praderie and Thomas is a valid representation of the solar chromosphere.

The result (8) for the mechanical energy dissipated in the chromosphere was given by Ulmschneider (1970, 1974). Praderie and Thomas (1972) determined the emergent flux $\delta F(0)$ given by equation (6) and asserted that it measured the dissipated wave energy. They also calculated the flux at the bottom of the chromosphere; from the sign of the flux $\delta F(\tau_c)$ they concluded that it implied mechanical cooling. They therefore rejected it and took $\delta F(0)$ as the total flux of the mechanical waves, stating (1976) that the expression obtained by Ulmschneider was wrong and was too large by a factor of 2.

The conclusion of Praderie and Thomas (1976, p. 341) is due to an inconsistency in the calculation of the radiative cooling rate (1972, p. 486): For the inward-directed intensity they used their chromospheric model, in which the source function is increased by the amount δB in the thin layer $0 \le \tau \le \tau_c$; for the outward intensity, however, the source-function increase extended throughout the atmosphere, $0 \le \tau < \infty$. The outward-directed intensity therefore contributed no net heating; and since the specific intensity in the chromospheric model discussed here is symmetric about the midplane, $\tau = \tau_c/2$, and the net flux antisymmetric, their neglect of net heating due to the outward radiation led to an error in the calculation of energy emitted by the slab of a factor of 2 exactly.

III. THE CHROMOSPHERIC TEMPERATURE RISE

The source function and the kinetic temperature in the chromosphere are increased by the dissipation of mechanical energy. The source-function rise is independent of the microscopic state of the gas; the corresponding temperature rise depends, however, on the state of the gas. We now determine the increase of the source function and of the temperature due to wave dissipation. For the temperature rise we assume that the gas is in statistical equilibrium. We then compare our result for the temperature rise in the limits of infinite and vanishing collision rates with the results of Praderie and Thomas.

The transfer equation may be written as (cf. Appendix)

$$\mu\frac{d}{dx}I_{\nu\mu} = -\kappa k_\nu(I_{\nu\mu} - S_{\nu\mu})\,, \qquad (9)$$

where κ is an average or typical opacity, such as the monochromatic opacity at 5000 Å, and k_ν is the absorption profile. For a single absorber with only one bound state, such as H$^-$ with only bound-free transitions, the source function $S_{\nu\mu}$ may be written as

$$S_{\nu\mu} = \frac{1}{b} W_\nu \left(1 + \frac{I_{\nu\mu}}{2h\nu^3/c^2}\right) , \tag{10}$$

where W_ν is the monochromatic Wien function (cf. eq. [A6]) and b is the H$^-$ departure coefficient (cf. eq. [A7]).

We define the average optical depth,

$$d\tau = -\kappa dx , \tag{11}$$

and the angle and frequency integrals of the intensity,

$$\begin{aligned} F_r &= 4\pi \int_0^\infty d\nu \frac{1}{2} \int_{-1}^{1} d\mu \mu I_{\nu\mu} , \\ J &= \int_0^\infty d\nu k_\nu \frac{1}{2} \int_{-1}^{1} d\mu I_{\nu\mu} , \\ S &= \int_0^\infty d\nu k_\nu \frac{1}{2} \int_{-1}^{1} d\mu S_{\nu\mu} . \end{aligned} \tag{12}$$

The integrated transfer equation for nongray opacity is then

$$\frac{dF_r}{d\tau} = 4\pi(J - S) . \tag{13}$$

In radiative equilibrium the radiative flux, F_r, is conserved. In an atmosphere with a mechanically induced chromospheric temperature rise, it is the sum of the radiative and mechanical fluxes that is constant:

$$F_r + F_m = \text{const} . \tag{14}$$

The dissipation of the wave flux F_m heats the gas. In the steady state the increased emission rate per unit optical depth and solid angle is expressed by the source term Q,

$$4\pi Q = \frac{d}{d\tau} F_m . \tag{15}$$

The energy equation for the mechanically heated atmosphere may therefore be written as

$$S = J + Q , \tag{16}$$

which balances the local rate of radiative cooling against the rates of radiative and mechanical heating.

Since the model chromosphere starts at a very shallow optical depth, the intensity J is practically unchanged from its value in radiative equilibrium for which heating, J, and cooling, S_0, balance each other. The chromospheric source function rise, δB, is therefore given by

$$\delta B \equiv S - S_0 = Q , \tag{17}$$

where S_0 and S are the source functions in the radiative equilibrium and the mechanically heated atmospheres, respectively. Thus the increase of the emission rate is equal to the dissipation rate per unit optical depth,

$$4\pi \delta B = dF_m/d\tau . \tag{18}$$

From equations (8) and (18) the total radiative cooling rate is seen to equal the rate at which mechanical energy is dissipated in the chromosphere,

$$\Phi(0, \tau_c) = \int_0^{\tau_c} d\tau \frac{dF_m}{d\tau} , \tag{19}$$

as is expected from energy conservation. The radiative flux (6) lost at the surface $\tau = 0$ of an optically thin chromosphere is

$$\delta F(0) = \tfrac{1}{2}\delta F_m , \qquad \delta F_m = (dF_m/d\tau)\tau_c . \tag{20}$$

Equation (20) shows that the slab emits only half the dissipated mechanical flux at the upper boundary, and it emphasizes again that Praderie and Thomas account for only half the wave energy dissipated in the chromosphere. Therefore, as noted by Athay (1965) and Ulmschneider (1974), estimates of the mechanical energy input to the chromosphere must take into account energy radiated at the front and back faces of the chromosphere.

The temperature rise caused by the mechanical heat input depends on the opacity and on the state of the gas. At the temperature minimum and the foot of the chromosphere the main opacity source is the H$^-$ ion. At the highest density, H$^-$ is nearly in local thermodynamic equilibrium (LTE) because of the collisional control of H$^-$ dissociation; in higher layers in the chromosphere the state of H$^-$ is strongly influenced by the intensity of the radiation field.

The thermal response of the gas to the mechanical energy input is bounded by the case of LTE in the high-density limit and that of statistical equilibrium without collisions in the low-density limit. In the high-density limit the dissipated energy is shared equally among all degrees of freedom of the particles and the temperature rise is relatively low; in the low-density limit, on the other hand, most of the added energy is transferred to the random motions of the particles but the internal degrees of freedom do not receive the full share and the degree of dissociation of H$^-$ is increased much less than it would be if the gas were in LTE; a much larger temperature rise is therefore needed in order to radiate away the dissipated wave energy.

We now determine the temperature rise in the general case of statistical equilibrium. We then specialize the solution for the high- and low-density limits and compare the results with the equations of Praderie and Thomas. In this discussion we assume that the opacity is due to H$^-$ bound-free transitions only. The temperature rise is obtained from the relation (17) between the source function rise δB and the mechanical heating rate Q. The integrated source function S, which is given by

$$S = \frac{1}{b} \int d\nu k_\nu W_\nu \left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right) , \tag{21}$$

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may be written as the ratio of the frequency integral B,

$$B = \int d\nu k_\nu W_\nu \left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right), \qquad (22)$$

which depends on kinetic temperature, and the departure coefficient, b, which in general depends on temperature and density:

$$S = B/b . \qquad (23)$$

We evaluate the frequency integral (22) with the aid of the polynomial expansion of Gingerich (1964) for the photodissociation cross section of H^-, using the correction given by Ulmschneider and Kalkofen (1978) of a misprint in Gingerich's formula. For the induced emissions we assume that the photospheric radiation field may be described by a dilute Planck function,

$$J_\nu = WB_\nu(T), \quad W = \tfrac{1}{2}, \quad T = 5800\ \mathrm{K}. \qquad (24)$$

Representing the frequency integral (22) by a power law for the kinetic temperature, the function B may be written as

$$B \sim T^m, \quad m = d\ln B/d\ln T . \qquad (25)$$

Column (2) of Table 1 lists the exponent m for several values of the kinetic temperature (see also the Appendix, following eq. [A13]).

The departure coefficient b is determined from the equation of statistical equilibrium for the H^- ion which balances the rates of formation and destruction of ions in the bound state. For solar conditions the important rates are those for photo dissociation and associative detachment and for the corresponding inverse processes. We write the net rate $\mathscr{R}$ of photodissociation as

$$\mathscr{R} = n_{H^-}R_i - n_{H^-}{}^*R , \qquad (26)$$

where R is the radiative recombination rate per ion, defined by equation (A11) in the Appendix, and R_i is the photo ionization rate without the correction for induced emission, which is included in the recombination rate. The net collisional rate $\mathscr{C}$ for associative detachment, $H + H^- \leftrightarrow H_2 + e$, is

$$\mathscr{C} = (n_{H^-} - n_{H^-}{}^*)n_H\Gamma , \qquad (27)$$

where n_H is the hydrogen density and Γ the rate coefficient for associative detachment, given by Browne and Dalgarno (1969) and quoted by Vernazza, Avrett, and Loeser (1973) as

$$\Gamma = 2.10 \times 10^{-9} . \qquad (28)$$

For the two processes considered, the equation of statistical equilibrium of H^- is

$$\mathscr{R} + \mathscr{C} = 0 , \qquad (29)$$

with the formal solution for the departure coefficient b,

$$d\ln b = (1 - \epsilon)d\ln R , \qquad (30)$$

where the collision parameter ϵ,

$$\epsilon = \frac{n_H\Gamma}{R + n_H\Gamma} , \qquad (31)$$

is defined in terms of the photo recombination rate R in the heated atmosphere. The rate R, defined by equation (A11), may be evaluated with the polynomial for the photodissociation cross section, giving

$$R \approx T^n , \quad n = d\ln R/d\ln T . \qquad (32)$$

Column (3) of Table 1 lists the exponent n for several values of the kinetic temperature and equation (A13) gives the value of R at $T = 4000$ K so that the recombination rate per ion at other values of the temperature can be calculated.

TABLE 1

TEMPERATURE DEPENDENCE OF THE COOLING RATE E AND OF THE RECOMBINATION RATE R

T(K) (1)	$m = d\ln E/d\ln T$ (2)	$n = d\ln R/d\ln T$ (3)
3000	5.34	4.97
3500	4.92	4.52
4000	4.60	4.18
4500	4.35	3.91
5000	4.15	3.69
5500	3.98	3.50
6000	3.85	3.35
6500	3.73	3.22
7000	3.63	3.11

Now, the energy equation (17) may be written in terms of the temperature rise as

$$\frac{dT}{T_0} = \frac{1}{d\ln S/d\ln T}\frac{Q}{S_0} , \qquad (33)$$

where T_0 is the kinetic temperature in the radiative equilibrium atmosphere. From the expression (23) for the source function S and the definitions (25), (30), and (32), the dependence of S on temperature and collision frequency is obtained as

$$d\ln S/d\ln T = m - (1 - \epsilon)n . \qquad (34)$$

The temperature rise in the statistical-equilibrium atmosphere due to mechanical energy dissipation is therefore given by

$$\frac{dT}{T_0} = \frac{1}{m - (1 - \epsilon)n}\frac{Q}{S_0} . \qquad (35)$$

The equation shows that the temperature rise is high when the dissipation rate Q is high. Even when the value of the mechanical energy flux F_m is small compared with that of the radiative flux F_r, the temperature increase is large if the mechanical flux is dissipated in a layer of sufficiently small optical thickness. In the case of the Sun, for example, the mechanical flux reaching the temperature-minimum region is only about 10^{-4} times the radiative flux. But since the chromosphere is optically thin in the continuum, with

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$\tau_{5000} \approx 10^{-4}$, the chromospheric temperature rise is large. Equation (35) also shows that the temperature rise is lowest when the gas is in LTE, where $\epsilon = 1$, and highest when the microscopic state of the opacity source is completely determined by the radiation field, with collisions playing no role, where $\epsilon = 0$; and, conversely, when the observed temperature increase above the radiative equilibrium value is used to estimate the mechanical dissipation rate Q and the mechanical flux F_m, the inferred value of F_m is highest in LTE and lowest in the collisionless case $\epsilon = 0$.

Equation (35) expresses the relative temperature rise as the ratio of the dissipation rate of mechanical energy flux in the chromosphere and the radiative flux emitted from the whole atmosphere. The scale factor depends on the microscopic processes that determine the state of the particles contributing the opacity. The source function S_0, which is proportional to the cooling rate per H^- ion in the radiative equilibrium atmosphere, is independent of the state of the gas since it balances the radiative heating rate J per ion. But the undisturbed temperature T_0 depends on the state of the gas. However, near the temperature minimum, where B is proportional to $T^{4.49}$ (see below), T_0 depends only on the 4.49th root of the departure coefficient b. The dependence of the equilibrium temperature T_0 on the state of the gas is therefore quite weak.

Consider now conditions in the solar atmosphere at the foot of the chromosphere, where the kinetic temperature $T_0 \approx 4200$ K. The temperature dependence of the cooling rate and of the photorecombination rate is given by $m = 4.49$ and $n = 4.06$, and the relative radiative recombination rate, listed in the last column of Table 2, is $(1 - \epsilon) = 0.21$. The fractional temperature rise for several values of the collision parameter ϵ is

$$\left(\frac{dT}{T_0}\right)_\epsilon \begin{aligned} &= 0.22Q/S_0\,, \quad \epsilon = 1\,,\\ &= 0.27Q/S_0\,, \quad \epsilon = 0.79\,,\\ &= 2.33Q/S_0\,, \quad \epsilon = 0\,, \end{aligned} \tag{36}$$

and the temperature rise relative to that in the LTE case is

$$\frac{(dT/T_0)_\epsilon}{(dT/T_0)_{\epsilon=1}} \begin{aligned} &= 1.23\,, \quad \epsilon = 0.79\,,\\ &= 10.5\,, \quad \epsilon = 0\,. \end{aligned} \tag{37}$$

Several interesting conclusions may be drawn from equation (37): The temperature rise in the collisionless case, $\epsilon = 0$, is 10 times as high as in the LTE case. But, at the foot of the chromosphere the actual temperature increase hardly differs from that in LTE so that the LTE description of the medium is a very good approximation. Higher in the chromosphere this is no longer true. At the level where the kinetic temperature is 5000 K, the assumption of LTE underestimates the temperature rise for a given dissipation rate by a factor of 3, and the collisionless case overestimates it by the same factor. However, the density at the 5000 K level is lower by an order of magnitude than at the temperature minimum. Therefore, the assumption that H^- is in LTE is a much better approximation to actual solar conditions than is the assumption that it is in statistical equilibrium with negligible collisional transitions. Of course, the correct description of H^- radiation throughout this region, in which H^- is the dominant opacity source, must treat H^- in statistical equilibrium, with collisions, provided the model on which the calculation is based allowed for deviations from LTE (see § IV).

TABLE 2

TEMPERATURE AND MASS DENSITY* AND RECOMBINATION RATES FOR H^- AS FUNCTIONS OF OPTICAL DEPTH AT 0.5 MICRONS

$\tau_{0.5}$	T(K)	ρ	R	$R/(R + n_H\Gamma)$
6.3×10^{-6}...	5590	8.11 (−11)	2.13 (6)	0.96
1.0×10^{-5}...	5300	2.00 (−10)	1.78 (6)	0.90
1.6×10^{-5}...	5040	4.16 (−10)	1.49 (6)	0.79
2.5×10^{-5}...	4790	8.28 (−10)	1.23 (6)	0.61
4.0×10^{-5}...	4530	1.48 (−9)	1.00 (6)	0.42
6.3×10^{-5}...	4280	2.31 (−9)	0.80 (6)	0.27
1.0×10^{-4}...	4170	3.24 (−9)	0.72 (6)	0.19

* From Gingerich *et al.* 1971.

The temperature rise in the limiting cases of $\epsilon = 1$ and $\epsilon = 0$ was studied by Praderie and Thomas (1972, 1976), whose results differ from ours in some details because they assumed that the H^- photoabsorption cross section was frequency independent, and, in a more basic respect, because they assumed that the kinetic temperature T_0 in the radiative equilibrium atmosphere was the same in the two cases of $\epsilon = 1$ and $\epsilon = 0$. However, since the cooling rate, described by the source function S_0, must balance the radiative heating rate in the radiative equilibrium atmosphere, the source function S_0, and not the temperature T_0, must be the same in the two cases. Because of the assumption of gray H^- opacity, the factor Praderie and Thomas found for the relation (36) between the temperature rise and the dissipation rate was 0.5 instead of our value of 0.22 for $\epsilon = 1$, and 2 instead of 2.33 for $\epsilon = 0$; and instead of the value 10.5 for the ratio (37) of temperature increases they obtained the value $4b_0$. The difference between the equations of Praderie and Thomas and those of this paper is most evident when the same data on kinetic temperature and departure coefficient at the foot of the chromosphere are used to infer the value of the mechanical dissipation rate, using in both instances the statistical equilibrium solutions: Praderie and Thomas would conclude that the chromosphere can be heated with only 14% of the dissipation rate required by our analysis. This difference is due partly to their neglect of the inward traveling radiative flux and partly to the temperature dependence of the source function, which grows linearly with kinetic temperature in their case and with the 3.64th power in ours.

We note that Athay (1976) has also discussed the relation between temperature rise and mechanical dissipation in the limiting cases of $\epsilon = 1$ and $\epsilon = 0$. His treatment of the radiative transfer takes account of radiation emitted in both the inward and outward directions. But he did not distinguish between the

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radiative equilibrium temperature in LTE and that in statistical equilibrium, which affects the difference in the required mechanical energy fluxes but not the estimate of the dissipation rate implied by observations. And Cram (1978) considered the temperature rise in a gray atmosphere for a mechanical dissipation rate that decreased exponentially with optical depth. He solved the transfer equation analytically for the source function using the method of discrete ordinates and determined the kinetic temperature for a collision parameter ϵ that had an exponential depth dependence.

In their discussion of the relation between temperature rise and mechanical energy dissipation Praderie and Thomas assumed that the H^- photodissociation cross section α_ν was frequency-independent. This assumption deserves further comment since it affects the temperature dependence of the source function S and the value of the H^- departure coefficient b.

When the cross section α_ν is constant, the cooling integral B and the photorecombination frequency R are proportional to T^4 and T^3, respectively; apart from corrections due to the finite threshold energy of the dissociation reaction, which Praderie and Thomas neglected. The source function S in statistical equilibrium is then proportional to temperature. However, near the temperature minimum the temperature dependence of B and R is different from that obtained for constant cross section (see Table 1); the T^4 law for B is found near $T = 5500$ K and the T^3 law for R near $T = 7500$ K. Thus the dependence on T assumed by Praderie and Thomas is reached at different temperatures for B and R, and the implied linear dependence of S is not found at any temperature of interest. This affects the numerical factor in the relation between temperature rise and dissipation rate, which at the foot of the chromosphere is in error by a factor of nearly 4. By comparison, the error made by neglecting deviations from LTE in H^- is much smaller.

The frequency dependence of the photo cross section α_ν also affects the H^- departure coefficient b. We can define an effective frequency dependence of the cross section by noting that if $\alpha_\nu \approx \nu^k$, then $B \approx T^{4+k}$ and $R \approx T^{3+k}$, if the correction for finite threshold energy is neglected. At a kinetic temperature of $T = 4200$ K the cooling integral B implies that the effective cross section behaves like $\sqrt{\nu}$, and R implies that $\alpha_\nu \approx \nu$. Thus the effective cross section rises with frequency. Now, in an investigation of deviations from LTE in a model atmosphere of one-level hydrogen atoms with radiative equilibrium in the Lyman continuum, hydrostatic equilibrium, and statistical equilibrium, Kalkofen, Klein, and Stein (1978) found that the departure coefficient was less than unity if the photoionization cross section rose sufficiently rapidly with frequency. The increase with frequency of the effective H^- cross section is not fast enough, as is shown by pure H^- models which give $b > 1$ (cf. Feautrier 1968). However, the heavy line blanketing in the solar atmosphere, which increases rapidly with frequency, is equivalent to a cross section increasing rapidly with frequency. Therefore, an underpopulated bound state may be expected, and is actually found (Vernazza, Avrett, and Loeser 1976), which is consistent with the low kinetic temperature of the empirical models at the foot of the chromosphere (Kalkofen 1977).

In the case of the gray Lyman continuum, Kalkofen *et al.* found the hydrogen departure coefficient to vary between $b = 2.2$, when the radiation field was not affected by the departures, and $b = 43$, when the intensity was affected to great depth. Clearly, there was no upper limit on b; $b = 2$ was a lower limit. Hence, we would expect the same to be true for a model solar atmosphere in radiative equilibrium, where $b \approx 2$ for $\epsilon = 0$ would be found under the assumptions that deviations from LTE occur only near the surface, where they are unable to change the intensity of the radiation field, and that the opacity—and not merely the H^- opacity—depends only weakly on frequency (a uniform picket-fence opacity would be similar to a gray opacity in its effect on deviations from LTE). The first assumption is satisfied in the solar atmosphere, but the second one is not. Lines are more prominent in the blue than in the red; their opacity lowers the intensity in the blue, which depresses the color temperature associated with the radiation field and reduces the departure coefficient. It is therefore not surprising that the value of the departure coefficient in the layers below the chromospheric temperature rise should be less than unity. We note that Vernazza, Avrett, and Loeser (1976) found $b = 0.9$ at the temperature minimum, a value that is incompatible with the limiting value of $b = 2$ given by Praderie and Thomas for negligible collisions. The value of $b = 0.9$ and the associated kinetic temperature also describe the source function in the higher layers where the H^- opacity dominates (cf. the Appendix following eq. [A9]).

IV. EMPIRICAL MODELS

The dissipation rate of mechanical energy flux is equal to the excess radiative emission of the solar atmosphere over the emission that would take place if the Sun were in radiative equilibrium. Thus the calculation of the radiative flux gradient depends on a model of the actual solar chromosphere and on a model of the solar atmosphere in radiative equilibrium. Since both types of models are uncertain, the resulting estimate of the mechanical dissipation rate is uncertain. We consider first the errors in the models and then ways to minimize the uncertainties in the computed radiative emission rates.

Semiempirical models of the solar atmosphere are constructed by choosing the temperature structure that matches observed and predicted intensities and the pressure structure that satisfies hydrostatic equilibrium (Gingerich *et al.* 1971; and Vernazza, Avrett, and Loeser 1976). The temperature structure at the temperature minimum and the foot of the chromosphere is determined from observations in the far-infrared at wavelengths between 0.1 and 0.2 mm and in the ultraviolet between 0.1 and 0.2 μm. In the

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infrared, the theory of radiative transfer is relatively straightforward since the emission takes place in LTE, except that the electron donors are not in ionization equilibrium at the local kinetic temperature (Vernazza, private communication), but the emitted intensity depends only weakly on temperature since the Planck function is in the Rayleigh-Jeans region; therefore, the temperature structure of the atmosphere is uncertain because of the low temperature sensitivity of the Planck function. In the ultraviolet, on the other hand, the Planck function is much more sensitive to temperature—by a factor of 20 to 30—but the emitted radiation depends on deviations from LTE of the emitting ions; therefore, the temperature structure is uncertain because of the difficulties inherent in the radiative transfer in statistical equilibrium. Consequently, the resulting temperature structure is uncertain, in the temperature minimum region by ± 100 K (Vernazza, Avrett, and Loeser 1976).

The theoretical model of Kurucz (1974) assumes radiative equilibrium but does not allow for deviations from LTE and may be incomplete in its opacity sources despite the large number of transitions included. Its temperature throughout the layer in which the actual solar temperature has a minimum is higher than that of the empirical models. Therefore it may not be used directly for the required calculation. That leaves the choice of either modifying the theoretical model or adapting an empirical model so as to exclude the temperature increase due to mechanical heating, e.g., by extending the kinetic temperature and the departure coefficient from the temperature minimum outward at constant value (see the discussion in the Appendix following eq. [A9]). The underlying assumption justifying this procedure is that the temperature-minimum region is a layer that is *not* heated by waves, except for a negligible amount of radiation-damping of the acoustic waves, and therefore represents the atmosphere without mechanical heating, i.e., an atmosphere in radiative equilibrium. Consequently the mechanical energy input is practically zero at the temperature minimum and rises outward according to the difference in kinetic temperature and departure coefficient between the depth in question and the temperature minimum.

In the calculation of the radiative losses implied by the empirical model it is imperative that the assumptions concerning the state of the gas be the same in building the model and in computing the radiative cooling. Thus, the emission from the HSRA (Gingerich *et al.* 1971) must be computed in LTE, that from the VAL model (Vernazza, Avrett, and Loeser 1976) in statistical equilibrium with the departure coefficients they determined. This procedure is necessary for the sake of consistency between the observed radiation, which led to the model, and the computed radiation. For the two empirical models under discussion it will lead to similar radiative cooling rates, as follows from the temperature profiles in the low chromosphere (see Vernazza, Avrett, and Loeser 1976, Fig. 20), which is steeper for the VAL model, and their increasing departure coefficient for H^- (quoted by Ulmschneider and Kalkofen 1978, Table 1), so that the source functions in the two models are similar.

In order to minimize the errors inherent in the models in the calculation of the chromospheric radiative cooling rate, Praderie and Thomas (1976) urged that the model be "obtained from the data in the same spectral features as those which determine the effective radiation losses." To this should be added the requirement that the emergent radiation be calculated under the same assumption regarding the state of the gas that was made in constructing the model. Praderie and Thomas (1976) failed to recognize this important point when they criticized Ulmschneider (1974) for assuming the H^- ion to be in LTE in his calculation of the cooling rate based on the HSRA. If he had followed their advice, he would have obtained a much lower estimate for the wave dissipation rate: At 5360 K, for example, the departure coefficient of the VAL model is $b = 1.53$. In the HSRA, b would have nearly the same value since the photospheric radiation field and the electron density at that temperature are nearly the same in the two models. Therefore, as follows from equation (A9), he would have estimated the net radiative emission rate reduced by a factor of more than 2, which clearly shows the importance of consistency between model construction and radiative transfer calculation when that radiation carries most of the flux.

V. SUMMARY AND CONCLUSIONS

We have derived an expression for the total radiative flux emitted by a heated slab overlying an atmosphere. Using the energy equation for a static atmosphere with radiative and mechanical energy transport we have shown that the mechanical energy dissipated is equal to the radiative energy emitted in the inward and outward directions, in agreement with Ulmschneider (1970). For radiation due to H^- transitions we have then obtained estimates of the temperature rise at the foot of the chromosphere due to wave dissipation for H^- in statistical equilibrium, and for the limiting cases of H^- in LTE and in statistical equilibrium without collisions. We found that the actual solar conditions at the temperature minimum are fairly well represented by LTE. Finally, we considered the use of empirical models for estimating the energy input into the chromosphere and stressed consistency in the assumed state of the gas for the calculation of the radiative cooling and for the construction of the model. In the Appendix we give an equation for estimating the mechanical energy input into the layers in which H^- is the dominant opacity source.

We considered in some detail two papers by Praderie and Thomas (1972, 1976), addressed to the same questions, and showed that the mechanical energy input estimated with their equations is too low by about one order of magnitude, leading them to conclude that the mechanical heat input to the low chromosphere is an insignificant fraction of the total mechanical input to the chromosphere and corona.

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We have presented arguments that lend support to the more traditional view that a large fraction of the mechanical energy traveling outward from the convection zone is dissipated in the low chromosphere.

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APPENDIX I

We derive an equation for estimating the mechanical energy input into the low chromosphere from the emitted radiation in bound-free and free-free transitions of H$^-$, and we define the rates for radiative cooling and photorecombination.

In an atmosphere with plane-parallel stratification, the transfer equation for H$^-$ opacity may be written

$$\mu \frac{d}{dx} I_{\nu\mu} = -n_{\mathrm{H}^-}\alpha_{\mathrm{bf}} I_{\nu\mu} + n_{\mathrm{H}^-}{}^*\alpha_{\mathrm{bf}} e^{-h\nu/kT}\left(\frac{2h\nu^3}{c^2} + I_{\nu\mu}\right) - n_{\mathrm{H}^-}{}^*\alpha_{\mathrm{ff}}\left[I_{\nu\mu} - e^{-h\nu/kT}\left(\frac{2h\nu^3}{c^2} + I_{\nu\mu}\right)\right], \tag{A1}$$

where x is the height in the chromosphere, $\alpha_{\mathrm{bf}}(\nu)$ is the bound-free photoabsorption cross section, $\alpha_{\mathrm{ff}}(\nu)$ is the free-free cross section, $n_{\mathrm{H}-}$ is the actual local density of H$^-$ ions, and $n_{\mathrm{H}^-}{}^*$ is the equilibrium density defined by the Saha equation in terms of the densities of electrons and hydrogen atoms in the ground state, n_e and n_1,

$$n_{\mathrm{H}^-}{}^* = 2.89 \times 10^{-22} n_1 n_e \theta^{3/2} \theta^{1.736\theta}, \quad \theta = 5040/T \tag{A2}$$

(cf. Vernazza, Avrett, and Loeser 1973).

The net radiative flux is defined as

$$\mathscr{F} = 2\pi \int_0^\infty d\nu \int_{-1}^{1} d\mu \mu I_{\nu\mu}, \tag{A3}$$

and its gradient is obtained from the transfer equation,

$$\frac{d}{dx}\mathscr{F} = -4\pi n_{\mathrm{H}^-} \int_0^\infty d\nu \alpha_{\mathrm{bf}} J_\nu + 4\pi n_{\mathrm{H}^-}{}^* \int_0^\infty d\nu \alpha_{\mathrm{bf}}(W_\nu + e^{h\nu/kT} J_\nu) - 4\pi n_{\mathrm{H}^-}{}^* \int_0^\infty d\nu \alpha_{\mathrm{ff}}[J_\nu - (W_\nu + e^{-h\nu/kT} J_\nu)], \tag{A4}$$

where J_ν is the mean monochromatic intensity,

$$J_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu I_{\nu\mu}, \tag{A5}$$

and W_ν the Wien function,

$$W_\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}. \tag{A6}$$

In radiative equilibrium the net flux $\mathscr{F}$ is independent of height. With the departure coefficient of H$^-$ defined by

$$b = n_{\mathrm{H}^-}/n_{\mathrm{H}^-}{}^* \tag{A7}$$

and the gas variables in radiative equilibrium denoted by the superscript 0, the radiative equilibrium condition may be written as

$$0 = -\int_0^\infty d\nu \alpha_{\mathrm{bf}} J_\nu + \frac{1}{b^0}\int_0^\infty d\nu \alpha_{\mathrm{bf}}(W_\nu{}^0 + e^{-h\nu/kT^0} J_\nu) - \frac{1}{b^0}\int_0^\infty d\nu \alpha_{\mathrm{ff}}{}^0[J_\nu - (W_\nu{}^0 + e^{-h\nu/kT^0} J_\nu)]. \tag{A8}$$

Since the mechanical flux dissipation is assumed to be confined to the surface layers of the atmosphere the increase in the intensity is insignificant. The intensity J_ν may therefore be assumed to be unaffected by the wave dissipation.

We combine the radiative equilibrium condition (A8) with the integrated transfer equation (A4) in such a way that the absorption rate due to bound-free transitions is canceled,

$$\begin{aligned}\frac{d}{dx}\mathscr{F} = 4\pi n_{\mathrm{H}^-}\Bigg[\int_0^\infty d\nu \alpha_{\mathrm{bf}}\left(\frac{W_\nu}{b} - \frac{W_\nu{}^0}{b^0}\right)\left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right) + \int_0^\infty d\nu\left(\frac{\alpha_{\mathrm{ff}}{}^0}{b^0} - \frac{\alpha_{\mathrm{ff}}}{b}\right)J_\nu \\ + \int_0^\infty d\nu\left(\alpha_{\mathrm{ff}}\frac{W_\nu}{b} - \alpha_{\mathrm{ff}}{}^0\frac{W_\nu{}^0}{b^0}\right)\left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right)\Bigg].\end{aligned} \tag{A9}$$

The intensity of the radiation field now appears only in the stimulated emission term for bound-free transitions and in the much less important terms for free-free transitions.

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The conditions in the outermost layers of the radiative equilibrium atmosphere are well described by the values of the kinetic temperature and of the departure coefficient at the temperature minimum even though they both may rise in the outward direction. If we neglect free-free transitions, it is clear from the assumed constancy of the intensity, and therefore of the absorption rate per ion, that the emission rate per ion must likewise be depth independent. Thus the kinetic temperature and the departure coefficient may change together only so as to balance the bound-free heating rate. We assume that these conclusions are essentially unchanged by the presence of the free-free transitions. Hence the cooling rate per H^- ion in the upper atmosphere in radiative equilibrium is equal to the corresponding rate at the temperature minimum. The superscript 0 therefore refers to conditions at the temperature minimum. The gradient of the radiative flux in the low chromosphere where H^- is the dominant opacity source is given by equation (A9) as the net cooling rate relative to the rate in the radiative equilibrium atmosphere; it measures the local dissipation rate of hydrodynamical waves.

The radiative cooling rate due to bound-free transitions is obtained from the transfer equation (A1) as

$$n_{\mathrm{H}^-}{}^*E = 4\pi n_{\mathrm{H}^-}{}^* \int_0^\infty d\nu\,\alpha_{\mathrm{bf}}(\nu)\, W_\nu\left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right), \tag{A10}$$

and the corresponding photorecombination rate as

$$n_{\mathrm{H}^-}{}^*R = 4\pi n_{\mathrm{H}^-}{}^* \int_0^\infty d\nu\,\alpha_{\mathrm{bf}}(\nu)\, \frac{W_\nu}{h\nu}\left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right). \tag{A11}$$

At a kinetic temperature of 4000 K the radiative cooling rate per ion is

$$E = 1.39 \times 10^{-6}\ \mathrm{ergs\ s^{-1}}\,, \tag{A12}$$

and the photorecombination frequency is

$$R = 6.02 \times 10^{5}\ \mathrm{s^{-1}}\,. \tag{A13}$$

From these numbers and the values of the exponents m and n of the corresponding power laws for the temperature dependence (eqs. [25] and [32]), listed in Table 1, the rates at arbitrary temperatures in the given range can easily be calculated to good accuracy. For stars with effective temperatures not too different from the Sun's this procedure may be used as well, since the contribution to the rates made by induced emissions is not very large; at 4000 K it amounts to about 4%. If instead of the dilute Planck function (24) representing light coming from the photosphere, the frequency R is computed with the intensity value equal to that of the local Planck function, giving R corresponding to thermal equilibrium at 4000 K, its value is changed from (A13) by less than 1%.

We note that the cooling rate is simply related to the source function (23) if only bound-free H^- opacity is treated. When the opacity coefficient κ is defined as the monochromatic opacity at frequency ν_0,

$$\kappa = n_{\mathrm{H}^-}\alpha_{\mathrm{bf}}(\nu_0)\,, \tag{A14}$$

the relation between the cooling rate per H^- ion and the source function S is

$$E = 4\pi\alpha_{\mathrm{bf}}(\nu_0)B\,, \tag{A15}$$

with $S = B/b$.

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WOLFGANG KALKOFEN: Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

PETER ULMSCHNEIDER: Institut für Astronomie und Astrophysik, Am Hubland, D-8700, Würzburg, Federal Republic of Germany

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